# Network-aware Co-optimisation of Residential DER in Energy and FCAS Markets

Ahmad Attarha, Paul Scott and Sylvie Thiébaux

College of Engineering and Computer Science

The Australian National University (ANU)

Canberra, ACT 2601, Australia

Email: firstname.lastname@anu.edu.au

*Abstract*—**With the increasing uptake of distributed energy resources (DER), such as rooftop PV and batteries, a significant amount of electricity is generated at a smaller scale. Consequently, the large-scale generators, responsible for frequency response, are having less share in our electricity markets. This makes it challenging to provide the grid with the required frequency response. To overcome this challenge, we introduce a network-aware co-optimisation approach that enables consumers to co-optimise their DER in energy and frequency markets. Our approach is based on the alternating direction method of multipliers where consumers and the grid negotiate on a receding horizon framework to obtain consensus solutions which satisfy the grid constraints under all operating conditions. Our experiments on 69 and 141 bus networks show a significant improvement over a case where decisions are not co-optimised. We also show how, compared to the majority of the literature, we can avoid network violations.**

*Index Terms*—**ADMM, Co-optimisation, DER, Frequency Response, Network-aware.**

## I. Introduction

### A. Motivation

The increasing penetration of renewable energy in our power systems is displacing synchronous generators while adding more volatility to the grid. This creates a greater need for frequency response in a system with generators typically less able to fulfil such need. For example, as the Australian energy market operator (AEMO) reported in [1], rooftop PV will be sufficient to cover all load in low-demand days in South Australia as early as in 2025, leaving the region without conventional generators to provide the required frequency response. Fortunately, with large uptake and appropriate controls, distributed energy resources (DER) owned by consumers, such as rooftop PV and battery storage, have the potential to fill this crucial role.

However, getting the required frequency response from numerous consumers is challenging as it requires solving a large-scale co-optimisation problem to find DER share in energy and frequency markets. More importantly, such co-optimisation needs to include network constraints to ensure that the DER decisions are network-aware, meaning that they do not exceed the distribution network limits. Consumers' privacy concerns as well as their uncertain data also contribute to the complexity of such a proposal. To simplify the problem, the state of the art mainly neglects the network impact of residential DER [2], [3], however, this may lead to infeasible solutions going beyond the network capabilities.

To meet the above challenges, we propose a decentralised approach based on the alternating direction method of multipliers (ADMM). Our approach extends our previous work [4] to obtain network-aware decisions in both energy and frequency markets by generating locational marginal prices (LMPs) that reflect the distribution network constraints. Using LMPs and the wholesale market prices, consumers co-optimise their participation in each market and communicate with the grid their demand/supply profiles for each market. We verify that the participation in different markets does not violate the network constraints using three separate AC optimal power flows (OPFs) and update the LMPs accordingly. When the algorithm converges, the DER have their optimum schedules and the retailer[1] has network-aware bids ready to submit to each market. Note that our decentralised approach also overcomes the computational complexity stemming from the large-scale nature of the problem, and mitigates consumers' privacy concerns. Moreover, to take uncertainty into account, we apply our approach within a receding horizon context in which the participants can update their uncertain parameters and use their latest (most accurate) information.

In this paper, we develop our approach under the National Energy Market (NEM) regulations, operated by AEMO in Australia. In line with NEM and AEMO terminology, hereafter, we use the term "frequency control ancillary service" (FCAS) instead of frequency response and "FCAS market" for the market in which FCAS is traded. Despite this focus, we expect our approach to be widely applicable to other electricity markets that trade in or optimise reserve capacity. Moreover, given the relatively low marginal cost of DER and in line with [2], [3], [5]–[10], we also assume that the retailer is a price-taker participant of the electricity market. We plan to study the market participation of a price-maker retailer in our future work.

### B. Related Work

The participation of DER in energy and FCAS markets can be categorised into two main groups; a) the works studying the market participation of a stand-alone DER [5]–[8]; and b)

---

[1]Here, the retailer is in charge of bidding in the market, However, this could be done by an aggregator or possibly even the owners themselves.

the works studying the market participation of several smaller-scale DER by an agent e.g., an aggregator [2]–[4], [9]–[14]. Modeling the participation of a single DER in electricity markets mainly leads to a small-scale optimisation problem which is not the focus of this paper. On the other hand, the second group of approaches, to which our work belongs, includes many DER, and given the potential conflicting interests of DER owners, leads to a complex large-scale co-optimisation problem.

More importantly, the synchronised action of many DER in a distribution network can exceed its limits, e.g., when they are responding to a price spike or during peak PV production. Thus, the distribution network constraints should be also taken into account. However, the nonlinear nonconvex nature of the network constraints, e.g., power flow equations, makes this problem more difficult to solve, especially within the time frame of the real-time electricity markets (e.g., every 5 minutes in Australia). Consumers privacy concerns and their uncertain data also exacerbate the complexity of this problem.

To simplify this problem, previous research works neglect either the FCAS market participation e.g., [4], [15], or the network e.g., [2], [9]–[11], [13] or both e.g., [12], [14]. However, as we show in the numerical results section, such simplifications can lead to non-optimum or even infeasible results. To overcome this issue, [16] includes linear load flow equations and aggregates a large-scale battery and an electric vehicle (EV) fleet to bid in day-ahead frequency regulation market. The work [16] can be extended to include AC-OPF instead of their linear load flow and consumers instead of their battery/EV-fleet; however, running a central AC-OPF when there are numerous consumers would lead to a large-scale non-linear optimisation problem which is computationally expensive for an online real-time setting. Also, [16] requires a central access to all of the information of all the consumers which is not practical and compromises consumers privacy.

