Smart Air-Conditioning Control by Wireless Sensors: An Online Optimization Approach

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ABSTRACT

One of the most prominent applications of smart technology for energy saving is in buildings, in particular, for optimizing heating, ventilation, and air-conditioning (HVAC) systems. Traditional HVAC systems rely on wired temperature regulators and thermostats installed at fixed locations, which are both inconvenient for deployment and ineffective to cope with dynamic changes in the thermal behavior of buildings. New generation of wireless sensors are increasingly becoming popular due to their convenience and versatility for sophisticated monitoring and control of smart buildings. However, there also emerge new challenges on how to effectively harness the potential of wireless sensors. First, wireless sensors are energy-constrained, because they are often powered by batteries. Extending the battery lifetime, therefore, is a paramount concern. The second challenge is to ensure that the wireless sensors can work in uncertain environments with minimal human supervision as they can be dynamically displaced in new environments. Therefore, in this paper, we study a fundamental problem of optimizing the trade-off between the battery lifetime and the effectiveness of HVAC remote control in the presence of uncertain (even adversarial) fluctuations in room temperature. We provide an effective offline algorithm for deciding the optimal control decisions of wireless sensors, and a 2-competitive online algorithm that is shown to attain performance close to offline optimal through extensive simulation studies. The implication of this work is to shed light on the fundamental trade-off optimization in wireless sensor controlling HVAC systems.

Categories and Subjects: [Computer systems organization]: *Embedded and cyber-physical systems*: *Sensors and actuators*

General Terms: Algorithms, Design, Management

Keywords: Wireless Sensors, Smart Buildings, HVAC, Air-Conditioning, Online Algorithms

1. INTRODUCTION

Buildings are among the largest consumers of energy, topping 40% of total energy usage in many countries [9]. A significant portion of energy use in buildings is attributed to the heating, ventilation, and air conditioning (HVAC) systems, which account for up to 50% of the total energy consumption in buildings [7]. Therefore, improving energy efficiency of buildings, in particular, optimizing HVAC system is critically important and will have a significant impact in reducing the overall energy consumption.

Usually, the air conditioning systems need to maintain room temperature within a certain desirable range. To detect the variations of temperature, traditional air conditioning systems rely on wired temperature regulators and thermostats installed at fixed locations. These classical controllers are both inconvenient for deployment and ineffective to cope with dynamic changes in the thermal behavior of buildings. In particular, the temperature distribution is not spatially uniform. Having sensors installed at fixed and limited locations cannot react to the rapidly varying room conditions due to transient and non-stationary human behavior.

New generation of wireless sensors are revolutionizing the design of HVAC systems. Wireless sensors, being not limited by wired installation, can be deployed strategically close to the fluctuating thermal sources in an ad hoc fashion (e.g., near to doors, windows and computers). With wireless sensors, demand responsive air-conditioning control can be developed that dynamically adjusts the room temperature according to intelligent monitoring and tracking of human behavior and room conditions. Furthermore, wireless sensors can be integrated with home security and infotainment systems, enabling more sophisticated smart home control systems.

Despite the promising potential, wireless sensors also introduce several new challenges:

1. **Battery Lifetime**: Wireless sensors are often battery-powered and typically have to operate for prolonged periods of time. Therefore, one of the primary goals is to maximize the battery lifetime of sensors. According to a survey of several commercial wireless sensors (see Appendix-C), the communication operations consume the most energy. Thus, an effective way to extend battery lifetime is to reduce the communication frequency, inducing limited communication among wireless sensors.

2. **Control Effectiveness**: Wireless sensors are also dis-

tributed autonomous computing devices. They can be programmed to intelligently optimize their energy consumption with respect to the effectiveness of their control operations. Intuitively, energy consumption is inversely proportional to the effectiveness (i.e., sleeping all the time can effectively reduce energy consumption, but is ineffective to satisfy the control requirement). The ability to balance the energy consumption and effectiveness is critical to the usefulness of these wireless sensors, particularly for smart home applications.

3. **Uncertain Deployment**: Wireless sensors are supposed to be deployed in an ad hoc fashion, without a-prior measurement or calibration. It is critical to ensure that wireless sensors operate robustly and reliably in the presence of uncertainty of new environments. They should be able to rapidly cope with dynamic displacements with minimal human supervision. An important question is to investigate the fundamental ability of wireless sensors to control room temperature without assuming any a-prior or stochastic knowledge of the temperature fluctuations.

In this paper, we study a fundamental problem of optimizing the trade-off between the lifetime of the wireless sensors and the effectiveness of HVAC remote control in the presence of uncertain (even adversarial) fluctuations in room temperature. The novelty of our work lies in the fact that unlike most intelligent HVAC control techniques (as summarized in the related work section), our approach is to solve the optimization problem in an online manner without stochastic modeling or machine learning methods. The key contributions of this work are summarized as follows.

1. We formulate a new online optimization problem of balancing the trade-off between communication frequency of wireless sensor and the effectiveness of HVAC remote control. Our goal is to simultaneously maintain thermal comfort and maximize the battery lifetime of the wireless sensor. In other words, we aim to maximize the sensor energy efficiency while meeting the required control performance. To the best of our knowledge, this specific problem has not been studied before.

2. We present an effective offline algorithm, which is based on dynamic programming, for determining the optimal control decisions by wireless sensors when all future temperature fluctuations are known in advance. The offline algorithm is useful to benchmark the online algorithm we propose.

3. We devise an online algorithm that optimizes the control decisions without the knowledge about future temperature fluctuations. We prove that our online algorithm is 2-competitive against offline optimal algorithm.

4. We experimentally evaluate the performance of our algorithm through simulations and show that our online algorithm can attain performance close to the offline optimal solution.

The rest of the paper is organized as follows. In Section 2, we present the background of online algorithmic approach, competitive analysis, and a related problem known as dynamic TCP acknowledgement problem. We present the models and

formulations of ambient room temperature and wireless sensor network control in Section 3. In Section 4, we provide the offline and online algorithms and competitive analysis. In Section 5, we evaluate the performance of our algorithms through extensive simulations. In Section 6, we present a review of related work. Finally, we summarize and discuss several future extensions in Section 7.

2. BACKGROUND

In this section, we present the background information about online algorithms and a well-known online problem known as dynamic TCP acknowledgment problem, which is closely related to our problem.

2.1 Online Algorithms

Online algorithms have received considerable attention in the literature for their fundamental principles and practical applications. In an online problem, a sequence of input is revealed gradually over time. The algorithm needs to make certain decisions and generate output instantaneously over time, based on only the part of the input that has been seen so far, without knowing the rest of the input to be revealed in the future. There are many practical problems studied in the online algorithmic setting that require real-time and instantaneous decisions, such as real-time resource allocation in operating systems, data structuring, robotics or communication networks [1,8]. The performance of online algorithms is evaluated using competitive analysis. The *competitive ratio* of an online algorithm is defined as the worst-case ratio between the cost of the solution obtained by the online algorithm versus that of an offline optimal solution obtained by knowing the all input sequence in the future [19].

Online algorithms have several practical implications. First, they do not require a-prior or stochastic knowledge of the input sequence, which makes them robust in any uncertain (even adversarial) environments. Second, online algorithms uses often simple decision-making mechanisms, without being hampered by inaccurate or slow convergent machine learning techniques. Third, online algorithms can give a fundamental characterization without further assumptions of the problems, which are useful to benchmark other sophisticated and more complicated decision-making mechanisms. In this paper, we adopt the online algorithmic approach to study the fundamental problem of optimizing the trade-off between the battery lifetime and the effectiveness of HVAC remote control in the presence of uncertain fluctuations in room temperature.

2.2 Dynamic TCP Acknowledgment

A well-known example involving online algorithms is the dynamic TCP acknowledgment problem as described as follows. A stream of packets arrives at a destination. The packets must be acknowledged in order to notify the sender that the transmission was successful. However, it is possible to simultaneously acknowledge multiple packets using a single acknowledgments packet. The delayed acknowledgment mechanism reduces the frequency of the acknowledgments, but it might also add excessive latency to the TCP connection and interfere with the TCP's congestion control mechanisms [10]. The problem is to find an optimal trade-off between the total number of acknowledgments sent and the latency cost introduced due to delaying acknowledgment.

