

Affinely Adjustable Robust ADMM for Residential DER Coordination in Distribution Networks

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Abstract—It is becoming increasingly important for network operators to coordinate the use of prosumer-owned DER so that their full value can be harnessed without violating the network’s technical limits. Unfortunately, this coordination is complicated by the highly volatile uncertainties of PV and prosumer load, as well as the distributed nature of the problem. To address this challenge, we present an affinely adjustable robust extension of the distributed ADMM algorithm that is resilient to forecast deviations. As with ADMM, every prosumer negotiates with the grid over a receding horizon to obtain locational marginal prices and their coordinated “here-and-now” decisions prior to the uncertainty realizations. However, our new AARO-ADMM approach robustly coordinates the first time step of each negotiation to enable prosumers to take local “wait-and-see” recourse decisions that compensate deviations from the forecast in real-time. Our experiments on a modified 69-bus network show a significant reduction in the frequency of negotiation and communication needed by ADMM to maintain grid security, with just a small cost increase over an idealized but unachievable baseline.

Index terms— Affinely Adjustable Robust, Distributed Optimization, ADMM, DER.

The main symbols used throughout this paper are:

| | |
|------------------------------------|--|
| $P_t^{(k)}$ | Power produced/consumed by prosumer i at t and iteration k . |
| $\lambda_t^{(k)}$ | LMPs of prosumer i at t and iteration k . |
| $P_t^{\prime(k)}$ | Network expectation from prosumer i at t and iteration k . |
| $\bar{P}_t^L / \bar{P}_t^{PV}$ | Forecasted load/PV of prosumer i at t . |
| P_t^{Ch} | Battery charge power of prosumer i at t . |
| P_t^{Dis} | Battery discharge power of prosumer i at t . |
| P_t^{Cur} | PV power curtailment of prosumer i at t . |
| $R_{Ch}^{max} / R_{Dis}^{max}$ | Battery maximum charge/discharge rate. |
| E_t | Battery state of charge of prosumer i at t . |
| η | Battery efficiency. |
| E^{min} / E^{max} | Min/max allowable battery energy. |
| P_t^{PV} / P_t^L | Uncertain PV power/load of prosumer i at t . |
| $\hat{P}_t^{L+}, \hat{P}_t^{L-}$ | Maximum allowable load deviation in positive/negative direction. |
| μ | Budget of uncertainty. |
| P_t^{L+}, P_t^{L-} | Load deviation in positive/negative direction. |
| P_t^{PV+}, P_t^{PV-} | PV power deviation in positive/negative direction. |
| $\hat{P}_t^{PV+}, \hat{P}_t^{PV-}$ | Maximum allowable PV power deviation in positive/negative direction. |

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I. INTRODUCTION

A. Research Motivation

THE advent of distributed energy resources (DER), is transforming power systems, dominated by central generation and passive consumers to a more decentralized one in which consumers are producing a notable amount of the overall energy. For example, in Australia, as of February 2018, 1.8 million small-scale solar PV systems are installed with one in every three consumers owning PV in South Australia [1]. This new demand-side generation can help the distribution networks increase capacity, improve performance, and defer augmentation. However, leaving the uncoordinated DER in the hands of prosumers, who do not have any grid visibility, creates significant issues for networks, such as voltage, rebound peak, and ramping problems. Therefore, approaches capable of coordinating DER taking into account network technical limits are crucially needed. Unfortunately, such coordination is a notoriously challenging large-scale optimization problem, plagued by non-linear constraints (AC power flows), the unavailability of consumers private information, and highly volatile uncertainties associated with residential demand and PV power.

Centralized approaches, relying on conventional optimal power flow (OPF) models [2], [3], would fail to appropriately coordinate DERs in a real setting as a) they struggle to scale with the number of DER, b) residential DER and the distribution network are operated by different parties who might have conflicting interests, and c) they require a central access to the relevant prosumer constraints and information which compromises prosumers privacy. More recently, the alternating direction method of multipliers (ADMM) has emerged as a promising approach for solving OPF and coordinating residential DER in a distributed manner [4], [5], [6], [7]. By breaking the large coordination problem into smaller-scale sub-problems solved in parallel, it is able to mitigate the complexity, conflicting interests, and privacy issues of a central tool. Through such a distributed algorithm, prosumers coordinate to optimally schedule their DER by individually negotiating with the network the power exchanged at their connection point for multiple time steps over an appropriate time horizon.

When enacted, the solution negotiated by ADMM (similar to the solution of a central tool) will keep its promise of optimality and feasibility as long as all prosumers are able to honour their commitment to the negotiated power. However, this assumption rarely holds in reality, due to the fact that prosumer parameters such as residential demand and PV power

are highly uncertain and can significantly deviate from the forecast used in the negotiation. To tackle this important limitation, we propose AARO-ADMM, an affinely adjustable robust ADMM approach in which prosumers optimize not only their connection point power (CPP) but also a set of affine control functions. These functions enable prosumers to honour the negotiated conditions by making adjustive “wait-and-see” recourse decisions capable of compensating for any deviation about the forecast within a given polyhedral uncertainty set. Unlike the decisions made in the negotiation phase, these recourse decisions can be made by the prosumer without the need for communication, and so can be enacted in real-time, compensating for short time-scale fluctuations.

Moreover, to improve the effectiveness of this robust approach and tighten the uncertainty set, we apply AARO-ADMM in a receding horizon framework enabling prosumers to periodically negotiate using the latest forecast.

In this receding horizon setting, AARO-ADMM is used to robustly coordinate the first time step of each negotiation and to obtain the maximum limits of the adjustable terms for the affine controller, whereas the rest of the time steps are coordinated deterministically. The decisions in the first time step are then enacted, and as the uncertain parameters are revealed, the adjustable terms of the AARO controller take recourse decisions to compensate from forecast deviation and hold the CPP at the negotiated value. The same process is then repeatedly shifted forward in time to robustly coordinate the next time step. We show that this robust approach avoids network constraint violations at similar computational cost to the standard ADMM, while substantially reducing the frequency of negotiations required by the latter (e.g., in our experiments from every 5min to every hour). This makes AARO-ADMM applicable in many situations where frequent negotiations of the standard ADMM would be too computationally expensive or would lead to excessive communication.

