MIMO Capacity for Spatial Channel Model Scenarios

Shuo Pan[†] and Salman Durrani
Department of Engineering,
The Australian National University, Canberra, Australia.

†Email: u4108445@anu.edu.au

Marek E. Bialkowski[‡]
School of Information Technology & Electrical Engineering,
The University of Queensland, Brisbane, Australia.
Email: meb@itee.uq.edu.au

Abstract—The capacity of Multiple Input Multiple Output (MIMO) systems has received much attention in recent years. In this paper, we analyse the capacity of MIMO systems for the 3GPP-3GPP2 Spatial Channel Model (SCM). We examine the impact of number of antennas, inter-element distance and mutual coupling, assuming both waterfilling and uniform transmit power allocation schemes, for different SCM scenarios. We also compare the simulation results with prior measurement results. The comparison provides insight into the accuracy of MIMO capacity predictions using SCM model.

I. INTRODUCTION

Multiple Input Multiple Output (MIMO) systems, which use multiple element antennas for signal transmission and reception, are expected to play a key role in improving the performance of future wireless communication systems [1], [2]. It has been shown that if the signal fading between pairs of transmit and receive antenna elements are independent and identically distributed (i.i.d.), the capacity of MIMO systems can increase linearly with the number of antennas [3]. These idealized conditions are, however, not fully met in practice and the performance of a real MIMO system is affected by non-ideal propagation conditions [4] and by mutual coupling effects due to finite spacing of antenna elements [5], [6]. In order to provide better assessment, MIMO capacity investigations have been carried out using more realistic channel models [7]-[10]. Many MIMO measurement campaigns have also been conducted and channel capacity examined in different propagation scenarios [11]-[15]. It is important to provide an assessment of these results against standardized channel model predictions for comparable MIMO system evaluations.

The Spatial Channel Model (SCM) is a standardized model developed by 3GPP-3GPP2 for evaluating MIMO system performance in outdoor environments [16], [17]. It incorporates important parameters such as phases, delays, doppler frequency, angle of departure (AOD), angle of arrival (AOA) and angle spread to provide a description of MIMO channels. It also takes into account spacing in transmitter and receiver arrays, which makes mutual coupling investigation feasible.

In this paper, we investigate the MIMO capacity using the Spatial Channel Model of [16]. We consider both waterfilling and uniform transmit power employed at the transmitter. We present simulation results to investigate the MIMO capacity as a function of number of antennas and inter-element distance, in

propagation scenarios covered by SCM. We also compare the SCM capacity predictions with prior work. The results provide insight into the accuracy of MIMO capacity predictions using SCM model.

This paper is organized as follows. The general system model is introduced in Section II. The SCM channel model is described in Section III. The simulation results and comparison to prior work are discussed in Section IV and Section V. Finally conclusions are presented in Section VI.

II. SYSTEM MODEL

Consider a narrow-band single user MIMO system with N_T transmit and N_R receive antennas. The overall MIMO inputoutput relationship can be represented in vector notation as

$$y = Hx + n \tag{1}$$

where \mathbf{y} is the $N_R \times 1$ received signal vector, \mathbf{x} is the $N_T \times 1$ transmitted signal vector, \mathbf{n} is the $N_R \times 1$ zero-mean complex Gaussian noise vector with independent, equal variance real and imaginary parts, and \mathbf{H} is the $N_R \times N_T$ normalised channel matrix. Each element H_{ij} represents the complex gains between the j^{th} transmit and i^{th} receive antenna.

We consider two channel scenarios. If the channel state information is known only at the receiver and the channel is i.i.d. Rayleigh fading, then uniform transmit power is optimal at the transmitter [3]. In this case, the capacity is given by

$$C_{EP} = \log_2 \left[\det \left(\mathbf{I} + \frac{\rho}{N_T} \mathbf{H} \mathbf{H}^{\dagger} \right) \right]$$
 bps/Hz (2)

where $\det(\cdot)$ denotes the determinant of a matrix, \mathbf{I} is an $N_R \times N_T$ identity matrix, ρ is the average received Signal to Noise Ratio (SNR), and \mathbf{H}^{\dagger} is the complex conjugate transpose of \mathbf{H} .