More specifically, Dooly et al. [6] formulated this trade-off as the dynamic TCP acknowledgement problem as follows.

In the dynamic TCP acknowledgement problem, a sequence of n packets $\sigma = (p_1, p_2, ..., p_n)$ arrive at a certain destination. An algorithm divides the received sequence σ into m subsequences $\sigma_1, \sigma_2, ..., \sigma_m$, where a single acknowledgment is sent at the end of each subsequence. All the packets contained in $\sigma_j (1 \leq j \leq m)$ are acknowledged together by the j-th acknowledgement at time t_j. The objective is to choose an optimal acknowledgment time sequence that minimizes the weighted sum of the cost for transmitting acknowledgements and the cost of the latency of delayed acknowledgements. The decision of transmitting an acknowledgment time is decided in an online fashion without knowing the future packet arrivals.
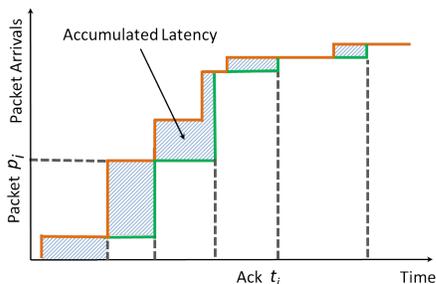
(a) Dynamic TCP acknowledgment
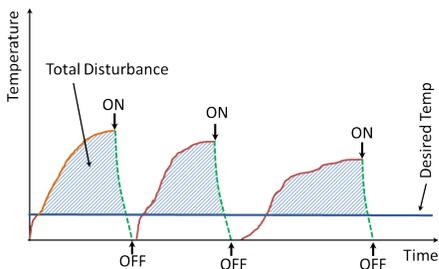
(b) Wireless sensor controlling AC system

Figure 1: A pictorial comparison between dynamic TCP acknowledgment and wireless sensor controlling AC system.

Comparison to Our Problem: Our problem is somewhat similar to the dynamic TCP acknowledgment problem. In TCP, random arrivals of packets are received, such that the receiver makes online decisions when to transmit acknowledgments considering the weighted total cost of number of acknowledgment and latency. In our problem, random fluctuations of temperature and external thermal sources are perceived by the wireless sensor, and the wireless sensor makes online decisions when to transmit control commands to remote air conditioning system considering the weighted total cost of transmissions and effectiveness (defined by the disturbance of temperature compared to a desirable temperature). A pictorial comparison between the two problems is provided in Fig. 1.

Despite the similarity, our results are not direct applications of the dynamic TCP acknowledgment problem. In particular, the dynamic TCP acknowledgment problem assumes latency as a linearly increasing function of time, whereas in our problem the total disturbance of temperature changes non-linearly with time. This requires a non-trivial extension of the original TCP acknowledgment problem to the new context of air-conditioning control. Furthermore, we present extensive simulation studies that are specific to the air-conditioning control setting for corroborating the usefulness of our online algorithms for this new problem.

3. MODEL AND FORMULATION

The goal of our study is to optimize the trade-off between the wireless sensor battery lifetime and the effectiveness of ambient room temperature control in the presence of uncertain fluctuations. In this section, we present the models of ambient room temperature and wireless sensor control. We note that a table of notations with explanations is provided in Table 5 in the Appendix. It is worth mentioning that we make several assumptions in order to improve the tractability of our models and for convenience of analysis.

3.1 Assumptions of Ambient Room Temperature

The thermal behavior of buildings is a complex system. The mathematical models in the literature typically involve several empirical constants, non-linear functions and uncertain factors such as heat flow and material properties [16]. Moreover, external factors, such as weather condition (e.g., temperature, humidity), soil temperature, radiation effects and other sources of energy (e.g., human activities, lighting and equipment), also play a critical role in determining the thermal behavior of buildings [16].

Tractable mathematical models of building thermal behavior are particularly useful for the design of intelligent controls and regulations of HVAC systems. Therefore, assumptions are often imposed to improve the tractability of the thermal models of buildings.

In this work, we employ a simple yet commonly used thermal model for a single room. This model considers several major factors, such as the outdoor environment, the thermal characteristics of the room, and the air-conditioning system. We mostly consider the setting of cooling, where the air-conditioning system is required to make continual adjustment to the room temperature for maintaining a (lower) desirable temperature level. We remark that our results can be applied to the setting of heating with minor modifications.

First, we list several common assumptions of the ambient room temperature in the literature [21] for improving the tractability:

- The air in the room is assumed to be fully mixed.
- The temperature distribution is assumed to be uniform and the dynamics can be expressed using a lump capacity model.
- The room behaves ideally, such that the effect of each wall is uniformly equivalent.
- The density of the air is constant and is not affected by the changes in temperature and humidity.

3.2 Dynamic Model of Ambient Room Temperature

Based on the above assumptions, a simple dynamic model of ambient room temperature can be formulated as follows.

We consider the setting of continuous time, and model the ambient room temperature at time t by a function $T(t)$, which depends on several major factors:

1. The *initial ambient room temperature* T_0 at time $t = 0$.

2. The *influence of outdoor temperature* $T_{od}(t)$, which is a function of time affected by time-of-day and weather. A simple example is a sinusoidal function depending on the time-of-day. We assume that the variation of $T_{od}(t)$ is relatively slow, as compared to the effect of air-conditioning system. Hence, we simply write $T_{od}(t)$ as a constant T_{od}.

3. The *external thermal sources* entering into the room, for example, due to human body heat or human activities (e.g., computers). We model the arrivals of thermal sources by a function $W(t)$, such that there is a level of thermal intensity $W(t)$ (measured by degree Celsius) arriving at time t.

4. The *heat absorptivity and insulation properties* of the materials in a room (e.g., walls). Heat can be retained in a room for a longer period of time in a well-insulated room with sufficiently absorptive materials.

5. The *air-conditioning system output.* This is the control variable we seek to optimize in order to maintain the ambient room temperature within a desirable range.

a) **Without external thermal sources**: Throughout this paper, we rely on a widely-used model of dynamic ambient room temperature [5]. First, we assume that there is no external thermal sources entering into the room (i.e., $W(t) = 0$ for all t). In particular, we denote the ambient room temperature without external thermal sources as $\tilde{T}(t)$. Given the initial ambient room temperature T_0 and outdoor temperature T_{od}, the dynamic behavior of $\tilde{T}(t)$ can be described by the following differential equations

$$\frac{d\tilde{T}(t)}{dt} = \frac{1}{c \cdot M_{air}} \cdot \left(\frac{dQ_{in}(t)}{dt} - \frac{dQ_{ac}(t)}{dt} \right) \tag{1}$$

$$\frac{dQ_{in}(t)}{dt} = \frac{T_{od} - \tilde{T}(t)}{R_{eq}} \tag{2}$$

$$\frac{dQ_{ac}(t)}{dt} = \frac{c \cdot M_{ac} \cdot (\tilde{T}(t) - T_{ac})}{E_{ac}} \tag{3}$$

where T_{ac} is the temperature output by the air-conditioning system, $Q_{in}(t)$ is the net heat transfer from outdoor, $Q_{ac}(t)$ is the net heat chilled by the air-conditioning system, M_{air}, M_{ac}, E_{ac}, c, R_{eq} are constants that model the heat absorptivity and insulation properties in the room (see Appendix for full explanations). By substitution, one can solve the differential equations by the following lemma.