B. Related Work

ADMM and other distributed optimization techniques have been widely used to solve OPF problems [8], [9], [10], [11], [12]. These distributed approaches are particularly well-suited to the task of DER coordination, given the intrinsically distributed nature of the data and the large numbers of participants with potentially competing interests. Another key characteristic of DER coordination is the large amount of uncertainty around load and distributed generation production, but which often gets overlooked by these distributed approaches. When not properly accounted for, such uncertainty can lead to decisions that are suboptimal or even violate the network’s constraints.

Common techniques for handling uncertainty, that have been used in both centralized and occasionally distributed settings, include stochastic programming (SP) [13], [14], robust optimization (RO) [15], [16], [17], [18], and online receding horizon optimization [5], [7], [19].

Two-stage stochastic programming was used in [13] and [14] to model uncertainties in OPF and demand response problems respectively. However, adapting such a two-stage

approach to a residential DER coordination setting is unlikely to scale due to the large number of scenarios required to capture the many active participants and sources of uncertainty, and thus avoid network constraint violations. Our approach uses robust optimization techniques to reduce the scenarios that need to be considered, and hence the computational burden.

Robust optimization [15]–[18] avoids having to consider large numbers of scenarios by finding and optimizing according to a worst-case scenario. Because it focuses on just one scenario, that could in practice be very unlikely to occur, RO often leads to solutions that are overly conservative. To address this problem, our approach combines both receding horizon optimization and robust optimization. Only the first time step in the horizon is robustly optimized, with the remainder optimized considering the expected value of the uncertain parameters. This produces solutions with a good balance between considering the worst possible outcome and what is likely to actually occur.

In addition to potentially overly conservative decisions, almost all existing RO approaches to solving OPF and coordinating DER are centralized, e.g., [15], [17], [18], and due to the nature of the techniques employed, i.e. using mixed-integer formulations with column and constraint generation, it would be difficult to efficiently distribute the computation among prosumers. The exceptions that have used ADMM to distribute RO problems include [16] where the decomposition is between a coupled electricity and gas network, and [20] and [21] where the decompositions is between 3 interconnected microgrids. None of these distributed RO approaches have considered the problem down at the residential DER level, and either rely on linear power flow models that perform poorly on distribution networks, or the same solving techniques as the centralized approaches which would be challenging to further decompose to the prosumer level. In contrast our approach decomposes the problem at the natural boundary between prosumers and the network while enabling the use of non-linear power flow models.

To add even more flexibility, our approach to DER coordination allows recourse decisions by way of affine functions that respond to realizations in the uncertain parameters. These functions are optimized to operate over the entire uncertainty set, represented by a convex polytope, rather than just for a single worst-case point as is the case for regular non-AARO approaches. This means that the recourse decisions can automatically respond for any realization within the uncertainty set, and can even be applied in real-time operation, something that the regular RO approaches do not provide.

Further to this, the uncertainty set we consider accounts for the impact of both too much and too little solar on the network, which when combined with load uncertainty can lead to voltages going too high or too low. The works [15]–[18] and [20]–[21] only consider recourse actions where overall demand is more than expected, whereas our flexible recourse actions cover the entire uncertainty set, and so are additionally optimized to take advantage of excess solar generation (e.g., through storing it, rather than always curtailing it).

Distributed online receding-horizon approaches [5], [7],

[22], which rely on up-to-date forecasts of the uncertain parameters, can avoid network violations so long as very frequent optimizations can be made. However, this is not always possible as computational and communication resources are limited in a residential DER setting. In particular communication delays can dominate, and may make ADMM impractical when frequent optimizations are necessary [23]. Coping with the communication requirements of ADMM is an essential topic of research, as we observed when deploying ADMM in real-world trials [22].

Asynchronous variants of ADMM have been proposed to help improve convergence time but have been found to only help under mild communication delays [24]. Our approach addresses this problem by instead reducing the need to frequently reoptimize. By utilising AARO in the first time step of each horizon, we deliver recourse actions that can be enacted in real-time without communication, and that are robust to uncertainty for the entire first time step, preventing network violations. This enables less frequent renegotiation and communication, enabling our distributed optimization problem to be solved in a practical setting.

C. Contributions

The major contributions of this work are:

- 1 an affinely adjustable robust extension of ADMM capable of handling uncertainty associated with PV power and residential demand when coordinating DER;
- 2 experimental results showing that applying AARO-ADMM to the first step of a receding horizon optimization leads to a practical number of negotiations, without compromising the feasibility of the solution nor significantly increasing its cost.

The rest of this paper is organized as follows. Section II provides an overview of our proposed algorithm. The two key sub-components of our algorithm are then detailed in Sections III and IV: an ADMM-based multi-period OPF optimization for a single horizon; and our proposed AARO approach for solving prosumer subproblems. Section V examines the performance of the proposed approach on a modified 69-bus distribution network. Finally, Section VI provides the main observations and conclusions of this study.

II. THE OVERALL ALGORITHM

The outermost layer of our approach is a receding horizon algorithm, that schedules the actions of all prosumers over a forward horizon of 24 hours, represented by the set T . A new horizon is optimized every hour, so as to take into account the latest operating state and forward forecasts for load and solar.

Within each horizon, a multi-period OPF problem is solved in a distributed manner using ADMM, where each prosumer subproblem robustly optimizes their connection-point power for the first time step using AARO. In this AARO approach, the prosumers optimize both their connection-point power and a set of individualized affine control functions, while taking into consideration their local uncertainty. These affine functions can then be enacted in real-time to guarantee the negotiated connection-point power over the next hour, for the chosen uncertainty set.

III. THE DISTRIBUTED ADMM ALGORITHM

We first present a high-level model of the multi-period OPF problem solved within each horizon, which is used to explain our ADMM decomposition. We then present a deterministic model of the prosumer subproblems, which we modify to handle uncertainty using AARO in Section IV.

The multi-period OPF problem is split between prosumers and the remainder of the network. Each prosumer $i \in C$ has a vector $P_i \in \mathbb{R}^{|T|}$ that represents the active power¹ they exchange with the network for each time step in horizon T . Each prosumer has objective f_i and constraint g_i functions which take these powers as inputs. We drop the subscript to represent all prosumer powers, $P \in \mathbb{R}^{|C| \times |T|}$, which is an input to the network's own objective f' and constraint g' functions (which represent AC power flows). Finally, X_i and Y represent any internal variables for the prosumers (e.g., battery state of charge (SoC)) and network (e.g., voltages), respectively. The combined multi-period OPF is:

$$\min_{P_i, X_i, P', Y} \sum_{i \in C} f_i(P_i, X_i) + f'(P', Y) \quad (1a)$$

s.t.

$$\forall i \in C : g_i(P_i, X_i) \leq 0 \quad (1b)$$

$$g'(P', Y) \leq 0 \quad (1c)$$

$$P - P' = 0 \quad (1d)$$

In the above, we have duplicated the prosumer power variables so that the network and prosumers have their own copies, P' and P , which get set equal through equation (1d). By relaxing this constraint, we are able to decompose the problem and then solve it iteratively using the ADMM algorithm.