To meet the above challenges mentioned for a central OPF, the distributed algorithm ADMM [17] has been used in the literature on OPF [18], [19] and DER coordination [4], [15]. Even though [4], [15], [18], [19] can mitigate the computational complexity and privacy issues of a central tool, they neglect FCAS market participation and only coordinate DER decisions in the energy market. However, as we show in our result section, this is making a poor use of DER flexibility which can be sold in highly-priced FCAS markets.

Therefore, in this paper, we extend our previous work [4] to provide network-aware bids in both energy and FCAS markets. Similarly to [4], the network and consumers negotiate to converge on a consensus solution; yet here, we modify the energy management system (EMS) of the consumers to co-optimise their DER in several markets rather than optimising them according to time-of-use tariffs (as we did in [4]). Moreover, unlike [4], here, our network model uses three AC-OPFs to guarantee that the consumers decisions in energy, raise and lower FCAS markets do not violate any network constraints. Thus, our contributions compared to the state of the art are:

- Bidding in both energy and FCAS markets by developing an ADMM-based approach in distribution networks which reflects the wholesale electricity market prices and the network LMPs.
- Developing a co-optimisation EMS problem for consumers to optimally schedule their DER in both energy and FCAS markets while not violating any network constraints.

*C. Paper organisation*

Section II provides a brief introduction on NEM and AEMO as well as their different FCAS markets. Section III gives a high level presentation of our proposed approach which is then detailed in Section IV; Section V numerically illustrates the effectiveness of the proposed approach; and finally, Section VI concludes the paper.

## II. NEM CONTINGENCY FCAS

We develop our network-aware co-optimisation approach to participate in the 5-minute real-time NEM. Therefore, in this section, we briefly introduce NEM and its markets before the design of our model in Section III. Under NEM frequency standards, AEMO must ensure that following a credible contingency event, the frequency deviation remains within the contingency band (e.g., 49.5 to 50.5 Hz) and returns to the normal operating threshold (e.g., 49.85 to 50.15 Hz) within 5 minutes[2]. To do so, AEMO uses 6 contingency FCAS markets, categorised in three main groups according to their response time:

- 6-second raise and lower: these services are the fastest responses to a major drop/rise in frequency aiming at arresting the frequency after an event.
- 60-second raise and lower: these services are the slow responses to stabilise frequency following a major drop/rise in frequency.
- 5-minute raise and lower: these services, also known as delayed responses, are used to recover frequency to the normal operating band following a major drop/rise in frequency.

A registered FCAS provider can participate in any or all 6 of these FCAS markets, for which they get paid their accepted bids regardless of whether or not a contingency actually occurs. In the case that a contingency does occur, they must respond up to their market accepted capacity, to correct the frequency deviation. In this paper, we assume the DER is equipped with the necessary metering and control systems to be able to enact the required frequency responses.

## III. THE PROPOSED APPROACH

The proposed approach consists of three main parts:
1) EMS subproblem: which enables consumers to optimally contribute to energy and all FCAS markets.
2) Network subproblem: which a) ensures the secure operation of the network in all different cases (i.e., a case in

---

[2]According to the event and/or location, the contingency frequency band and recovery time might differ [20].

which only energy is traded, as well as the cases in which the consumers are providing AEMO with their raise or lower FCAS support); b) obtains the optimum bids to submit to each market.

3) ADMM approach: which coordinates consumers action (EMS subproblem) with the grid (Network subproblem) and and creates visibility of both the market and the grid by reflecting the wholesale market prices and the distribution network constraints.

The first part of our approach relates to the consumers. The retailer is in charge of the second part (i.e., bidding in the different markets); while both the consumers and the retailer contribute to the ADMM coordinator of the third part. In the following, we separately explain each part of the proposed approach. To increase the readability, we start with a high level presentation of the ADMM algorithm in section III-A and then thoroughly model the first and second parts of the proposed approach in sections IV-A and IV-B, respectively.

### A. The General High Level Problem

We treat each consumer $n \in C$ as a generating unit exchanging with the grid active power[3] $P_n^e \in \mathbb{R}^{|T|}$ at each time step in horizon $T$. Consumers can increase / decrease their output (with respect to $P_n^e$) to provide raise / lower FCAS $P_n^r$ / $P_n^l \in \mathbb{R}^{|T|}$ leading to their maximum / minimum power exchange with the grid. We put $P_n^e$, $P_n^r$ and $P_n^l$ in a vector $P_n \in \mathbb{R}^{3 \times |T|}$ to increase readability. Consumers also have their own internal variables $Y_n$ as well as the objective and constraint functions $f_n$ and $g_n$ which take $P_n$ and $Y_n$ as inputs. We drop the subscript to represent all prosumer exchange powers, $P \in \mathbb{R}^{3 \times |C| \times |T|}$, which together with the network internal variables $X$ are inputs to the network's own constraint function $h$. The multi-period OPF can be written as:

$$\min \sum_{n \in C} f_n(P_n, Y_n) \tag{1a}$$

$$\text{s.t.} \quad \forall n \in C : g_n(P_n, Y_n) \leq 0 \tag{1b}$$

$$h(P', X) \leq 0 \tag{1c}$$

$$P - P' = 0 \tag{1d}$$

In the above, we have duplicated the active power exchange variables, so, the consumers and the network have their own copies ($P$ and $P'$). The duplicated variables are enforced to have the same values through (1d) which is the equation we will now relax to decompose the consumers from the network.