LEMMA 1. *In the above model, the solution to Eqns. (1)-(3) is given by*

$$\tilde{T}(t) = \frac{C_1}{C_2} - \left(\frac{C_1}{C_2} - \tilde{T}(0) \right) \cdot e^{-C_2 \cdot t} \tag{4}$$

where

$$C_1 = \frac{c \cdot T_{ac} \cdot M_{ac} \cdot R_{eq} + E_{ac} \cdot T_{od}}{c \cdot M_{air} \cdot R_{eq} + E_{ac}} \tag{5}$$

$$C_2 = \frac{E_{ac} + c \cdot M_{ac} \cdot R_{eq}}{c \cdot E_{ac} \cdot M_{air} \cdot R_{eq}} \tag{6}$$

We provide the proof in Appendix-A.

b) **With external thermal sources**: Next, we consider the setting with external thermal sources. We consider $W(t)$ as a sequence of *impulsive thermal sources*, such that

$$W(t) = \sum_{i=1}^{m} w_i \cdot \delta(t - t_i) \tag{7}$$

where $\delta(t)$ is Dirac delta function, and w_i is the level of thermal intensity entering into the room at time t.

Impulsive thermal sources are a reasonable assumption for modeling short-lived thermal sources (e.g., temporarily opening a door). Further, any arbitrary $W(t)$ can be approximated by a sequence of appropriately placed impulsive thermal sources by taking $w_i = W(t_i)$ (see Fig. 2 for an illustration). Note that, in this paper, we do not assume any a-priori knowledge of the stochastic property of $W(t)$. We
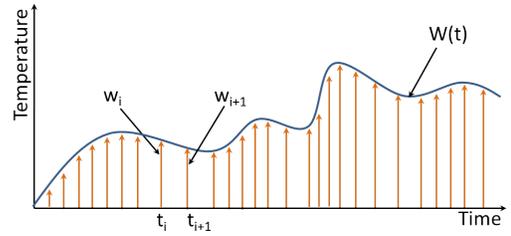
Figure 2: **An illustration for using impulsive heat sources to approximate arbitrary** $W(t)$.

denote $\mathbf{a} \triangleq ((w_i, t_i) : i = 1, ..., m)$ for a sequence of arrivals of impulsive thermal sources, where m is the total number of arrivals. Given $\mathbf{a}$, the ambient room temperature at time t can be obtained recursively as follows. For $i \in \{1, ..., m\}$, we note that there is no external thermal source during interval $t_{i-1} < t < t_i$. We denote the ambient room temperature during interval $t_{i-1} \le t < t_i$ by $\tilde{T}_i(t)$. Thus, following by Lemma 1, we obtain

$$\tilde{T}_i(t) = \frac{C_1}{C_2} - \left(\frac{C_1}{C_2} - \tilde{T}_{i-1}(t_{i-1}) - w_{i-1} \right) \cdot e^{-C_2 \cdot (t - t_{i-1})} \tag{8}$$

where $\tilde{T}_{i-1}(t_{i-1}) + w_{i-1}$ is the initial temperature at t_{i-1}.

For completeness, we let $t_0 = 0$, $w_0 = 0$ and $\tilde{T}_0(t_0) = T_0$. Hence, we obtain the ambient room temperature for given external thermal sources $\mathbf{a}$ and initial ambient room temperature T_0 as

$$T(t; \mathbf{a}, T_0) = \tilde{T}_i(t), \text{ if } t_{i-1} \le t < t_i \tag{9}$$

3.3 Model of Wireless Sensor Control

To model wireless sensor control, we consider a wireless sensor deployed in the target zone for sensing the ambient temperature. The wireless sensor issues control commands to a remote air-conditioning system when the locally sensed ambient temperature exceeds a certain desirable temperature range. There are several issues considered in our sensor model.

a) **Trade-off**: Since wireless sensors are energy constrained and often powered by batteries, the wireless sensor is required to optimize the battery lifetime without affecting the thermal comfort. Although various operations are performed in wireless sensors (e.g., computations and sensing), the wireless communication operations typically consumes

most of the energy in a wireless sensor (see Appendix-C). Hence, it is crucial to reduce the number of wireless communication operations for extending the battery lifetime.

There are two prominent conflicting factors that a wireless sensor needs to optimize:

1. The *update frequency* of control commands to remote air-conditioning system in the presence of random fluctuating thermal sources, which characterizes the effectiveness of ambient room temperature control.

2. The *communication operations* for transmitting the control commands, which critically governs the wireless sensor battery lifetime.

Note that increasing of the number of communication operations will reduce the battery lifetime. This naturally gives rise to an online decision problem, where the wireless sensor decides the update frequency in an online manner without a-prior information of random fluctuating arrivals of thermal sources.

b) Air-conditioning Operations: Let $T_{\text{des}}^{\max}$ be the maximally desirable temperature (e.g., 25 degree Celsius), and $T_{\text{des}}^{\min}$ be the minimally desirable temperature (e.g., 21 degree Celsius). The desirable ambient room temperature is aimed to be retained within $[T_{\text{des}}^{\min}, T_{\text{des}}^{\max}]$.

A simple setting of control command by wireless sensor is the "ON/OFF" or hysteresis control, such that when the ambient room temperature is sufficiently higher than $T_{\text{des}}^{\max}$, an ON command is communicated to air-conditioning system, whereas when the sensed ambient room temperature is sufficiently lower than $T_{\text{des}}^{\min}$, an OFF command is communicated to air-conditioning system[1]. This induces an ON/OFF cycle of air-conditioning operations (see Fig. 3 for an illustration), which is one of the most commonly used control strategy in today's air-conditioning systems [13].
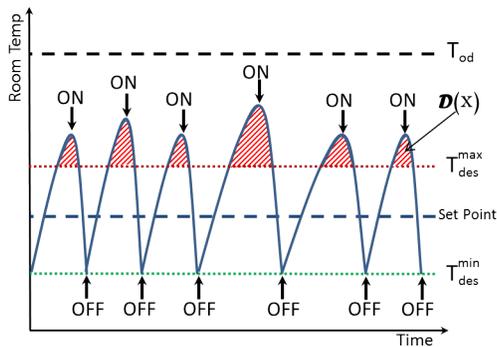
Figure 3: An illustration of the ON/OFF cycle of air-conditioning. Note that we may allow the ambient room temperature to exceed $T_{\text{des}}^{\max}$ temporarily.

Furthermore, for the sake of tractability, we assume that an OFF command is automatically issued when the ambient room temperature drops below $T_{\text{des}}^{\min}$, and the cooling process is rather efficient, i.e.,cooling can be achieved in a relatively short time. However, we may allow the ambient room temperature to exceed $T_{\text{des}}^{\max}$ temporarily. Hence, our study is simplified to only optimize the ON command decisions in order to balance the trade-off between the wireless sensor

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[1]In our ambient room temperature model, the air-conditioning system can be disabled by letting $M_{\text{ac}} = 0$

battery lifetime and the effectiveness of ambient room temperature control, without considering the OFF commands.

We consider a finite time horizon for any $t \in [0, B]$. We define the *decision variables* as $\mathbf{x} = (x_k \in [0, B])_{k=1}^K$, where each x_k is the time that the k-th ON command is issued by the wireless sensor, while K is the total number of ON commands which the wireless sensor needs to optimize without affecting the thermal comfort.

c) Disturbance of Temperature: We characterize the thermal comfort by a metric defined as the total disturbance of ambient temperature exceeding the desirable temperature range.

For given time τ, we let $\mathbf{a}_\tau$ be the sub-sequence, such that

$$\Big((w_i, t_i - \tau), (w_{i+1}, t_{i+1} - \tau), (w_{i+2}, t_{i+2} - \tau), ...\Big) \quad (10)$$

where t_i is defined such that $t_{i-1} < \tau \le t_i$. Namely, $\mathbf{a}_\tau$ is a truncated sequence of $\mathbf{a}$ starting at τ.

We define $T_\tau(t)$ to be the temperature function $T(t; \mathbf{a}, T_0)$ starting at time τ with initial temperature $T_0 = T_{\text{des}}^{\min}$ and sequence of thermal sources $\mathbf{a}_\tau$. That is, for any $t \ge \tau$,

$$T_\tau(t) \triangleq T\Big(t - \tau; \mathbf{a}_\tau, T_{\text{des}}^{\min}\Big) \quad (11)$$

Hence, the total *disturbance* given decision variables $\mathbf{x}$ is defined by (also shown in Fig. 3)

$$\mathcal{D}(\mathbf{x}) \triangleq \sum_{k=1}^K \int_{t=x_k}^{x_{k+1}} [T_{x_k}(t) - T_{\text{des}}^{\max}]^+ dt \quad (12)$$

where $[x]^+ = \max(x, 0)$ and $T_{\text{des}}^{\max}$ is the maximal desirable temperature threshold.