A. ADMM Algorithm

The first step to applying the ADMM algorithm is to take an augmented Lagrangian relaxation of (1d), which results in the following penalty term component:

$$\mathcal{L}^*(P, P', \lambda) = \lambda^\top (P - P') + \frac{\rho}{2} \|P - P'\|_2^2 \quad (2)$$

where $\lambda \in \mathbb{R}^{|C| \times |T|}$ is a vector of dual variables for the constraint and ρ is the penalty parameter of the augmented Lagrangian.

The ADMM algorithm [4] can then be used to iteratively solve (1a)–(1d) to its optimum. Fig. 1 shows how the problem is split up into subproblems that exchange data, and which ADMM solves in three parts per iteration k :

$$P^{(k)} := \min_{P_i, X_i} \sum_{i \in C} [f_i(P_i, X_i) + \mathcal{L}_i^*(P_i, P_i^{(k-1)}, \lambda_i^{(k-1)})] \\ \forall i \in C : g_i(P_i, X_i) \leq 0 \quad (3a)$$

$$P'^{(k)} := \min_{P', Y} [f'(P', Y) + \mathcal{L}^*(P^{(k)}, P', \lambda^{(k-1)})] \\ g'(P', Y) \leq 0 \quad (3b)$$

¹Here, we focus just on the real power exchanged with the network; however, reactive power could be similarly modeled.

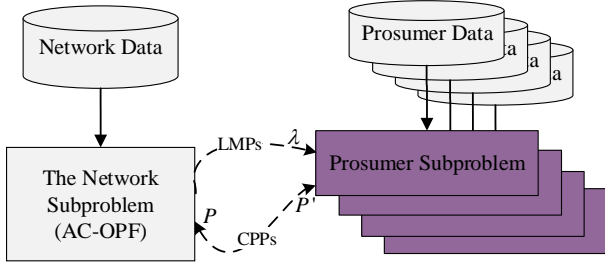


Fig. 1: The proposed decomposition configuration.

$$\lambda^{(k)} := \lambda^{(k-1)} + \rho^{(k)} \cdot (P^{(k)} - P'^{(k)}) \quad (3c)$$

In the first phase (3a), the prosumers are optimized for P , while holding P' and λ constant at their $k-1$ -th value. In the second phase (3b), the network is optimized for P' , while holding P and λ constant at their k and $k-1$ -th values respectively. Finally, the dual variables λ are updated in (3c), completing the k -th iteration. Note that λ is the vector of the Lagrangian multipliers for the energy conservation constraint at each node. So, when the algorithm converges to its optimum, λ represents the locational marginal prices (LMPs), i.e., it reflects the cost of serving energy to that node in the network. Through (3a)–(3c), within each iteration, the prosumers selfishly optimize their own objective function independently of the network and all other prosumers. Moreover, as shown in (3a) and (3b), such representation enables each subproblem (stakeholder) to have a different objective function reflecting their interests to the problem.

B. Stopping Criteria and Convergence of the Algorithm

In line with [4], we define the stopping criteria using primal $R_p^{(k)}$ and dual residuals $R_d^{(k)}$ as follows:

$$R_p^{(k)} := ((P_1^{(k)} - P_1'^{(k)}), (P_2^{(k)} - P_2'^{(k)}), \dots)^\top \quad (4a)$$

$$R_d^{(k)} := (\rho(P_1'^{(k)} - P_1^{(k-1)}), \rho(P_2'^{(k)} - P_2^{(k-1)}), \dots)^\top \quad (4b)$$

The primal residuals (4a) represent the constraint violation at the current solution and the dual residuals (4b) the violation of the KKT stationarity constraint [4]. In this paper, we consider the problem to have converged when the scaled 2-norms of both the primal and dual residuals are smaller than 10^{-6} . Moreover, to improve the convergence we adaptively improve ρ based on the primal and dual residual balance, presented in [25], as follows:

$$\rho^{(k+1)} = \begin{cases} \rho^{(k)} \cdot (1 + \sigma_k), & \text{if } \|R_p^{(k)}\|_2 > \nu \|R_d^{(k)}\|_2. \\ \rho^{(k)} / (1 + \sigma_k), & \text{if } \|R_d^{(k)}\|_2 > \nu \|R_p^{(k)}\|_2 \\ \rho^{(k)}, & \text{Otherwise.} \end{cases} \quad (5)$$

We adaptively change ρ for the first 50 iterations, but then continue with the latest penalty i.e., $\rho^{(k=50)}$, in order to guarantee convergence, as discussed in [25].

C. Network Subproblem

In this paper, we model a feeder exposed to time-varying wholesale prices for real power energy import. The network constraints including AC power flow equations and any capacity or voltage constraints represented by (3b) in our model. We use a conic relaxed branch flow model for power flows which is proved to be exact under some mild conditions [26]. In the final solution, when our AARO-ADMM converges, the primal residuals in the ADMM algorithm approach zero, which translates to a feasible solution for the OPF problem, therefore satisfying all the network constraints (3b). It is worth mentioning that our OPF subproblem enables the operators to thoroughly examine their networks using the available tools, proposed for the well-studied OPF problem.

We refer the readers to [26] for the detail network model and focus on the prosumer subproblem, which we model and solve using AARO in Section IV.