The penalty term of the augmented Lagrangian applied to the equality connection constraint (1d) is:

$$\mathcal{L}^*(P, P', \lambda) = \lambda^{\mathsf{T}}(P - P') + \frac{\rho}{2} ||(P - P')||_2^2 \tag{2}$$

---

[3]Here, we focus on active power exchange, the reactive power exchange can be modelled similarly.

$\lambda := [\lambda_e \ \lambda_r \ \lambda_l]^{\mathsf{T}}$ is the vector of dual variables for the relaxed constraints (1d) and $\rho$ is the penalty parameter of the augmented Lagrangian.

### B. The ADMM Algorithm

We use the ADMM algorithm [17] to iteratively solve (1a)−(1d) to its optimum. The ADMM algorithm has three phases per iteration $k$:

$$P^{(k)} := \min_{P, Y} \sum_{n \in C} \left[ f_n(P_n, Y_n) + \mathcal{L}_n^*(P_n, P_n'^{(k-1)}, \lambda_n^{(k-1)}) \right]$$

$$\text{s.t. } \forall n \in C : g_n(P_n, Y_n) \leq 0 \tag{3a}$$

$$P'^{(k)} := \min_{P', X} \mathcal{L}^*(P^{(k)}, P', \lambda^{(k-1)})$$

$$\text{s.t. } h(P', X) \leq 0 \tag{3b}$$

$$\lambda^{(k)} := \lambda^{(k-1)} + \rho^{(k)} \cdot (P^{(k)} - P'^{(k)}) \tag{3c}$$

In the first phase (3a), the consumers optimise for $P$, holding $P'$, and $\lambda$ constant at their $k-1$-th values. In the second phase (3b), the network optimises for $P'$, holding $P$ and $\lambda$ constant at their $k$-th and $k-1$-th values, respectively. Finally, the dual variables $\lambda$ are updated in (3c), completing the $k$-th iteration.

Through (3a)–(3c), consumers schedule their appliances and communicate with the network their preferred load/generation for each market. Network then checks if consumers behaviour satisfies the network constraints and updates $\lambda$ accordingly. When the algorithm converges, $\lambda$ represents the price of having the network constraints satisfied at each node i.e., represents LMPs. Compared to a central approach which requires to know *all* of the information of *all* of the consumers[4], our approach only requires to know the consumers connection point power (bids). Thus, not only does our approach provide a higher privacy level for consumers, but also (from the data communication prospective) the necessary data exchange is much simplified.

### C. Stopping criteria and Convergence of the Algorithm

In line with [17], we define the stopping criteria using primal and dual residuals as follows:

$$R_p^{(k)} := \left( (P_1^{(k)} - P_1'^{(k)}), (P_2^{(k)} - P_2'^{(k)}), ... \right)^{\mathsf{T}} \tag{4a}$$

$$R_d^{(k)} := \left( \rho(P_1'^{(k)} - P_1'^{(k-1)}), \rho(P_2'^{(k)} - P_2'^{(k-1)}) ... \right)^{\mathsf{T}} \tag{4b}$$

The primal residuals (4a) represent the constraint violation at the current solution and the dual residuals (4b) the violation of the KKT stationarity constraint [17].

To feed the proposed approach with the latest uncertainty information and obtain more accurate results, we implement (3b)–(3c) within a receding horizon framework. More explanation on our receding horizon implementation is given in Section IV-C.

---

[4]This information includes consumers' battery SoC, PV power, their uncontrollable load and any other appliances a consumer might have.

---

## IV. Consumer and Network Subproblems

In this section, we first model the consumers EMS subproblem[5]; then develop the network subproblem to make sure that consumers behaviour does not go beyond the network capabilities; and finally, we present the whole approach on a receding horizon framework.

### A. The Proposed Consumer Subproblem Model

We break the EMS constraints into two groups: constraints for linking the absolute raise and lower quantities to biddable values for the FCAS markets, and those related to the physical and operating constraints of the DER. Note that as we work with one prosumer at a time, here, we drop the superscript $n$.

*1) Market Linking Constraints:* We introduce a pair of variable vectors $\Delta P^r_m, \Delta P^l_m \in \mathbb{R}^{|T|}_{\geq 0}$ for each frequency market $m \in M$, to represent the amount of raise and lower capacity to bid into the market. The corresponding power exchanged with the network in each case is given by the related variables $P^r_m, P^l_m \in \mathbb{R}^{|T|}$, where:

$$P^r_m = P^e + \Delta P^r_m \tag{5a}$$

$$P^l_m = P^e - \Delta P^l_m \tag{5b}$$

Response is only required from one market at a time, so when checking network feasibility we only need to consider the most extreme raise and lower values. This can be written as the max and min over the markets, which we relax as follows:

$$P^r = \max_{m \in M}(P^r_m) \rightsquigarrow P^r \geq P^r_m \quad \forall m \in M \tag{5c}$$

$$P^l = \min_{m \in M}(P^l_m) \rightsquigarrow P^l \leq P^l_m \quad \forall m \in M \tag{5d}$$

We expect this to be an exact relaxation since larger $P^r$ or smaller $P^l$ will lead to more active network constraints (relative to $P^e$) rather than any benefit to the network subproblem.