DEFINITION 1. *Formally, we define the decision problem for wireless sensor controlling air-conditioning (WSAC) as follows:*

WSAC problem:

$$\min_{\mathbf{x}} \ \text{Cost}(\mathbf{x}) \triangleq \min_{\mathbf{x}} \ \eta \cdot K + (1 - \eta) \cdot \mathcal{D}(\mathbf{x}) \quad (13)$$

where $\eta \in [0, 1]$ is a weight assigned to balance the update frequency and the thermal comfort.

In the offline decision setting, $\mathbf{x}$ is decided given a-priori information of $\mathbf{a}$ and T_{od} without any restriction; whereas in the online decision setting, we require $\mathbf{x}$ to be decided such that x_k only considers the thermal sources before time x_k: $\{(w_i, t_i) \mid t_i \le x_k\}$.

Let $\mathbf{x}^*$ be the offline optimal solution to WSAC problem, while $\mathbf{x}_\mathcal{A}$ is the output solution given by an online algorithm $\mathcal{A}$. We define the competitive ratio as

$$\text{CR}(\mathcal{A}) \triangleq \max_{\mathbf{a}, T_{\text{od}}} \frac{\text{Cost}(\mathbf{x}_\mathcal{A})}{\text{Cost}(\mathbf{x}^*)} \quad (14)$$

In our problem, we seek to find an optimal online algorithm $\mathcal{A}$ to solve WSAC problem with the minimal $\text{CR}(\mathcal{A})$.

4. RESULTS

In this section, we provide an effective offline algorithm to solve WSAC problem, and a 2-competitive online algorithm.

4.1 Offline Algorithm

While the rest of paper considers online algorithm, we first devise an effective offline algorithm to solve WSAC problem

based on dynamic programming. The ramifications are that (1) the offline algorithm will enable us to compute the competitive ratio under diverse simulation settings; (2) the offline algorithm is useful in the setting with predictable $\mathbf{a}$. For example, based on the past history and statistics of $\mathbf{a}$, one can effectively solve WSAC problem by offline algorithm.

In the offline decision setting, we assume that all future temperature fluctuations are given in advance. We present our offline algorithm ($\mathcal{A}_{\text{OFL}}$) in Algorithm 1 that gives an optimal solution to WSAC problem.

Algorithm 1 Optimal Offline Algorithm $\mathcal{A}_{\text{OFL}}$, Input(a)

1: $\text{Cost}_{\min}[0] \leftarrow 0$
2: $\text{Cost}[1,1] \leftarrow 1 \cdot \eta + (1-\eta) \cdot \left[\int_{t=0}^{t_1} [T_{t_0}(t) - T_{\text{des}}^{\max}]^+ \mathrm{d}t \right]$
3: $\text{Cost}_{\min}[1] \leftarrow \text{Cost}[1,1]$, $\text{idx}[1] \leftarrow 1$
4: **for** $i \in [2,m]$ **do**
5:      **for** $j \in [1,i]$ **do**
6:          $\text{Cost}[i,j] \leftarrow 1 \cdot \eta$
                 $+ (1-\eta) \cdot \left[\int_{t=t_{i-j}}^{t_i} [T_{t_{i-j}}(t) - T_{\text{des}}^{\max}]^+ \mathrm{d}t \right]$
                 $+ \text{Cost}_{\min}[i-j]$
7:          **if** $\text{Cost}[i,j] < \text{Cost}_{\min}[i]$ **then**
8:              $\text{Cost}_{\min}[i] \leftarrow \text{Cost}[i,j]$
9:              $\text{idx}[i] \leftarrow j$
10:          **end if**
11:      **end for**
12: **end for**
13: $y_1 \leftarrow t_m$, $k' \leftarrow 1$, $r \leftarrow m$ ▷ *backtrack to find* $\mathbf{x}^*$
14: **while** $r > 1$ **do**
15:      $r \leftarrow r - \text{idx}[r]$, $k' \leftarrow k' + 1$
16:      $y_{k'} \leftarrow t_r$
17: **end while**
18: $K \leftarrow k'$
19: Output $(x_k = y_{K-k+1})_{k=1}^{K}$

The basic idea of $\mathcal{A}_{\text{OFL}}$ is based on dynamic programming, which relies on solving a sub-problem to decide when the previous ON command should be transmitted, assuming all the previous ON commands can be decided optimally.

Recall that t_i is the arrival time of the i-th external thermal source in sequence $\mathbf{a}$. Let $\text{Cost}[i,j]$ be the minimum cost when the last ON command is transmitted at time t_i and the second to last ON command is transmitted at time t_{i-j}, over all possible $\mathbf{x}$ with fixed $x_K = t_i$ and $x_{K-1} = t_{i-j}$. Also, let $\text{Cost}_{\min}[i]$ be the minimum cost when the last ON command is transmitted at time t_i. We note that $\text{Cost}[i,j]$ and $\text{Cost}_{\min}[i]$ can be computed recursively in Algorithm 1.

Once $\text{Cost}_{\min}[m]$ is found, the optimal decision $\mathbf{x}^*$ can be determined by backtracking. To enable backtracking, we maintain indices $\text{idx}[i]$ to record j when $\text{Cost}_{\min}[i] \leftarrow \text{Cost}[i,j]$.

THEOREM 1. $\mathcal{A}_{\text{OFL}}$ *in Algorithm 1 outputs an optimal solution to WSAC problem*

PROOF. The proof can be achieved in two steps.

     (i) WSAC problem exhibits the optimal sub-structure property;

     (ii) $\mathcal{A}_{\text{OFL}}$ explores all sub-problems and thus gives an optimal solution.

To prove (**i**), we consider a subsequence of thermal sources

$$\Big((w_1, t_1), (w_2, t_2), ..., (w_i, t_i), \Big) \qquad (15)$$

where the last ON command is transmitted at time $x_k = t_i$. Let us assume that we know that (perhaps told by an oracle) the second to last ON command is transmitted after the $(i-j)$-th arrival of thermal sources (i.e., $x_{k-1} = t_{i-j}$) is optimal, then we only need to optimize the subsequence $\Big((w_1, t_1), (w_2, t_2), ..., (w_{i-j}, t_{i-j}) \Big)$ in order to obtain the full optimal solution. Thus, the problem exhibits the optimal sub-structure property.

To prove (**ii**), we need to examine the execution of $\mathcal{A}_{\text{OFL}}$. We note that there are two FOR-loops. For each iteration of the outer loop (i.e., upon arrival of each new thermal source), the inner loop is executed from start to i (i.e., all subsequences in $\Big((w_1, t_1), (w_2, t_2), ..., (w_i, t_i), \Big)$ are traversed). This process is repeated for each new thermal source until we reach the end of the sequence. By doing so, $\mathcal{A}_{\text{OFL}}$ is able to explore all subsequences and, therefore, all sub-problems. □

4.2 Online Algorithm

In this section, we present a deterministic online algorithm that optimizes the trade-off between the frequency of ON commands and the thermal comfort. Our online algorithm achieves so by balancing the cost of transmitting the ON command immediately with the cost of delaying the ON command.

We assume that a wireless temperature sensor continuously tracks the change of temperature. Without the arrival of external thermal sources, the change in ambient temperature occurs smoothly as given by the differential equations Eqns. (1)-(3). However, when there is an arrival of external thermal source, the wireless sensor will be able to detect a sudden spike (because we assume impulsive thermal sources) in temperature, and hence, infer the arrival time of thermal source.

Recall that the j-th thermal source arrives at t_j. Let

$$\sigma_k \triangleq \{ i \in \{1, ..., m\} \mid x_{k-1} < t_i \le x_k \} \qquad (16)$$

Namely, σ_k is the set of thermal sources arrived between the $(k-1)$-th and the k-th ON commands. Upon each new arrival of thermal source, our online algorithm sets a timer such that the total cost (i.e., sum of transmission and disturbance costs) for σ_k if an ON command is transmitted immediately is equal to the disturbance cost for σ_k if an ON command is transmitted after waiting for some time τ.