D. Deterministic Prosumer Subproblem

In the detailed prosumer formulation we drop the i subscript, as we only work with one prosumer at a time, and index the variable vectors and other quantities by time $t \in T$, where T is the set of time points for the horizon. Here, we provide an explicit representation of (3a) for a single prosumer owning a background load, PV system and battery². The background load and solar production are the uncertain parameters in this paper; we forward their forecasts, \bar{P}_t^L and \bar{P}_t^{PV} , to make the deterministic model as follows:

$$P^{(k)} := \operatorname{argmin}_{P, X} \sum_{t \in T} \left[\lambda_t^{(k-1)} (P_t - P_t'^{(k-1)}) + \frac{\rho^{(k-1)}}{2} (P_t - P_t'^{(k-1)})^2 \right] \quad (6a)$$

$$P_t - \bar{P}_t^L - P_t^{Ch} + P_t^{Dis} + \bar{P}_t^{PV} - P_t^{Cur} = 0 \quad (6b)$$

$$P_t^{Cur} \leq \bar{P}_t^{PV} \quad (6c)$$

$$P_t^{Ch} \leq R_{Ch}^{max} \quad (6d)$$

$$P_t^{Dis} \leq R_{Dis}^{max} \quad (6e)$$

$$E_{t+1} = E_t + \eta P_t^{Ch} - P_t^{Dis} / \eta \quad (6f)$$

$$E_t \in [E^{min}, E^{max}] \quad (6g)$$

The objective (6a) consists solely of minimising the real power augmented Lagrangian terms. Here, the prosumer has a zero cost function $f(P, X) = 0$, however they can include any convex cost function such as a time of use tariffs (ToU). Note that since our network is exposed to wholesale market prices, our algorithm will end up reflecting in the LMPs λ_t ; and prosumers are charged $\sum_{t \in T} \lambda_t P_t$.

Equation (6b) fixes the prosumer CPP P_t to be the sum of the battery power, PV power after curtailment and house background load. Constraint (6c) limits the curtailment to the maximum available PV power. The battery charge and discharge rates are restricted by (6d) and (6e), respectively.

²Note, that (3a) is a general model which can include other appliances such as EV, heating ventilation and air conditioning (HVAC), and deferrable loads.

The SoC equation of the battery is given in (6f). Finally, (6g) keeps the SoC of the battery within its limits. In this explicit representation, the vector X is expanded into variables for battery charging and discharging P_t^{Ch} and P_t^{Dis} , PV curtailment P_t^{Cur} , and battery SoC E_t . The general constraint $g(P, X) \leq 0$ is replaced by (6b)–(6g).

The solution of the ADMM algorithm in (3a)–(3c) produces the network operating state and prosumer DER schedules over the horizon. However, when using the above deterministic prosumer subproblem, there is no guarantee that prosumers can stick to their schedules. Errors in load and PV forecasts can force prosumers to deviate from their negotiated CPP, which can cause network constraint violations. To overcome this, we propose prosumers solve an affinely adjustable robust optimization instead of this deterministic model, and then apply the affine rules in real-time to hold the CPP steady at the negotiated value.

IV. THE PROPOSED AARO APPROACH

Our approach integrates ideas from *adjustable robust optimization* [27] and *protection functions* [28] to obtain DER schedules that adjust themselves in real-time, achieving the negotiated CPP in practice. To clearly present our AARO approach, we first present a general compact formulation in Section IV-A; then we present an explicit extended formulation for the problem (6a)–(6g) in Section IV-B.

A. The Compact AARO prosumer Model

The deterministic prosumer subproblem can be re-written as:

$$P^{(k)} := \min_{P, X, Z} \mathcal{L}^*(P, P^{(k-1)}, \lambda^{(k-1)}) \quad (7a)$$

$$A \cdot P + B \cdot X + C \cdot Z + D \cdot \bar{U} + \mathcal{E} \leq 0 \quad (7b)$$

where P is the negotiated active power between the prosumers and the network; X represents the “here-and-now” variables which are made before the realization of the uncertainty; Z includes the variables that can take “wait-and-see” recourse decisions; \bar{U} represents a realization of the uncertain parameters; and finally, $A - \mathcal{E}$ represent the certain parameters.

To obtain the AARO model from (7a)–(7b), we first model the uncertain parameters \bar{U} , in a polyhedral uncertainty set (US) as follows:

$$US = \left\{ \bar{U} \geq 0; \quad F \cdot \bar{U} \leq G : \quad \zeta \text{ (dual variable)} \right\} \quad (8)$$

The aim of the proposed AARO-ADMM coordination approach is to immunize prosumers’ decisions against any uncertainty realization in US (8). Thus, (7b) can be written as:

$$\forall \bar{U} \in US : \quad A \cdot P + B \cdot X + C \cdot Z + D \cdot \bar{U} + \mathcal{E} \leq 0 \quad (9)$$

To allow recourse actions, we substitute the vector Z with its affine functions as:

$$Z = Z^n + Z^a \cdot \bar{U} \quad (10)$$

where Z^n is the “here-and-now” part of Z which is made before the realization of uncertainty while $Z^a \cdot \bar{U}$ adjusts itself to varying data when the uncertain parameters are revealed. Note that since all the constraints at the prosumer level, i.e.,

(7b), are linear, the relation between all the variables are already affine. Therefore, affine representation of recourse variables does not restrict our problem. Substituting (10) into (9) leads to:

$$\forall \bar{U} \in US : \quad A \cdot P + B \cdot X + C \cdot (Z^n + Z^a \cdot \bar{U}) + D \cdot \bar{U} + \mathcal{E} \leq 0 \quad (11)$$

Due to the requirement $\forall \bar{U} \in US$, (11) consists of an infinite number of constraints. However, the “wait-and-see” recourse term $Z^a \cdot \bar{U}$ in (11) can adjust itself to account for all scenarios in US if it can compensate the most extreme cases. This can be included to the model by using a max operator which enables the adjustable terms to compensate the extreme scenarios of US as follows:

$$A \cdot P + B \cdot X + \max_{\bar{U}} \left\{ C \cdot (Z^n + Z^a \cdot \bar{U}) + D \cdot \bar{U} \right\} + \mathcal{E} \leq 0 \quad (12)$$

The maximization term in (12) is analogous to the protection functions introduced in [28] to find a robust solution immunized against any realization of uncertain parameters belonging to the uncertainty set. To solve (12) efficiently and in one go, we use duality theory to eliminate the max operator, which results in:

$$P^{(k)} := \min_{P, X, Z, \zeta} \mathcal{L}^*(P, P^{(k-1)}, \lambda^{(k-1)}) \quad (13a)$$

$$A \cdot P + B \cdot X + C \cdot Z^n + G \cdot \zeta + \mathcal{E} \leq 0 \quad (13b)$$

$$F \cdot \zeta \leq C \cdot Z^a \cdot \bar{U} + D \quad (13c)$$

The AARO prosumers subproblem (13a)–(13c) is a convex quadratic problem with linear constraints, which can be easily solved via available solvers, such as Gurobi.

B. The AARO Prosumer Model

In this section, we follow the steps (7)–(13) to derive an AARO model for prosumer subproblem (6a)–(6g).