The cost associated with the market participation is given by the following component to the objective function:

$$\sum_{t \in T} \delta_t \left( \pi^e_t P^e_t + \sum_{m \in M} \left( \pi^r_{m,t} \Delta P^r_{m,t} + \pi^l_{m,t} \Delta P^l_{m,t} \right) \right) \tag{5e}$$

where $\delta_t$ is the time step duration in hours; $\pi^e_t$ is the wholesale energy market price at $t$; and $\pi^r_{m,t}$ / $\pi^l_{m,t}$ are the prices of the raise / lower FCAS market $m$ at $t$. These amounts are payable whether or not a contingency actually occurs. However, a contingency can have a small impact on the state-of-charge of the battery. To approximate any lost or gained energy due to FCAS deployment, we model the probability of a contingency event occurring for each market $\mu^l_{m,t}$ and $\mu^r_{m,t}$[6], and assume that the value of the lost / gained energy is at the energy market price $\pi^e_t$:

$$\sum_{t \in T} \sum_{m \in M} \pi^e_t \cdot \delta'_m \left( \mu^r_{m,t} \cdot \Delta P^r_{m,t} - \mu^l_{m,t} \cdot \Delta P^l_{m,t} \right) \tag{5f}$$

[5]Note that the consumer subproblem, modelled via (3a), is general and independent of any load type. Yet to show the performance of our approach, here, we assume our consumers own a PV, a battery and a background load.

[6]This value can be updated every five minutes as our receding horizon moves forward.

where $\delta'_m$ is the worst case number of seconds we would need to be deployed for each market $m$ for a single contingency. The significance of these deployment costs shrink to zero as contingencies become more rare.

In our example, (5e), (5f) and the Lagrangian penalty term are the only values in the objective; however, other forms of DER might have additional costs associated with operation and there maybe other electricity charges such as network tariffs.

*2) DER Constraints:* We need to ensure the DER constraints are satisfied under participation in each market, including the case where no frequency support is required. We present the DER constraints for a single time step, using a placeholder variable $P \in \mathbb{R}^{|T|}$ to represent the power sent to the network. The DER variables and constraints are then duplicated for each scenario of interest, along with substituting the appropriate $P^e$, $P^r_m$, or $P^l_m$ variable for the placeholder.

*Solar PV:* Solar PV has a single variable $P^{PV}_t \in [0, P^F_t]$, which can be curtailed down to zero from the forecast available solar $P^F_t$.

*Battery Storage:* A battery has variables for charge and discharge powers $P^{Ch}_t, P^{Dis}_t \in [0, R]$, and for the state of charge $E_t \in [E^{min}, E^{max}]$. These are linked with the state of charge (SoC) at the previous time step through the equation:

$$E_t = E_{t-1} + \delta_t(\eta P^{Ch}_t - P^{Dis}_t/\eta) \tag{5g}$$

where $\eta$ is the battery efficiency and $\eta^2$ gives the round-trip efficiency. Note that since the FCAS bids are capacities, there is no guarantee that they are deployed. Therefore, the SoC constraint for raise and lower FCAS scenarios, uses the previous SoC form the energy market, i.e., $E_{t-1}$ in each time step.

*Combined Power:* The combined household power is then:

$$P^e_t = P^{Dis}_t - P^{Ch}_t + P^{PV}_t - P^U_t \tag{5h}$$

where we have included a parameter for the forecast household uncontrollable load $P^U_t \in \mathbb{R}$. Note that the uncertainty for both $P^U_t$ and $P^F_t$ are taken care of through our receding horizon implementation.

*3) The Combined EMS Subproblem:* In summary, the EMS subproblem for a single customer is to minimise the sum of (5e), (5f) and the associated Lagrangian penalty term (2). The constraints consist of the market linking constraints (5a–5d), and 7 copies of the DER variables and constraints (5g–5h), one for each market: $P^e$, $P^r_m$ and $P^l_m$, where $m \in \{1, 2, 3\}$.

### B. The Proposed Network Operation Model

Since there is no time coupling constraints in the network subproblem (unlike in the EMS subproblem due to the battery SoC constraint), here, we drop the index $t$ to increase readability. The objective value for the network subproblem consists of just the corresponding Lagrangian penalty term (2).

Our network model includes three OPFs: one for obtaining schedules when consumers participate in energy market $P^e$, and one for each extreme case $P^r$ and $P^l$. In the following,

we model the distribution network constraints, using distflow equations [21], for the energy market participation; we generate similar constraints for the maximum raise and lower cases.

We use $i, j, k \in N$ for nodes in a tree network; $F_i^e$, $Q_i^e$ and $I_i^e$ are the active power, reactive power and the current flowing into node $i$ from the parent node $k$, where the line has resistance $r_i$, reactance $x_i$ and impedance $z_i$. $D_i$ represents the children nodes of node $i$; and $C_i$ is the set of consumers at node $i$. The network constraints can be written as:

$$F_i^e - r_i I_i^e + \sum_{n \in C_i} P_n'^e = \sum_{j \in D_i} F_j^e \quad \forall i \in N \tag{6a}$$

$$Q_i^e - x_i I_i^e = \sum_{j \in D_i} Q_j^e \quad \forall i \in N \tag{6b}$$

$$V_i^e = V_k^e - 2\left(r_i F_i^e + x_i Q_i^e\right) + z_i^2 I_i^e \quad \forall i \in N \tag{6c}$$

$$v_{min}^2 \le V_i^e \le v_{max}^2 \quad \forall i \in N \tag{6d}$$

$$F_i^{e2} + Q_i^{e2} = V_i^e I_i^e \quad \forall i \in N \tag{6e}$$

$$0 \le I_i^e \le i_i^{max2} \quad \forall i \in N \tag{6f}$$

Active and reactive power flow equations are given through (6a)–(6b); The voltage of each node is calculated through (6c) and is enforced to be within its safe limits ($v_{min}^2$ and $v_{max}^2$) through (6d). The complex power, flowing in each line, is given in (6e) and finally, (6f) limits the current of each line to the maximum line capacity $i_i^{max2}$.