To be specific, suppose the last ON command is transmitted at time x_k. We decide the transmission time of the next ON command (x_{k+1}). The cost incurred if an ON command is transmitted immediately (i.e., at time t_j) is given by

$$\eta + (1-\eta) \cdot \int_{t=x_k}^{t_j} [T_{x_k}(t) - T_{\text{des}}^{\max}]^+ \mathrm{d}t \qquad (17)$$

On the other hand, the total cost if an ON command is transmitted after waiting for time τ (i.e., at $t_j + \tau$) is given by

$$(1-\eta) \cdot \left[\int_{t=x_k}^{t_j} [T_{x_k}(t) - T_{\text{des}}^{\max}]^+ \mathrm{d}t + \int_{t=t_j}^{t_j+\tau} [T_{x_k}(t) - T_{\text{des}}^{\max}]^+ \mathrm{d}t \right] \qquad (18)$$

Equating Eqn. (17) and Eqn. (18), we obtain τ as a solution to the following equation.

$$\frac{\eta}{(1-\eta)} = \int_{t=t_j}^{t_j+\tau} [T_{x_k}(t) - T_{\text{des}}^{\max}]^+ \mathrm{d}t \qquad (19)$$

However, if there is an arrival of a new thermal source (at t_{j+1}) before timer expires, then we have to reset the timer and obtain a new τ as a solution to the following equation.

$$\frac{\eta}{(1-\eta)} = \int_{t=t_j}^{t_{j+1}+\tau} [T_{x_k}(t) - T_{\text{des}}^{\max}]^+ \mathrm{d}t \qquad (20)$$

Thus, upon each new arrival, we increment the upper integration limit in Eqn. (20) and get a new τ. The complete algorithm is presented in Algorithm 2 ($\mathcal{A}_{\text{ONL}}$).

Algorithm 2 OnlineAlgorithm $\mathcal{A}_{\text{ONL}}$, Input(t_{now})

1: Global variables: τ, timer
2: Initialization: $\tau \leftarrow 0$, timer $\leftarrow 0$

3: **if** $t_{\text{now}} >$ timer **then**   ▷ *upon the beginning or after each OFF command*
4:     Find τ such that

$$\frac{\eta}{(1-\eta)} = \int_{t=t_{\text{now}}}^{t_{\text{now}}+\tau} [T_{x_k}(t) - T_{\text{des}}^{\max}]^+ \mathrm{d}t$$

5:     timer $\leftarrow t_{\text{now}} + \tau$
6: **end if**
7: **if** $t_{\text{now}} =$ timer **then**   ▷ *timer has expired*
8:     Transmit an ON command
9: **else if** $t_{\text{now}} <$ timer **then** ▷ *timer has not expired yet*
10:     **if** j-th new thermal source is detected at t_{now} **then**
11:       Let t_j be the time after the last ON command
12:       Find τ such that   ▷ *decrease the timer due to new thermal source*

$$\frac{\eta}{(1-\eta)} = \int_{t=t_j}^{t_{\text{now}}+\tau} [T_{x_k}(t) - T_{\text{des}}^{\max}]^+ \mathrm{d}t$$

13:       timer $\leftarrow t_{\text{now}} + \tau$
14:     **else**
15:       Do not transmit   ▷ *wait for timer expiry*
16:     **end if**
17: **end if**
18: **if** Room Temperature $\leq T_{\text{des}}^{\min}$ **then**
19:     Transmit an OFF command
20: **end if**

Selecting the timer in such a manner will make $\mathcal{A}_{\text{ONL}}$ behave as follows. Upon the arrival of a each new temperature command, the algorithm sets a timer such that the expiry of timer will indicate that the comfort level threshold has reached and an ON command needs to be transmitted to the air-conditioning system. If an additional thermal source arrives before the timer expires, then a new smaller timer is set because the comfort level threshold will reach sooner due to the additional thermal source. In any case, whenever the timer expires, an ON command is transmitted and the current outstanding sequence is ended.

Example: We provide an example to illustrate the operations of offline optimal and online algorithms. In the example, the outdoor temperature is assumed to follow sinusoidal pattern. The input temperature sampled by the wireless sensor as a result of thermal sources entering the room at at random intervals are given by Table. 1. For convenience, we restrict the example to 10 input samples (i.e., $m = 10$). The maximally desirable temperature $T_{\text{des}}^{\max}$ is 24 degree Celsius.

Table 1: Arrivals of impulsive thermal sources

t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
4	12	15	21	26	30	34	35	40	43
w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9	w_{10}
24	23	24	25	23	29	24	27	25	28

For the arrivals shown in Table. 1, we execute $\mathcal{A}_{OFL}$. Table. 2 lists the entries $\text{Cost}[i, j]$, where the minimum costs (i.e.,$\text{Cost}_{\min}[i]$) are highlighted in yellow.

Table 2: $\text{Cost}_{\min}[i]$ and $\text{Cost}[i, j]$ for offline optimal algorithm

i,j	1	2	3	4	5	6	7	8	9	10
1	1.4									
2	4.1	4.5								
3	7.4	7.1	7.9							
4	11.5	12.0	12.6	14.4						
5	17.1	17.2	18.5	20.0	22.5					
6	23.9	23.8	24.6	26.6	28.7	31.8				
7	31.0	31.1	31.8	33.2	35.9	38.6	42.3			
8	37.4	36.6	36.7	37.4	38.8	41.5	44.2	47.9		
9	44.8	46.3	47.2	49.1	51.4	54.4	58.6	62.7	67.8	
10	54.0	53.2	54.6	55.5	57.4	59.7	62.7	66.9	71.0	76.1

After obtaining $\text{Cost}_{\min}[m]$, we use backtracking to determine the optimal decision variables $\mathbf{x}^*$ as

$$\mathbf{x}^* = (t_1, t_3, t_4, t_6, t_8, t_{10})$$

where each t_i is the time to transmit an ON command.

For the same arrivals, the online algorithm online algorithm $\mathcal{A}_{\text{ONL}}$ gives the following solution

$$\mathbf{x}_{\text{ONL}} = (t_6, t_{10})$$

The decision made by both algorithms are illustrated in Fig. 4.
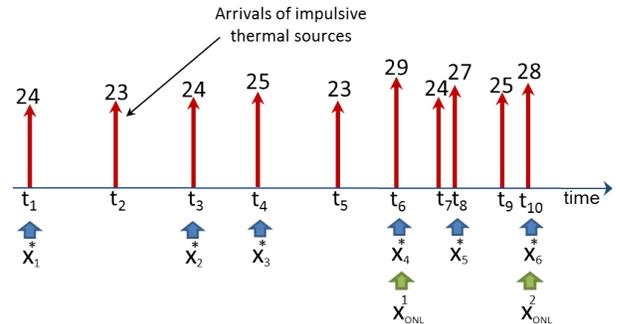
Figure 4: An illustration of the decisions by the offline optimal and online algorithms

Finally, the costs of both algorithms and the competitive ratio are computed as:

$$\text{Cost}(\mathbf{x}^*) = 53 \qquad \text{Cost}(\mathbf{x}_{\text{ONL}}) = 63 \qquad \text{CR}(\mathcal{A}_{\text{ONL}}) = 1.19$$

4.3 Competitive Analysis

Let $\mathbf{x}^*$ be the offline optimal solution, while $\mathbf{x}_{\text{ONL}}$ is the output solution given by online algorithm $\mathcal{A}_{\text{ONL}}$. We define the competitive ratio as

$$\mathsf{CR}(\mathcal{A}_{\text{ONL}}) \triangleq \max_{\mathbf{a}, T_{\text{od}}} \frac{\text{Cost}(\mathbf{x}_{\text{ONL}})}{\text{Cost}(\mathbf{x}^*)} \tag{21}$$

We show that the competitive ratio i.e., $\mathsf{CR}(\mathcal{A}_{\text{ONL}}) \leq 2$.