1) *Uncertainty Characterization*: As mentioned earlier, we apply our proposed approach within a receding horizon context. This enables us to use the latest (most accurate) electricity prices, PV power and residential demand for every negotiation. In addition to that we apply a robust treatment to PV power and residential demand as their variation, along a single horizon, can change the negotiated CPP and can lead to grid infeasibility. Similar to (8), we capture these uncertainty sets through a series of constraints, where P_t^{PV} and P_t^L are variables representing the potential realizations:

$$P_t^{PV} = \bar{P}_t^{PV} + P_t^{PV+} - P_t^{PV-} \quad (14a)$$

$$P_t^L = \bar{P}_t^L + P_t^{L+} - P_t^{L-} \quad (14b)$$

$$0 \leq P_t^{PV+} \leq \hat{P}_t^{PV+} : \zeta_t^{PV+} \quad (14c)$$

$$0 \leq P_t^{PV-} \leq \hat{P}_t^{PV-} : \zeta_t^{PV-} \quad (14d)$$

$$0 \leq P_t^{L+} \leq \hat{P}_t^{L+} : \zeta_t^{L+} \quad (14e)$$

$$0 \leq P_t^{L-} \leq \hat{P}_t^{L-} : \zeta_t^{L-} \quad (14f)$$

Inequalities (14c)–(14f) set the bounds on the solar and load uncertainty sets. These bounds can be chosen wide enough to compensate any uncertainty mismatches but in practice they are limited according to the recourse resources (i.e., battery capacity in our case).

To control the robustness of the proposed AARO approach, we use a parameter μ , called the degree of robustness:

$$\sum_{t \in T} \left[\left(\frac{P_t^{PV+}}{\hat{P}_t^{PV+}} + \frac{P_t^{PV-}}{\hat{P}_t^{PV-}} \right) + \left(\frac{P_t^{L+}}{\hat{P}_t^{L+}} + \frac{P_t^{L-}}{\hat{P}_t^{L-}} \right) \right] \leq \mu : \omega \quad (15)$$

Setting μ to zero in (15) leads to the deterministic values $P_t^{PV} = \bar{P}_t^{PV}$ and $P_t^L = \bar{P}_t^L$ via (14a) and (14b). Larger values of μ increases the size of the uncertainty set and hence the robustness of our AARO solution. For sufficiently large values, for which all uncertain variables can vary (i.e., $\mu = 100\%$), μ does not constrain the uncertainty set any more than (14c)–(14f) already do. For later reference, we group the uncertain variables into the tuple $u := (P_t^{PV+}, P_t^{PV-}, P_t^{L+}, P_t^{L-})$, and the dual variables associated with constraints (14c)–(14f) and (15) into $\Omega := (\zeta_t^{PV+}, \zeta_t^{PV-}, \zeta_t^{L+}, \zeta_t^{L-}, \omega)$.

2) *Affine Functions*: Our AARO approach takes into consideration the uncertainty set, and optimizes the parameters of affine functions used to control DER in real-time. These functions take real-time realizations of the available solar and load u as an input, and output battery charge and discharge actions that keep the negotiated CPP constant:

$$P_t^{Ch}(u) := c_t^0 + c_t^1 P_t^{PV+} + c_t^2 P_t^{PV-} + c_t^3 P_t^{L+} + c_t^4 P_t^{L-} \quad (16a)$$

$$P_t^{Dis}(u) := d_t^0 + d_t^1 P_t^{PV+} + d_t^2 P_t^{PV-} + d_t^3 P_t^{L+} + d_t^4 P_t^{L-} \quad (16b)$$

where c stands for charge and d for discharge. The nonadjustable terms c_t^0 and d_t^0 (similar to Z^n in (10)) represent the first stage “here-and-now” decisions, while the remaining adjustable terms c_t^{1-4} and d_t^{1-4} (similar to Z^a in (10)) represent the second stage “wait-and-see” recourse decisions [27].

The use of affine functions for recourse and our characterization of the uncertainty set about the forecast, enables us to compensate for fluctuations in multiple directions in the uncertainty set. Previous robust works, e.g., [17] and [18], are only able to account for uncertainty in available solar in one direction (less solar than expected), relying on curtailment alone for the other direction (excess solar). Our proposed approach is able to use the battery to compensate in both directions, leading to less wasted renewable energy.

3) *AARO Subproblem Formulation*: To obtain the proposed AARO model, we take the deterministic prosumer subproblem (6a)–(6g), and replace the forecasts and deterministic battery decisions \bar{P}_t^{PV} , \bar{P}_t^L , P_t^{Ch} and P_t^{Dis} with their uncertain and affinely adjustable function equivalents P_t^{PV} , P_t^L , $P_t^{Ch}(u)$ and $P_t^{Dis}(u)$ (similar to what we did in previous section to obtain (11)). These affine functions need to be able to compensate the most extreme uncertainty realizations within the uncertainty set, which requires us to select their parameters so that our constraints are feasible under all conditions. Similarly to what we did in (12), we enforce this on a constraint-by-constraint basis, as explained further below. The overall objective is to optimize the CPP of the house, and affine function coefficients c_t^{0-4} and d_t^{0-4} , so that the constraints

are feasible over all values $u \in US$ in the uncertainty set:

$$\min_{P, X, c^{0-4}, d^{0-4}} \sum_{t \in T} \left[\lambda_t^{(k-1)} (P_t - P_t^{(k-1)}) + \frac{\rho^{k-1}}{2} (P_t^{(k)} - P_t^{(k-1)})^2 \right] \quad (17a)$$

$$\min_{u \in US} \{ P_t - P_t^L - P_t^{Ch}(u) + P_t^{Dis}(u) + P_t^{PV} - P_t^{Cur} \} = 0 \quad (17b)$$

$$\max_{u \in US} \{ P_t - P_t^L - P_t^{Ch}(u) + P_t^{Dis}(u) + P_t^{PV} - P_t^{Cur} \} = 0 \quad (17c)$$

$$\max_{u \in US} \{ P_t^{Cur} - P_t^{PV} \} \leq 0 \quad (17d)$$

$$\max_{u \in US} \{ P_t^{Ch}(u) \} \leq R_{Ch}^{max} \quad (17e)$$

$$\max_{u \in US} \{ P_t^{Dis}(u) \} \leq R_{Dis}^{max} \quad (17f)$$

$$E_{t+1} = E_t + \min_{u \in US} \{ \eta P_t^{Ch}(u) - P_t^{Dis}(u) / \eta \} \quad (17g)$$