Remark: As suggested in [21], the Conic relaxed convexification of the network subproblem (6a)–(6f) can be obtained by replacing "=" with "≤" in (6e). However, the OPF results of such a relaxed problem is exact only if there is no upper bound limit on the voltages. In other words, when the voltage upper-bound limit is binding, the obtained results of such a relaxation do not lie within the feasible region. To avoid such an infeasible solution, here, we use the exact non-convex model (6a)–(6f) for our network sub-problem. As we show in the numerical results section, such a network model can be efficiently solved by the IPOPT solver and performs well within the ADMM context.

### C. Receding Horizon

To use the latest information and forecast (the most accurate PV power, residential demand and the wholesale energy and FCAS price forecasts), we apply our proposed network-aware co-optimisation approach within a receding horizon context. In this method, we run the proposed approach inline with the electricity market time-frame (i.e., every 5 minutes as in AEMO). This also allows us to update the SoC value in every re-optimisation (as the SoC might changed when consumers respond to the frequency deviations). Note that even though consumers act in the first five minutes of every re-optimisation, we solve a multi-period problem in every horizon. This ensures that the decisions in the first five minutes are not shortsighted.

## V. NUMERICAL RESULTS

For our experimental results we compare the costs and feasibility that our proposed approach achieves relative to 3

alternative approaches on a 69-bus distribution network with 207 consumers. We then apply our approach on a 141-bus distribution network with 1400 consumers to get a sense of how our distributed approach scales.

In both test networks we utilise 5 kW rooftop PV, and 5 kW / 10 kWh batteries with round-trip efficiencies of $\eta^2 = 90\%$, at a subset of the consumers. We use anonymised solar and demand data for 27 consumers in Tasmania, Australia, provided by Reposit Power [22], and randomly assign this data to the consumers in our networks. These consumers are then scaled in order to demonstrate performance during periods of network congestion. We take the 5 minute wholesale energy and FCAS prices from AEMO [23]. To clearly report our results, we use a 30 minute time granularity in sections V-A and V-B, while in section V-D we use 5-minute time granularity to investigate the performance of our variable time-discretisation receding horizon approach.

In all experiments, the ADMM penalty parameter $\rho$ is chosen to be 1 and we consider the problem to have converged when the infinity norms of the primal and dual residuals are both smaller than $10^{-5}$. The probability of a contingency event is considered to be 8% in every interval.[7] We use the Gurobi and IPOPT solvers in JUMP, Julia [24] to respectively solve the consumer and network subproblems on a laptop computer with a 2.50 GHz Intel[(R)] Core[(TM)] i7 and 8 GB of memory.

### A. 69-bus Distribution Network

We modify the 69-bus distribution network [25] with 3 consumers at each node. To obtain a more realistic case, 48 consumers are equipped with a rooftop PV and a battery (PV-BAT consumers); 48 consumers are equipped with just a rooftop PV but not a battery (PV consumers); and the remaining 111 consumers do not own any DER (NoDER consumers).

*1) Different approaches:* We compare the performance of the following four approaches:

1) Energy: in which consumers just participate in the energy market.
2) Seq: in which consumers sequentially participate in energy and FCAS markets, meaning that they first optimise their decisions in energy market, and then bid the rest of their capacity in the FCAS markets.
3) Co-opt: in which consumers co-optimise their decisions in energy and FCAS markets ignoring the network.
4) Proposed: in which the consumers use our network-aware co-optimisation approach to participate in both energy and FCAS markets.

Table I gives the average total cost of NoDER, PV, and PV-BAT consumers as well as the overall total cost, obtained using the above approaches for a day. Consumers owning DER (PV and PV-BAT) obtain larger benefits when participating in FCAS markets. The improvement in Co-opt and Proposed over

---

[7]Note that given the rare probability of event, 8% is a conservative estimate which provides a lower bound on consumers benefit. Thus in reality, the consumers are expected to obtain even more benefit than the ones reported in this section.

Seq demonstrates the value in co-optimising energy and FCAS participation. Co-opt is able to achieve the best objective; however, as discussed in Section V-A3, it violates the network constraints.

TABLE I: Overall cost and average consumer cost (AUD) for each approach.

| Approach | Overall | NoDER | PV | PV-BAT |
|----------|---------|-------|-----|--------|
| Energy | 1168.90 | 16.91 | -2.20 | -12.54 |
| Seq | -531.82 | 16.91 | -2.20 | -47.97 |
| Co-opt | -3140.60 | 16.91 | -5.80 | -98.66 |
| Proposed | -2982.84 | 16.91 | -5.58 | -95.85 |

*2) Energy and FCAS breakdown:* Fig. 1 breaks the costs down into their market components for the Seq, Co-opt and Proposed approaches. Since the Seq approach has no visibility on the highly-priced FCAS markets, it focuses on lowering costs in the energy market, which leads to a worse overall outcome.
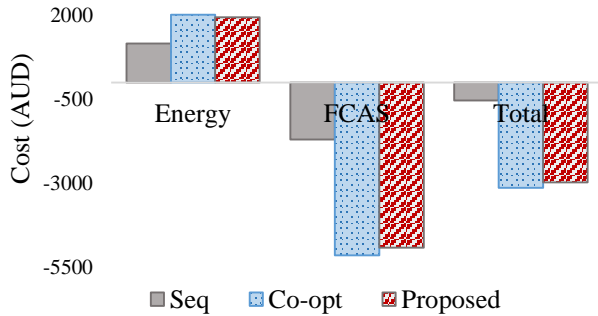


Fig. 1: Detailed cost/benefit in energy and FCAS markets.