THEOREM 2. $\text{Cost}(\mathbf{x}_{\text{ONL}}) \leq 2 \cdot \text{Cost}(\mathbf{x}^*)$

PROOF. Assume that $\mathcal{A}_{\text{ONL}}$ sends a total of m ON commands for certain external thermal source arrivals, thus partitioning the sequence into m subsequences, where each subsequence ends with an ON command being transmitted to the air-conditioning system. The total cost by $\mathcal{A}_{\text{ONL}}$ for the input $\mathbf{a}$ is the sum of the cost for transmitting m ON commands and the extra latency cost for each subsequence, which can be calculated as follows. First, as shown previously, $\mathcal{A}_{\text{ONL}}$ sets τ, such that

$$\frac{\eta}{(1-\eta)} = \int_{t=t_j}^{t_j+\tau} [T_{x_k}(t) - T_{\text{des}}^{\text{max}}]^+ \mathrm{d}t \tag{22}$$

Note that $\int_{t=t_j}^{t_j+\tau} [T_{x_k}(t) - T_{\text{des}}^{\text{max}}]^+ \mathrm{d}t$ is a strictly increasing function in τ. Hence, the solution τ always exists and is uniquely defined. Also, it can be seen from Eqn. (22) that the timer is set in a manner that equalizes the total thermal disturbance of the subsequence to $\eta/(1-\eta)$. Thus, the disturbance cost for each subsequence is $\frac{\eta}{(1-\eta)} \cdot (1-\eta) = \eta$. The total cost incurred by $\mathcal{A}_{\text{ONL}}$, therefore, is

$$
\begin{aligned}
\text{Cost}(\mathbf{x}_{\text{ONL}}) &= \text{cost of } m \text{ ON commands} \\
&\quad + \text{disturbance cost for } m \text{ subsequences} \\
&= m\eta + m\eta = 2m\eta
\end{aligned} \tag{23}
$$

To calculate $\text{Cost}(\mathbf{x}^*)$, let m^* be the number of ON commands transmitted to the air-conditioning system in an optimal solution. When $m \leq m^*$, it immediately follows that $\text{Cost}(\mathbf{x}^*) \geq m^*\eta \geq m\eta$. Thus $\text{Cost}(\mathbf{x}_{\text{ONL}})/\text{Cost}(\mathbf{x}^*) \leq 2$.

We now consider the case when $m > m^*$. Since the m^* optimal ON commands are distributed over the m subsequences partitioned by $\mathcal{A}_{\text{ONL}}$. Thus, at least $m - m^*$ subsequences in online algorithm partition have no ON command at their end from the corresponding optimal solution. We claim that for each such a sequence, the disturbance cost is at least η in $\mathcal{A}_{\text{ONL}}$, because $\mathcal{A}_{\text{ONL}}$ decides ON command in such a way that the disturbance cost is equal to weighted cost of ON command (i.e., η). It is straightforward to see that disturbance cost of such a subsequence is at least η, because $\mathcal{A}_{\text{ONL}}$ resets the room temperature to $T_{\text{des}}^{\text{min}}$ at the beginning of each subsequence, whereas offline optimal algorithm does not. This induces a total disturbance cost of at least $(m - m^*)\eta$ to the optimal solution. The total cost of offline optimal algorithm is:

$$\text{Cost}(\mathbf{x}^*) \geq m^*\eta + (m - m^*)\eta = m\eta \tag{24}$$

Thus, $\text{Cost}(\mathbf{x}^*) \geq m\eta$, which is at least half of $\text{Cost}(\mathbf{x}_{\text{ONL}})$. □

5. SIMULATION STUDIES

In this section, we present the results of the simulations to experimentally evaluate the performance of our algorithms. We have used the classical ON/OFF algorithm as a baseline control model. In the classical ON/OFF technique (also known as bang-bang or hysteresis control), the wireless sensor sends an ON command to the air-conditioner whenever the room temperature reaches $T_{\text{des}}^{\text{max}}$ and OFF command when the temperature drops to $T_{\text{des}}^{\text{min}}$. First we compare the online solution against the baseline algorithm. We, then, provide a detailed cost comparison between the online and offline algorithms under different models of random thermal sources and different values of η.

In the first experiment, all three algorithms were run multiple times for different values of η to determine their relative performance against each other. Fig. 5 shows the results of the experiment. The input size during all experiments was 1000. As can be seen, the average cost ratio of the online algorithm against offline algorithm is always below 1.5 which is much better than the theoretical ratio of 2. We can also see that our algorithm always perform better than than classic ON/OFF control technique.
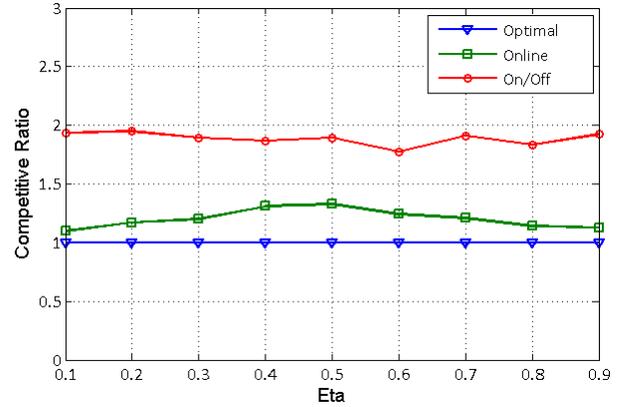
Figure 5: Simulation results showing the performance comparison between the online, the offline, and the classical ON/OFF algorithms.

We now compare the performance of the online algorithm with the optimal offline algorithm under different models of random thermal sources. For the next experiment, we draw the random thermal sources from Poisson distribution. Poisson distribution is with one parameter, where parameter, λ, is both the mean and the variance of the distribution. Thus, we can change the behaviour of random thermal source by changing λ. Poisson distribution is suitable in situations that involve counting the number of times a random event occurs in a given interval (e.g.,time, distance, area etc.). We ran the simulations for different models of random thermal sources generated by varying the parameter λ. Fig. 6 shows the simulations results for $\lambda \in \{10, 20, 30\}$. and $\eta \in \{0.1, 0.2, ..., 0.9\}$. The vertical axis gives the ratio of the cost of the online algorithm's solution to the cost of the optimal solution and the horizontal axis represents the relative cost weighting of sending a control signal to the air-conditioner. By looking at each line, it can be seen that the cost ratio gets closer to one when the value of η approaches either zero or one. *This means that the online algorithm performs better when the relative weighting of sending a control signal is either very low or very high. It can also be observed that the performance of the algorithm improves as we decrease λ (i.e.,reducing the random thermal disturbances).*

Similar results were observed when the experiment was re-
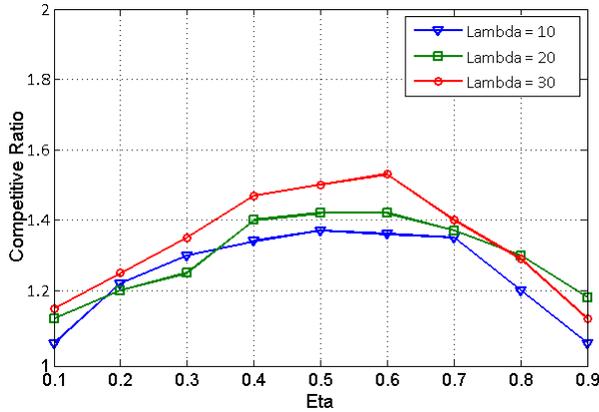
Figure 6: Competitive ratio of the online algorithm against the optimal algorithm when random thermal sources are drawn from Poisson distribution

peated, with random thermal sources drawn from Binomial distribution (see Fig. 7). Binomial distribution requires a parameter p, the probability of success. In our case, p is the probability of a random thermal source entering the room at a certain time. The results shown are for $p \in \{0.2, 0.5, 0.75\}$ and $\eta \in \{0.1, 0.2, ..., 0.9\}$. *Once again, as expected, the algorithm's performance improves as we reduce the value of p (i.e.,the probability of occurrence of thermal disturbances).*
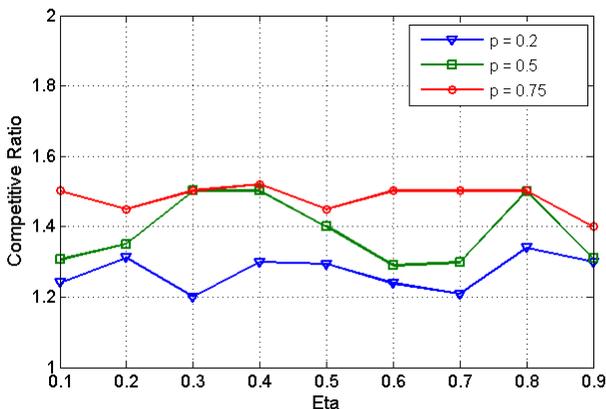
Figure 7: Competitive ratio of the online algorithm against the optimal algorithm when random thermal sources drawn from Binomial distribution.