$$E'_{t+1} = E_t + \max_{u \in US} \{ \eta P_t^{Ch}(u) - P_t^{Dis}(u) / \eta \} \quad (17h)$$

$$E_t, E'_t \in [E^{min}, E^{max}] \quad (17i)$$

For the inequality constraints (6c)–(6e), the worst case is when we maximize the left hand side (LHS) over the uncertainty set, producing (17d)–(17f). We treat the equality constraints in two different ways. We ensure the CPP constraint (6b) LHS remains zero in both directions: a min (17b) and max (17c) over the uncertainty set. This ensure the CPP remains fixed for all values in the uncertainty set. For the battery SoC constraint (6f), we allow the SoC to be a recourse variable. In this case we need to make sure all potential SoC values remain within the battery capacity. We apply a min (17g) and max (17h) to the right hand side of this constraint over the uncertainty set, and introduce a second recourse variable E'_{t+1} to represent the SoC in the max case. The smaller recourse variable, E_{t+1} is used in the constraints for the next time step, while the real SoC is used in every negotiation as we further discuss in Section IV-C.

The optimization problem in (17a)–(17i), with embedded max and mins over the uncertainty set, is not directly solvable with standard optimization tools. We use the dual variables Ω associated with constraints (14c)–(14f) and (15), and duality theory to eliminate the embedded max and min terms. A separate set of these dual variables needs to be introduced for each embedded max and min. In the following, we reformulate (17e), by first substituting in (16a), and then replacing the max over the uncertainty set with its dual equivalent:

$$c_t^0 + (\hat{P}_t^{PV+} \zeta_t^{PV+} + \mu \omega) + (\hat{P}_t^{PV-} \zeta_t^{PV-} + \mu \omega) + (\hat{P}_t^{L+} \zeta_t^{L+} + \mu \omega) + (\hat{P}_t^{L-} \zeta_t^{L-} + \mu \omega) \leq R_{Ch}^{max} \quad (18a)$$

$$\zeta_t^{PV+} + \omega / \hat{P}_t^{PV+} \geq c_t^1, \quad \zeta_t^{PV-} + \omega / \hat{P}_t^{PV-} \geq c_t^2 \quad (18b)$$

$$\zeta_t^{L+} + \omega / \hat{P}_t^{L+} \geq c_t^3, \quad \zeta_t^{L-} + \omega / \hat{P}_t^{L-} \geq c_t^4 \quad (18c)$$

This equivalent set of linear constraints (μ and \hat{P}_t^* are parameters) are used in place of (17e). When the same approach is applied to (17b)–(17d) and (17f)–(17h), we obtain a convex quadratic program over linear constraints, forming our final AARO subproblem, similar to (13a)–(13c) in the compact representation. Note that DER schedules and affine control parameters are obtained when our AARO-ADMM algorithm (3a)–(3c) converges. Since both prosumers and network subproblems are convex, the convergence and thus feasibility of our ADMM algorithm is guaranteed [4].

C. Receding Horizon Integration and Guarantees

While the AARO subproblem was presented in the previous section for all t , when integrated within a receding horizon context we only apply the AARO treatment for the first time step (1 hour) and instead use the original deterministic model with forecast values for the remaining 23 hours. This combination ensures that we can handle the fluctuations between now and the next reoptimization, and that those decisions are neither short-signed (as they would be if the rest of the horizon was simply ignored), nor overly conservative (as they would be if AARO was applied to the entire horizon). After an hour has passed, as in a model-predictive control approach, the real battery SoC is measured and used in optimizing the next horizon (rather than the worst case value used in other RO work [17]) Moreover, using the real SoC in every reoptimization indirectly adds the cost of the recourse actions to our problem.

Algorithm 1 together with Function AAROADMM show the implementation of the proposed AARO-ADMM approach. Through the coordination function AAROADMM, prosumers obtain their DER schedules as well as their affine functions while the network obtains its operating state for the horizon T . Algorithm 1 calls the function AAROADMM on a receding horizon basis to coordinate the prosumers and the grid frequently. Given the affine functions and the uncertainty realizations, prosumers take recourse actions every τ minutes³ between the two negotiations. Finally, at the end of the first time step of the horizon T , and just before the next negotiation, SoC is updated and is used for the next negotiation.

A solution to the AARO subproblem guarantees for the next hour a constant CPP equal to the negotiated value, as long as the uncertainty realization is within the uncertainty set. In some cases it might be desirable or necessary to restrict the uncertainty set (used with the AARO approach) to be smaller than the real-world uncertainty set. This could be a compromise to make the decisions less conservative, a way to guarantee feasible subproblems when the uncertainty volatility is greater than the available recourse actions (e.g., battery capacity), or as a result of limited information. In such cases the affine functions will still do their best effort to keep the CPP constant, and so the fluctuations experienced at the network level will be dampened.

Since our approach notably reduces the need for frequent negotiations (here from every 5 minute to every hour), it is

³Here, we take recourse actions every 5 minutes until the whole hour is covered.

also more robust to communication failure than the standard ADMM. However, in case the communication network fails for over 1 hour, our approach can follow common practices such as falling back on the previous actions or program the EMS to keep their CPP close to zero to avoid any network impacts.

Function AAROADMM

Input: A horizon T to optimize over
Output: DER schedules and affine functions for each prosumer. The operating state for the network.
Initialize $P^{(0)}, P^{(0)}, \lambda^{(0)}$;
 $k \leftarrow 1$;
do
 $P^{(k)} \leftarrow$ prosumers solve AARO subproblems using $P^{(k-1)}, \lambda^{(k-1)}$ and $\rho^{(k)}$ communicated from network;
 $P^{(k)} \leftarrow$ network solves multi-period OPF subproblem using $P^{(k)}$ communicated from prosumer, $\lambda^{(k-1)}$ and $\rho^{(k)}$;
 $\lambda^{(k)} \leftarrow \lambda^{(k-1)} + \rho^{(k)}(P^{(k)} - P^{(k-1)})$;
 $R_p^{(k)}, R_d^{(k)} \leftarrow$ calculate residuals;
 $k \leftarrow k + 1$;
while $R_p^{(k-1)} > \epsilon$ or $R_d^{(k-1)} > \epsilon$;

Algorithm 1 Proposed AARO-ADMM approach

$t \leftarrow 1$ **loop**
 $T \leftarrow \{t, \dots, t - 1 + |T|\}$
 Coordination \leftarrow AAROADMM (T)
 Recourse \leftarrow Prosumers affine functions, obtained from AAROADMM (T)
 foreach τ min period in first hour of T **do**
 Reveal uncertainty and take *Recourse* action locally
 for each prosumer
 Update SOC of batteries at end of first hour in horizon after applying recourse decisions
 $t \leftarrow t + 1$
end;

V. NUMERICAL RESULTS

To illustrate the effectiveness of the proposed approach, we use a radial 69-bus distribution network (data of which can be found in [29]). The wholesale electricity prices for the feeder import are taken from the Australian Electricity Market Operator (AEMO) [30]. Prosumers connecting to the grid are consuming 20 kWh per day on average and own a 5 kW rooftop PV and a 10 kWh battery with charge and discharge rates of 5kW and efficiency of 85%. We use anonymized solar and demand data for 28 consumers in Tasmania, Australia, provided by Reposit Power [31], and randomly assign this data to 96 prosumers in our 69-bus network.