*3) Network effect of DER:* To study the importance of accounting for network constraints, we check the network feasibility for the results obtained by the Co-opt approach (the case which neglects the network). To do so, we fix the connection point bid of consumers to the obtained results from the Co-opt approach, and solve three separate power flows (for energy, raise and lower) to see whether these bids satisfy the network constraints. We plot the voltage of node 27 in the energy case (normal operating condition) and when the raise FCAS is deployed (i.e. assuming a worst-case contingency occurs in each time step), for Co-opt and the Proposed approaches, in figures 2 and 3, respectively. As shown in these two figures, while the proposed network-aware co-optimisation approach managed to keep the voltage within the safe limits in all operating conditions, the Co-opt approach failed to do so in several time steps. This highlights the importance of providing network-aware bids.

To further show how the proposed network-aware co-optimisation approach works, we plot the energy bids of two similar batteries; one located at node 7 and the other at node 27 in Fig. 4. Without the proposed approach both batteries would bid similarly as they have similar price inputs. However, when consumers' decisions would violate the network constraints,
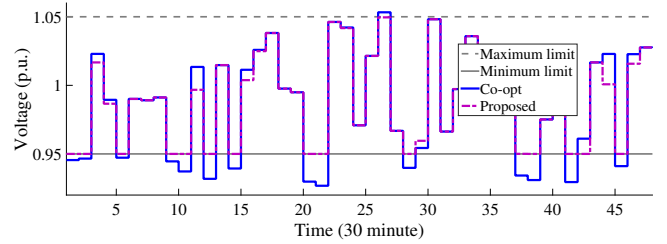


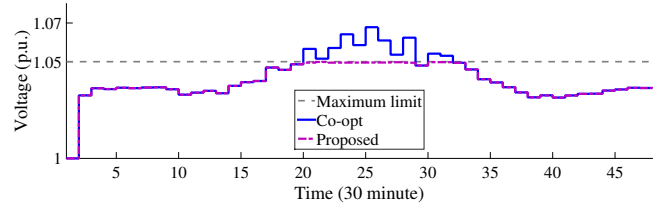Fig. 2: The voltage of node 27 in energy case.



Fig. 3: The voltage of node 27 when raise FCAS is deployed.

the proposed approach changes the LMPs to provide incentives for participants to change their schedules, leading to a difference in behaviour.
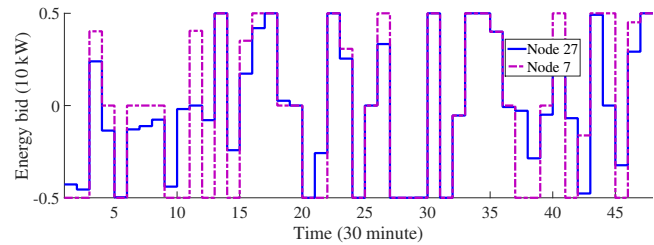


Fig. 4: Battery bids at node 7 vs. 27.

### B. 141-bus Distribution Network

To illustrate the scalability of the proposed approach, we modify a 141 bus distribution network [26] by allocating 10 consumers to each node except the root node.

Due to the decomposition we have chosen, within a single iteration all our EMS subproblems can be solved separately (either in parallel or sequentially), and similarly our network subproblem can be separated into an OPF for each time step[8] and each power flow case under consideration: energy, FCAS raise, FCAS lower. With a half-hourly time discretisation this results in 144 OPFs. In a practical setting we expect all our EMS problems to be solved in parallel on dedicated EMS hardware or in the cloud, while the OPFs could similarly be solved in parallel.

We report for a single horizon both the total computational time of our sequential implementation, and the expected fully parallel time (considering the slowest separated subproblems in each iteration). In a real setting, while the parallel computation, reported for the consumer subproblem, is representative, for the network subproblem, we expect a compromise between sequential and parallel solve times (depending on the available

---

[8]Since there is no time coupling constraints in the network subproblem.

cores). Therefore, the overall solve time stands somewhere between these two, and an additional overhead due to any communications latency. Table II reports the model size and solve time for the subproblems in a single horizon, and their contribution to the overall solve time in both sequential and parallel cases.

As can be seen in Table II, the total convergence time for a single horizon of our approach, in parallel computation, is 2.6s and 36.7s for our 69 and 141 bus distribution networks respectively. These times are reasonable, given the problem sizes as well as our strict convergence tolerance (i.e. $10^{-5}$) which represents at most 1 W error. Yet, in practice, we could run our approach with weaker tolerances in order to greatly reduce the number of iterations and thus overall time. In addition to this, when employed in a receding horizon context, warm-starting solutions can lead to further significant iteration reductions.