6. RELATED WORK

Recently, many studies have explored the use of intelligent methods to control HVAC systems. These methods vary from simple manipulation of set-point temperatures to more sophisticated techniques such as fuzzy logic, neural networks, genetic algorithms etc. In this section, we first summarize a few papers that are relatively simple extensions to the classical HVAC control techniques, then we discuss several state-of-the-art intelligent control techniques employed in HVAC systems. We also present a brief survey of the recent works on HVAC control through WSN. We conclude this section by discussing a paper that is somewhat related to our work in that it also aims to optimize the wireless sensors cost while maintaining the control performance within an acceptable range.

Extensions of Classical Techniques: In [12], the authors proposed a relatively simple way of controlling the HVAC systems in which the set-point temperature of the regulator and thermostat is manipulated. They developed an adaptive module of classical regulator to control the peak consumption and provide thermal comfort. Their regulator is based on varying temperature set-point of the air conditioning in response to maximum permissible power. Similar approach has been used in [13], where an optimal control scheme for compressor ON/OFF cycling operations has been proposed.

Intelligent HVAC Control: The design of an intelligent comfort control system by using human learning strategy for an HVAC system was proposed in [14]. Based on a standard thermal comfort model, a human learning strategy was designed to tune the user's comfort zone by learning the specific user's comfort preference. The integration of comfort zone with the human learning strategy was applied for thermal comfort control. The authors in [22] proposed a multi-objective particle swarm optimization algorithm, embedded in a controller. The algorithm was used to determine the amount of energy dispatched to HVAC equipment based on utilizing swarm intelligence technique.

A method based on fuzzy logic controller dedicated to the control of HVAC systems has been proposed in [2]. They obtained the initial knowledge-base required by fuzzy logic controller from human experts and control engineering knowledge which they subsequently tuned by a genetic algorithm. In [17], a hierarchical structure for the control of an HVAC system using the Model Predictive Control (MPC) algorithms and fuzzy control algorithms has been proposed. The main task of the proposed hierarchical control system is to provide thermal comfort and minimize energy consumption. Their technique showed a good comparison between two conflicted objectives: thermal comfort and energy consumption. The authors of [3] used model-predictive control technique to learn and compensate for the amount of heat due to occupants and equipment. They used statistical methods together with a mathematical model of thermal dynamics of the room to estimate heating loads due to inhabitants and equipment and control the AC accordingly. *However, majority of the existing intelligent HVAC control techniques rely on stochastic knowledge about the input which makes them less robust in uncertain environments.*

WSN-based AC Control: In [11], an air-conditioning control system for a dynamical situation in wide public spaces has been proposed. They tracked people movement through multiple large scale scanners. Also, networked temperature sensors were deployed in the target space for temperature monitoring. The obtained temperature distribution was integrated with the results of people tracking in real-time to direct HVAC to locations with high population density and insufficient temperature. In [20], the authors presented the conceptual design of an adaptive multi zone HVAC control system that utilized WSN for predicting the occupancy pattern of people in a building. Their control strategy involved turning off the AC in unoccupied zones and manipulating the set-point temperature. A multi-sensor non-learning control strategy has been proposed in [18]. This paper evaluates the energy and comfort performance of three multi-sensor control strategies that use wireless temperature and humidity sensors and that can be applied to existing ON/OFF central HVAC system. The multi-sensor control strategies

adjust the temperature set point of a thermostat to (i) control the average of all room temperatures using a temperature threshold logic, (ii) minimize aggregate discomfort of all rooms, or (iii) maximize the number of rooms within a comfort zone. The strategies were evaluated in a real occupied house and were found to outperform single-sensor control strategies.

In [15], the authors proposed somewhat similar approach to our work. They introduced a co-design methodology that optimizes the sensor network cost while maintaining the control performance within an acceptable range. They applied the developed co-design methodology to a distributed control for building lighting systems. They empirically compared the developed system for building lighting control with a baseline control method and reported significant reduction in energy use and saving in the network cost while maintaining the user comfort.

7. CONCLUSION AND FUTURE WORK

While intelligent systems for smart buildings have been a popular research topic, online optimization approach has been explored to a lesser extent. This paper investigates a new breed of research problems by applying online algorithms to wireless sensor based smart building control. We provide the first study of optimizing the trade-off between the battery lifetime of wireless sensor and the effectiveness of HVAC remote control in the presence of uncertain fluctuations in room temperature. We present both an effective offline optimal algorithm and a 2-competitive online algorithm.

There are plenty of research opportunities to extend the results of this work to a more general context. So far, we devised a deterministic online algorithm. It is well-known that randomized online algorithms can exhibit both improved theoretical competitive ratio and practical performance. For the on-going work, we will study randomized online algorithms for wireless sensor controlling air-conditioning systems, and evaluate their performance.

In this paper, we only consider a single sensor control setting. In a general setting, there may be multiple sensors and multiple air conditioning systems. The interaction among multi-input and multi-control systems in a networked setting will be a challenging yet important research problem.

Finally, we are implementing our control algorithms in real-world air-conditioning systems. More empirical studies will be presented in the extended version of this work.

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APPENDIX

A. PROOF OF LEMMA 1

In this subsection, we prove Lemma 1 that we used in Section 3.2. The differential equations are again listed here:

$$\frac{d\tilde{T}(t)}{dt} = \frac{1}{c \cdot M_{air}} \cdot \left(\frac{dQ_{in}(t)}{dt} - \frac{dQ_{ac}(t)}{dt} \right) \quad (25)$$

$$\frac{dQ_{in}(t)}{dt} = \frac{T_{od} - \tilde{T}(t)}{R_{eq}} \quad (26)$$

$$\frac{dQ_{ac}(t)}{dt} = \frac{c \cdot M_{ac} \cdot (\tilde{T}(t) - T_{ac})}{E_{ac}} \quad (27)$$

where $\frac{dQ_{in}(t)}{dt}$ is the heat flowing into the room from outside environment and and $\frac{dQ_{ac}(t)}{dt}$ is the chilled air flowing from air conditioning system into the room. By substituting, Eqn. (26) and Eqn. (27) into Eqn. (25), we obtain

$$\frac{d\tilde{T}(t)}{dt} = \frac{1}{c \cdot M_{air}} \cdot \left(\frac{T_{od} - \tilde{T}(t)}{R_{eq}} - \frac{c \cdot M_{ac} \cdot (\tilde{T}(t) - T_{ac})}{E_{ac}} \right)$$

$$= \frac{E_{ac} \cdot T_{od} - E_{ac} \cdot \tilde{T}(t) - R_{eq} \cdot c \cdot M_{ac} \cdot (\tilde{T}(t) - T_{ac})}{c \cdot M_{air} \cdot R_{eq} \cdot E_{ac}}$$

$$= \frac{E_{ac} \cdot T_{od} + R_{eq} \cdot c \cdot M_{ac} \cdot T_{ac}}{c \cdot M_{air} \cdot R_{eq} \cdot E_{ac}}$$

$$- \frac{E_{ac} + R_{eq} \cdot c \cdot M_{ac}}{c \cdot M_{air} \cdot R_{eq} \cdot E_{ac}} \cdot \tilde{T}(t) \quad (28)$$