A. Approaches Compared

To assess the performance of AARO-ADMM, we study and compare four different approaches as follows:

Deterministic: The grid components negotiate hourly over a receding horizon through the standard multi-period ADMM algorithm (3a)–(3c). This approach assumes that the forecast of the uncertain parameters over the next hour is accurate enough to enable all negotiations to be done deterministically. However, if the uncertain parameters deviate notably between negotiations, this approach cannot make any corrective action.

Reactive Controller: Similarly to the deterministic approach, the negotiations are on an hourly basis. However, this approach additionally includes a droop reactive controller which tries its best to avoid infeasibility by compensating for uncertainty mismatches between negotiations. This approach shares with AARO-ADMM the ability to apply corrective actions to compensate for deviations from forecast in real-time, however, since the controller’s decision have not been obtained robustly, there is no guarantee that these corrective actions succeed in delivering the negotiated CPP.

Proposed: This is the proposed AARO-ADMM approach. Similarly to the deterministic and reactive controller approaches, the negotiations are done hourly. However, instead of assuming an accurate forecast, the first time step of each negotiation robustly optimizes affine functions using AARO. Between negotiations, we take recourse decisions using these affine functions every 5 min, and are guaranteed to successfully compensate for any mismatch within the uncertainty set.

Perfect: This approach is intended to provide a perfect but unachievable baseline reflecting the ideal costs that could be obtained by ADMM if there were no uncertainty, and if computation and communication time were not an issue.

Due to the time granularity of our available data, both the Reactive Controller and the Proposed approach, w.l.o.g., re-optimize every hour and make recourse decisions every 5 minutes. However, operators can increase or decrease these timing parameters to match the volatility in their network. Note that since the recourse actions are made locally by the fast-responding inverter-based technologies, both approaches can act within much smaller time steps, e.g., in terms of seconds.

B. Convergence of the Proposed ADMM-AARO Approach

The average number of iterations as well as the CPU time of the various approaches for a horizon are reported in Table I. As can be seen from this table, the Deterministic and Reactive Controller approaches have the same performance in terms of computational effort. The reason is that the reactive controller is independent of the ADMM algorithm and adds negligible overhead. On the other hand, the Perfect approach uses 5 minute resolution data, 288 time steps per horizon, so it takes much longer to converge on our test case. This incompatibility between computational requirements and available time highlights the limitations of frequent negotiation with high resolution discretisation of time. In contrast, the computation time and the number of iterations of the Proposed AARO approach is close to those of Deterministic and Reactive Controller since all these use an hour time discretisation, and only renegotiate a new horizon every hour. The convergence

of the primal and dual residuals for the Proposed approach is shown in Fig. 2.

TABLE I: Average convergence and CPU time of different cases per horizon.

| Cases | No. Iterations | CPU time (mins) |
|---------------------|----------------|-----------------|
| Deterministic | 62 | 3.4 |
| Reactive controller | 62 | 3.4 |
| Proposed | 77 | 4.2 |
| Perfect | 356 | 26.1 |

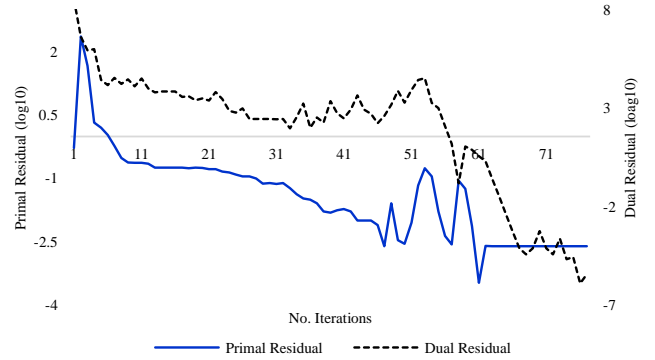


Fig. 2: Primal and dual residuals convergence.

C. Detailed Behaviour of the Recourse Decisions

To clearly illustrate the behavior of the proposed AARO-ADMM coordination approach, we consider a prosumer located on bus 27. The load forecast along with the actual five-minute load realization for hour 20 for this prosumer is shown in Fig. 3. As can be seen from the figure, the load realization during this peak hour significantly differs from the forecast.

Since the hourly forecast is used for the negotiations, these discrepancies could negatively affect the grid and even lead to constraint violations, especially in the case of correlated uncertainties such as those due to solar. However, the Proposed approach which uses the hourly forecast to robustly negotiate on a receding horizon basis is capable of taking “wait-and-see” recourse decisions through its adjustable terms and compensate for the uncertainty mismatches. Fig. 3 also shows the adjustable terms applied to the battery charge for the mentioned prosumer and hour 20. The positive values in this figure mean that the battery charges more than anticipated while the negative values stand for charge reduction. As shown in the figure, a proper recourse decision is made to compensate for deviation from the forecast every five minutes during the whole hour. As a result, the forecast deviations do not lead to deviation from the negotiated CPP.