TABLE II: Problem size and computational time.

| System | Subprob. | #Var. | #Cons. | #Iter. | Time (Parallel) |
|--------|----------|-------|--------|--------|-----------------|
| 69-bus | EMS | 427k | 325k | 38 | 193s (1.1s) |
|        | Network | 69k | 39k |  | 205s (1.5s) |
| 141-bus | EMS | 2,889k | 2,200k | 268 | 7895s (7.1s) |
|         | Network | 284k | 81k |  | 3419s (29.6s) |

### C. Convergence of the Proposed Approach

The proofs for ADMM convergence typically require the problem to be convex [17]. In practice however, it converges and performs well with the non-convex network model [18]. Here, we use the exact non-convex network model (6a)–(6f) to avoid any inexactness that a convex relaxation (such as Conic relaxation [21]) might bring to our problem (especially in the hours with peak PV generation and reverse power flow). Despite using a non-convex network subproblem, in all of our simulations, our ADMM approach converged with reasonable number of iterations. The convergence of the primal and dual residuals for the 69-bus test system is plotted in Fig. 5.
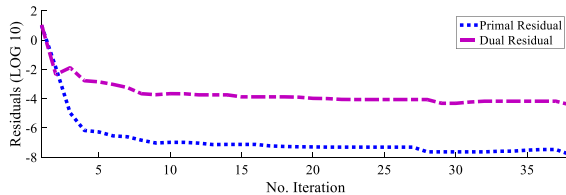


Fig. 5: The primal and dual residual convergence

### D. The Receding Horizon Implementation

As mentioned earlier, to mitigate the effect of uncertainty, the network and consumers negotiate every five minutes in a receding horizon framework. While the first time step (the first 5 minutes) is acted on, the other time steps are just to make sure that the decisions are not shortsighted. However, considering multiple time steps increases the size of the problem and the computational burden. Therefore, here we use the idea of a variable time-discretisation [4] and apply it to our multi-period problem. Rather than using constant 5 minute time steps, we use 5 minutes for the first time step and 30 minutes for the remaining steps, as shown in Fig. 6. This leads to a problem with 49 time steps per horizon, almost the same number as used in the previous section (i.e. 48), so we expect the computational results there to be representative.



Fig. 6: Time-discretisation of our receding horizon approach.

In our experiments, the variable time-discretisation technique reduced the convergence time by 300% with less than 2% increase in the total cost. Note that since the optimisation problem reruns every five minutes, it takes the latest battery SoC into account. Therefore, the feasibility of our decisions are guaranteed. In other words, such a variable time-discretisation approach does not lead to infeasible solutions, it just reduces the problem size at the expense of a higher cost. Fig. 7 reports the bids made in energy and raise FCAS markets obtained by the variable time-discretisation and five-minute yet constant time-discretisation approaches (shown by Variable and Constant, respectively) for our 69-bus network (lower FCAS bids were zero).
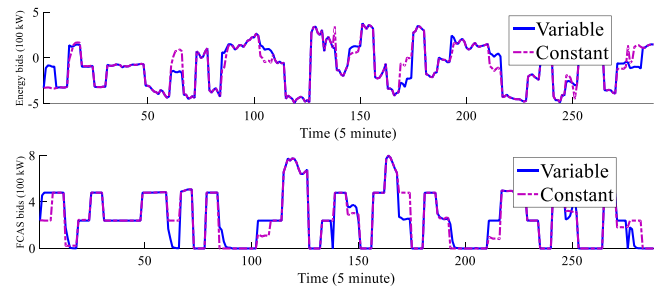


Fig. 7: The energy (top) and the raise FCAS (bottom) bids.

### E. Central Approach

In this section, we assume that the network has access to the required information of consumers to run a central OPF and schedule all the DER. This case study provides an insight on the problem size and the required time to solve such a complex co-optimisation problem without the proposed approach. Here, the network operator uses three AC-OPFs both to schedule consumers in different markets and to guarantee the network feasibility for any market participation (i.e, energy, raise and lower FCAS markets). Similar to our approach, we implement this case within a receding horizon framework to include the latest forecast information and increase the solution quality. Note that due to the time coupling constraints of DER (such

as battery SOC), each horizon needs to solve a multi-period AC-OPF. We use our variable time-discretisation technique, to reduce the size of this multi-period problem. Table III reports the problem size and the time taken to solve the problem centrally.

TABLE III: Problem size and computational time of the central approach.

| System | #Var. | #Cons. | Time (s) |
|--------|-------|--------|----------|
| 69-bus | 477k | 364k | 434.4 |
| 141-bus | 3,012k | 2,326k | 12,544.1 |

As reported in Table III, the central approach respectively took 434.4 s (7.3 min) and 12,544.1 s ($3^h, 30^{min}$) to solve our 68-bus and 141-bus test systems. The reported run-time highlights the limitations of applying a central approach to a bidding strategy with a 5- minute time-discretisation. However, as seen in table II, the parallel implementation of the proposed approach can solve such problem within few seconds.

## VI. CONCLUSION

We developed an ADMM-based approach in the distribution network to enable residential consumers to participate in both energy and FCAS markets. In our approach, the network and consumers negotiate frequently using the ADMM algorithm and converge on a consensus solution which does not violate consumers or the network constraints.

We first illustrated the effectiveness of our approach using 207 consumers, served through AEMO, within a 69-bus network. Our results show significant improvements over the case when the decisions in energy and FCAS markets are not co-optimised. Also, through a voltage analysis, we compared the voltages on the network when the co-optimisation neglects the network constraints with the proposed approach. The results revealed that neglecting the network can lead to infeasible solutions violating the voltage safe limits at different times of the day. Using 1400 consumers on a 141-bus network, we showed that the proposed approach is scalable as it distributes the computational burden on different components of the whole problem which can solved in parallel. To further improve the computational of the proposed approach, we enhanced our model to have variable time-discretisation across the horizon.