Let

$$\mathsf{C}_1 = \frac{E_{ac} \cdot T_{od} + R_{eq} \cdot c \cdot M_{ac} \cdot T_{ac}}{c \cdot M_{air} \cdot R_{eq} \cdot E_{ac}}$$

$$\mathsf{C}_2 = \frac{E_{ac} + R_{eq} \cdot c \cdot M_{ac}}{c \cdot M_{air} \cdot R_{eq} \cdot E_{ac}}$$

Then, Eqn. (28) can be written as:

$$\frac{d\tilde{T}(t)}{dt} = \mathsf{C}_1 - \mathsf{C}_2 \cdot \tilde{T}(t)$$

By rearrangement,

$$\frac{d\tilde{T}(t)}{\frac{\mathsf{C}_1}{\mathsf{C}_2} - \tilde{T}(t)} = \mathsf{C}_2 \cdot dt$$

Integrating both sides with respect to t,

$$-\log \left| \frac{\mathsf{C}_1}{\mathsf{C}_2} - \tilde{T}(t) \right| = \mathsf{C}_2 \cdot t + C$$

By substituting $t = 0$ (i.e.,initial condition), we obtain

$$-\log \left| \frac{\mathsf{C}_1}{\mathsf{C}_2} - \tilde{T}(t) \right| = \mathsf{C}_2 \cdot t - \log \left| \frac{\mathsf{C}_1}{\mathsf{C}_2} - \tilde{T}(0) \right|$$

$$e^{\mathsf{C}_2 \cdot t} = \frac{\frac{\mathsf{C}_1}{\mathsf{C}_2} - \tilde{T}(0)}{\frac{\mathsf{C}_1}{\mathsf{C}_2} - \tilde{T}(t)}$$

$$\tilde{T}(t) = \frac{\mathsf{C}_1}{\mathsf{C}_2} - \left(\frac{\mathsf{C}_1}{\mathsf{C}_2} - \tilde{T}(0) \right) \cdot e^{-\mathsf{C}_2 \cdot t} \quad (29)$$

This concluded the proof as Eqn. (29) is the same as Eqn. (4).

B. CALCULATION OF ROOM THERMAL RESISTANCE

The building thermal model used in this paper (both during the theoretical part and simulations) requires the total equivalent (also called lumped) thermal resistance, R_{eq}, of the entire room. Therefore, we include a simple example on how to calculate R_{eq} using the rooms dimensions, number and sizes of windows and the type of insulation used in walls. Table 3 shows the room geometry and insulation details used for calculation of R_{eq}.

Table 3: Room geometry and insulation details

Description	Value
Room length (Len_{room})	10 m
Room width (Wid_{room})	5 m
Room height (Ht_{room})	4 m
Roof pitch (Pit_{roof})	40
Number of windows ($Num_{windows}$)	4
Height of windows ($Ht_{windows}$)	1 m
Width of windows ($Wid_{windows}$)	1 m
Wall insulation having glass wool (L_{walls})	0.2 m
Window insulation ($L_{windows}$)	0.01 m
Thermal conductivity of walls (K_{walls})	0.038
Thermal conductivity of windows ($K_{windows}$)	0.78

From the values in Table 3, we can calculate the equivalent resistances of the walls as follows.

$$R_{Wall} = \frac{L_{Wall}}{k_{Wall} \times Wall_{area}} \quad (30)$$

Where,

$$Wall_{area} = (2 \cdot Len_{room} \cdot Ht_{room}) + (2 \cdot Wid_{room} \cdot Ht_{room})$$
$$+ [2 \cdot (1/\cos(Pit_{roof}/2)] \cdot (Wid_{room} \cdot Len_{room})$$
$$+ [(\tan(Pit_{roof}) \cdot Wid_{room})] - Window_{area}$$

Similarly, the equivalent resistance of windows is calculated as:

$$R_{Window} = \frac{L_{Window}}{k_{Window} \times Window_{area}} \quad (31)$$

Where,

$$Window_{area} = Num_{windows} \cdot Ht_{windows} \cdot Wid_{windows}$$

From Eqns. 31 and 30, R_{eq} is calculated as.

$$R_{eq} = \frac{R_{Wall} \times R_{Window}}{R_{Wall} + R_{Window}} \quad (32)$$

C. SENSOR POWER CONSUMPTION

In order to maximize the battery life-time of wireless sensors, it is important to understand the energy consumed by each component of a wireless sensor node. Therefore, we provide power consumption data for each unit (i.e.,transceiver, micro-controller, and sensor) in common wireless sensor nodes (see Tables 4-6). From the tables, it is evident that radio communication is most energy-intensive among the three operations (i.e.,sensing, processing, and communication). Specifically, the transceiver power consumption can get as high as 28 times compared the power consumption of micro-controller (see Table 4 and 5). The ratio becomes even higher when compared to the power consumption of the sensor modules. For these reasons, we aim to maximize the battery life-time of the wireless sensor by optimizing the update frequency of the control commands sent to the air-conditioner.

Table 4: Power Consumptions of Transceivers and in Common Wireless Sensors. [4]

Transceiver Model	Transmission (mA)	Reception (mA)	Sleep (mA)
TR1000	12	3.8	0.0007
CC1000	10.4	7.4	0.03
CC2500	21.6	12.8	0.0004
nRF2401A	10.5	18	0.0004
CC2420	17.4	18.8	0.4
RF230	14.5	15.5	0.00002
MC13192	30	37	0.5
JN5121	45	50	0.0004

Table 5: Power Consumptions of MCUs in Common Wireless Sensors. [4]

MCU Model	Active (mA)	Sleep (mA)
AT163	5	0.025
AT128	5.5	0.015
80c51	4.3	0.19
MSP430	1.8	0.00512
HCS08	4.3	0.0005

Table 6: Power Consumptions of Sensor Module in Common Wireless Sensors. [4]

Sensor Module	Function	Current (mA)
SHT15	Humidity, Temperature	0.55
TSL2561	Light	0.24
ADXL202	Accelerometer	0.6

Table 7: Key Notations in This Paper

Notation	Definition
$\tilde{T}(t)$	Ambient room temperature at time t (unit: degree Celsius)
T_0	Initial ambient room temperature at time $t = 0$
T_{od}	Outdoor temperature
T_{ac}	Temperature of the cold air from air conditioner
M_{air}	Total air mass inside the room
M_{ac}	Air mass flow through air conditioner (Kg/hr)
E_{ac}	Air conditioner efficiency
c	Heat capacity of the air at constant pressure
R_{eq}	Equivalent thermal resistance of the entire room
$W(t)$	Sequence of impulsive thermal sources
w_i	Level of thermal intensity entering the room at time t
a	Sequence of arrivals of impulsive thermal sources
T_{des}^{max}	Maximal desirable temperature
T_{des}^{min}	Minimal desirable temperature
$T_\tau(t)$	Temperature of thermal sources
X	Set of decision variables
x_k	Time that the k_{th} ON command is issued by the wireless sensor
$\mathcal{D}(\mathbf{x})$	Thermal disturbance given decision variable x
$[x]^+$	max(x, 0)
$T_{t_k}(t)$	Temperature of the room after k_{th} ON command
η	Weight assigned to balance the update frequency and the thermal comfort
$\mathcal{A}_{OFL}$	Offline Algorithm
$Cost[i, j]$	Minimum cost when the last and second to last ON command are transmitted at time t_i and t_{i-j} respectively
$Cost_{min}[i]$	Minimum cost when the last ON command is transmitted at time t_i
$idx[i]$	Array to record j when $Cost_{min}[i] \leftarrow Cost[i, j]$
σ_k	Set of thermal sources arrived between the $(k-1)$-th and the k-th ON commands
$\mathcal{A}_{ONL}$	Online Algorithm
$\mathcal{D}_{ij}(\tau)$	Total thermal disturbance accumulated from the start of the subsequence to the latest arrival
t_j	The time when the timer was first set after transmission of the last ON command
λ	Mean and variance of the Poission distribution
p	Success probability. A parameter required by Binomial distribution