To further demonstrate the effectiveness of the Proposed approach, we compare the voltage profile of bus 27 at hour 20 for the different approaches in Fig. 4. Voltages are considered to be safe when they are in the interval $[0.95, 1.05]$ p.u., and as shown in Fig. 4, the lower limit is exceeded by some of the approaches during this peak hour. In particular the voltage for the Deterministic approach is too low since there is

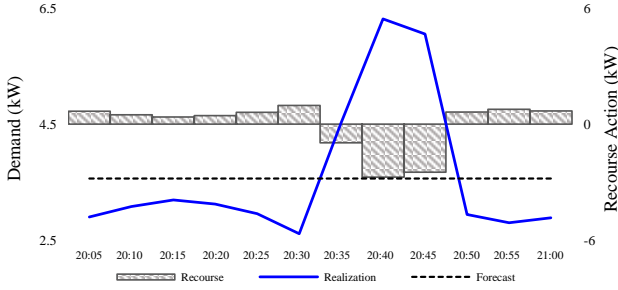


Fig. 3: Load forecast vs. actual realizations as well as recourse action on battery charge (hour 20).

no “wait-and-see” recourse decision. Whilst it improves over Deterministic, even the Reactive Controller is unable to keep the voltage within the safe limit during this peak hour. In contrast, the voltage of the Proposed approach is always the same as was negotiated. As the Perfect approach uses perfect information instead of a forecast, it is able to keep the voltage within the safe limit.

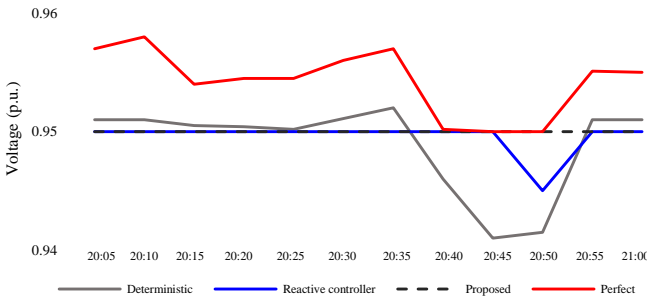


Fig. 4: Voltage profile of node 27 at hour 20.

To evaluate the performance and economic benefits of these approaches on a longer time frame, we use five-minute solar and demand data for our 28 prosumers over 30 days. Table II reports their total and relative costs (wrt the Perfect approach) as well as the the number of time steps during which they led to constraint violations occurring. Comparing the results of Table I, Fig. 4, and Table II leads to the following observations:

- 1) The Proposed approach is able to match the Perfect case in ensuring grid constraints are met. As reported in Table II, 6% better schedules are obtained when perfect information is available, but this is not possible in practice.
- 2) Although the Deterministic and Reactive Controller approaches require the least computational effort, they lead to constraint violations as shown in Fig. 4 and Table II. Moreover, despite, their lower “expected” cost reported in Table II, they are likely to yield the most expensive “real” costs, once expensive load shedding penalties are factored in. In contrast, the Proposed approach has similar computational performance as the Reactive controller case but provides a principled way to automatically and dynamically configure the control parameters, guaranteeing the absence of constraint violations within the limits afforded by the uncertainty set.

TABLE II: Performance of different approaches on 30 days.

| Cases | Expected Cost (\$) | Rel. to perfect % | Const. dev. |
|---------------------|--------------------|-------------------|-------------|
| Deterministic | 152537 | -20 | 447 |
| Reactive controller | 161439 | -15 | 128 |
| Proposed | 201012 | +6 | 0 |
| Perfect | 190451 | — | 0 |

All in all, the proposed AARO-ADMM coordination approach obtains feasible solutions, at a cost just slightly higher than the unachievable Perfect baseline, while enjoying the low computation and communication burden of approaches that use fewer renegotiations and low discretization of time. For comparison, an online non-AARO approach that reoptimizes every 5 minutes will take at least 12 times as much communication and computational effort, will not be able to ensure constraints remain satisfied, and, based on our experiments, in ideal conditions will only be able to improve the costs by at most 6%, as bounded by our perfect approach. This added cost would differ depending on how hard the constraints are, the type of constraints (e.g., linear or non-linear) and whether or not these constraints are binding. Network data (e.g., R and X, thermal limits), prosumers DER and load profile, as well as the uncertainty in the whole problem, can also increase/decrease such cost.

D. A comparison between SP and AARO

Table III compares AARO with SP on the problem (3a)–(3c). The results for SP are averaged over 50 runs, each of which uses 20 scenarios to model prosumer load and PV uncertainties. The table reports the cost and run time of each approach (averaged over the 50 runs for SP), and the number of runs during which SP was able to deliver the agreed CPP— for AARO, the agreed CPP is always delivered.

TABLE III: SP and AARO comparison

| | SP | AARO |
|-----------------|--------------|--------|
| Total cost (\$) | 5.373 | 5.372 |
| Time (s) | 2.98 | 0.74 |
| CPP | 46/50 agreed | agreed |

E. AARO-ADMM under PV penetration levels

PV penetration level (PPL) can affect the power system differently based on battery availability. Therefore, for this experiment, we developed two different cases as follows: PV) in which prosumers are equipped with a PV panel but not a battery; and PV+Batt) where prosumers are equipped with a PV-battery pair. Table IV compares the results obtained by the proposed coordinated approach (C) with those obtained without coordination (NC):

TABLE IV: AARO-ADMM under different PPL

| PPL (%) | Total Costs (\$ 1k) | | | |
|---------|---------------------|------|---------|------|
| | C | PV | PV+Batt | |
| | | NC | C | NC |
| 0 | 19.7 | 19.7 | 19.7 | 19.7 |
| 25 | 13.4 | 13.4 | 9.75 | Inf. |
| 50 | 7.178 | Inf. | 0.19 | Inf. |
| 75 | 1.25 | Inf. | -8.80 | Inf. |
| 100 | -4.24 | Inf. | -16.96 | Inf. |

As shown in Table IV, without the proposed coordination, the synchronized behaviour of the prosumers exceeds the network capabilities, leading to infeasible results (shown by Inf.) in most cases. On the contrary, the proposed approach varies the LMPs to avoid such infeasibilities, i.e., the LMPs will become negative at the point where the network is operated at its edges to create incentive for prosumers to curtail their PV power.

VI. CONCLUSION

We proposed AARO-ADMM, an affinely adjustable robust DER coordination approach based on the distributed algorithm ADMM. This approach addresses the privacy concerns of the prosumers, handles network constraints and AC power flows, and deals with uncertainty in PV generation and residential demand. To do so, it iteratively coordinates DER on a receding horizon basis and accounts for forecast deviations in the first time step of each renegotiation using AARO optimization. AARO-ADMM coordination negotiates “here-and-now” decisions in longer time steps and compensates the uncertainty mismatches using “wait-and-see” recourse decisions in shorter steps until the next negotiation. We demonstrate that this achieves an excellent compromise between computational cost and solution quality, whilst guaranteeing the feasibility and robustness of solutions.

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