## ACKNOWLEDGEMENT

## REFERENCES

[1] AEMO and Energy Networks Australia 2018, "Open energy networks consultation paper," *Energy Networks Australia*, 2018.
[2] D. Zhu and Y.-J. A. Zhang, "Optimal coordinated control of multiple battery energy storage systems for primary frequency regulation," *IEEE Transactions on Power Systems*, vol. 34, no. 1, pp. 555–565, 2019.
[3] J. Iria, F. Soares, and M. Matos, "Optimal bidding strategy for an aggregator of prosumers in energy and secondary reserve markets," *Applied Energy*, vol. 238, pp. 1361–1372, 2019.
[4] P. Scott, D. Gordon, E. Franklin, L. Jones, and S. Thiébaux, "Network-aware coordination of residential distributed energy resources," *IEEE Transactions on Smart Grid*, 2019.
[5] M. Kazemi, H. Zareipour, N. Amjady, W. D. Rosehart, and M. Ehsan, "Operation scheduling of battery storage systems in joint energy and ancillary services markets," *IEEE Transactions on Sustainable Energy*, vol. 8, no. 4, pp. 1726–1735, 2017.
[6] B. Xu, A. Oudalov, J. Poland, A. Ulbig, and G. Andersson, "Bess control strategies for participating in grid frequency regulation," *IFAC Proceedings Volumes*, vol. 47, no. 3, pp. 4024–4029, 2014.
[7] Q. Zhai, K. Meng, Z. Y. Dong, and J. Ma, "Modelling and analysis of lithium battery operations in spot and frequency regulation service markets in australia electricity market," *IEEE Trans Ind Inform*, vol. 13, no. 5, pp. 2576–2586, 2017.
[8] B. Xu, Y. Shi, D. S. Kirschen, and B. Zhang, "Optimal battery participation in frequency regulation markets," *IEEE Transactions on Power Systems*, vol. 33, no. 6, pp. 6715–6725, 2018.
[9] E. Yao, V. W. Wong, and R. Schober, "Optimization of aggregate capacity of PEVs for frequency regulation service in day-ahead market," *IEEE Transactions on Smart Grid*, vol. 9, no. 4, pp. 3519–3529, 2018.
[10] S. Ø. Ottesen, A. Tomasgard, and S.-E. Fleten, "Multi market bidding strategies for demand side flexibility aggregators in electricity markets," *Energy*, vol. 149, pp. 120–134, 2018.
[11] J. P. Iria, F. J. Soares, and M. A. Matos, "Trading small prosumers flexibility in the energy and tertiary reserve markets," *IEEE Transactions on Smart Grid*, vol. 10, no. 3, pp. 2371–2382, 2018.
[12] M. G. Vayá and G. Andersson, "Optimal bidding strategy of a plug-in electric vehicle aggregator in day-ahead electricity markets under uncertainty," *IEEE Transactions on Power Systems*, vol. 30, no. 5, pp. 2375–2385, 2014.
[13] S.-J. Lee, J.-H. Kim, C.-H. Kim, S.-K. Kim, E.-S. Kim, D.-U. Kim, K. K. Mehmood, and S. U. Khan, "Coordinated control algorithm for distributed battery energy storage systems for mitigating voltage and frequency deviations." *IEEE Trans. Smart Grid*, vol. 7, no. 3, pp. 1713–1722, 2016.
[14] Y. Wang, H. Liang, and V. Dinavahi, "Two-stage stochastic demand response in smart grid considering random appliance usage patterns," *IET Generation, Transmission & Distribution*, vol. 12, no. 18, pp. 4163–4171, 2018.
[15] A. Attarha, P. Scott, and S. Thiébaux, "Affinely adjustable robust ADMM for residential der coordination in distribution networks," *IEEE Transactions on Smart Grid*, 2019.
[16] B. Vatandoust, A. Ahmadian, M. Golka, A. Elkamel, A. Almansoori, and M. Ghaljehei, "Risk-averse optimal bidding of electric vehicles and energy storage aggregator in day-ahead frequency regulation market," *IEEE Transactions on Power Systems*, 2018.
[17] S. Boyd, N. Parikh, E. Chu, B. Peleato, J. Eckstein *et al.*, "Distributed optimization and statistical learning via the alternating direction method of multipliers," *Foundations and Trends® in Machine learning*, vol. 3, no. 1, pp. 1–122, 2011.
[18] P. Scott and S. Thiébaux, "Distributed multi-period optimal power flow for demand response in microgrids," in *Proceedings of the 2015 ACM Sixth International Conference on Future Energy Systems*. ACM, 2015, pp. 17–26.
[19] S.-C. Tsai, Y.-H. Tseng, and T.-H. Chang, "Communication-efficient distributed demand response: A randomized ADMM approach," *IEEE Transactions on Smart Grid*, vol. 8, no. 3, pp. 1085–1095, 2017.
[20] "The Frequency operating standard," https://www.aemc.gov.au, 2017.
[21] M. Farivar and S. H. Low, "Branch flow model: Relaxations and convexification—part I," *IEEE Transactions on Power Systems*, vol. 28, no. 3, pp. 2554–2564, 2013.
[22] "Reposit Power," http://www.repositpower.com.
[23] "Australian Energy Market Operator." http://www.aemo.com.au.
[24] I. Dunning, J. Huchette, and M. Lubin, "Jump: A modeling language for mathematical optimization," *SIAM Review*, vol. 59, no. 2, pp. 295–320, 2017.
[25] J. Savier and D. Das, "Impact of network reconfiguration on loss allocation of radial distribution systems," *IEEE Transactions on Power Delivery*, vol. 22, no. 4, pp. 2473–2480, 2007.
[26] R. D. Zimmerman, C. E. Murillo-Sánchez, and R. J. Thomas, "Matpower: Steady-state operations, planning, and analysis tools for power systems research and education," *IEEE Transactions on power systems*, vol. 26, no. 1, pp. 12–19, 2010.