If the channel state information known at both the transmitter and receiver and the channel is i.i.d. Rayleigh fading, then waterfilling is optimal at the transmitter [4]. The resulting capacity is given by

$$C_{WF} = \sum_{i=1}^{m} \log_2(\mu \lambda_i)^{+} \text{ bps/Hz}$$
 (3)

where μ is chosen from the waterfilling algorithm, which is

$$\rho = \sum_{i=1}^{m} (\mu - \lambda_i^{-1})^+ \tag{4}$$

where $(\cdot)^+$ denotes taking only those terms which are positive and $\lambda_1, \lambda_2, \ldots, \lambda_m$ are the eigenvalues of $\mathbf{H}\mathbf{H}^{\dagger}$ with $m = \min(N_T, N_R)$.

A. Mutual Coupling

When antenna elements are placed close to each other, the electromagnetic field generated by the current flowing in one antenna causes a voltage to be induced in neighbouring antennas. This is called mutual coupling. For half-wavelength dipoles, analytical expressions can be used to model the effect of mutual coupling in MIMO system.

The coupling matrix C is defined as [18]

$$\mathbf{C} = (Z_A + Z_T)(\mathbf{Z} + Z_T \mathbf{I}_N)^{-1} \tag{5}$$

where Z_A is the antenna's impedance in isolation (for $l=\lambda/2$ dipole, $Z_A=73+j42.5\Omega$), \mathbf{I}_N is the identity matrix, Z_T is the impedance of the receiver at each antenna element, chosen as the complex conjugate of Z_A to obtain an impedance match for maximum power transfer, and \mathbf{Z} is the $N\times N$ mutual impedance matrix. For the side by side configuration and dipole lengths $l=\lambda/2$, an element of the mutual impedance matrix Z_{mn} , where $1\leq m,n\leq N$, is given by [18]

$$Z_{mn} = \begin{cases} 30[0.5772 + \ln(2Kl) - C_i(2Kl)] & m = n \\ +j[30(S_i(2Kl))], & \text{(6a)} \\ 30[2C_i(u_0) - C_i(u_1) - C_i(u_2)] & m \neq n \\ -j[30(2S_i(u_0) - S_i(u_1) - S_i(u_2))], & \text{(6b)} \end{cases}$$

where $u_0 = Kd_h$, $u_1 = K(\sqrt{d_h^2 + l^2} + l)$, $u_2 = K(\sqrt{d_h^2 + l^2} - l)$, d_h is the horizontal distance between the two dipole antennas and $C_i(u)$ and $S_i(u)$ are the Cosine and Sine Integrals respectively defined as $C_i(u) = \int_{\infty}^u \frac{\cos(x)}{x} dx$ and $S_i(u) = \int_0^\infty \frac{\sin(x)}{x} dx$.

Taking mutual coupling into account, the channel matrix \mathbf{H} can be modified by multiplying the coupling matrix \mathbf{C}_R and \mathbf{C}_T for transmitter and receiver respectively.

$$y = H_{MC}x + n \tag{7}$$

where $\mathbf{H}_{MC} = \mathbf{C}_R \mathbf{H} \mathbf{C}_T$ is the modified channel matrix.

B. Normalization

In order to investigate the effect of inter-element distance and mutual coupling, channel matrix should be properly normalized. There are two main normalization methods. The first one normalizes \mathbf{H}_{MC} such that

$$\|\mathbf{H}_{MC}\|_F^2 = N_T N_R \tag{8}$$

where $\|(\cdot)\|_F^2$ is the Frobenius norm.

This normalization is performed on each realization of the channel matrix, which includes the propagation channel and antennas. The limitation of this normalization is that the differences in the channel gain due to antennas are removed. However this type of normalization permits investigation of correlation between the channel matrix entries and gives good indication of the richness of the multipath environment [5], [10], [19].

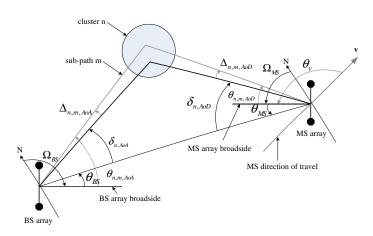


Fig. 1. Important parameters in the 3GPP SCM for a cluster of scatterers [16].

In the second normalization,

$$\|\mathbf{H}\|_F^2 = N_T N_R \tag{9}$$

This normalization is also performed on each realization of the channel matrix, which includes the propagation channel only. This normalization permits investigation of the effects of instantaneous changes of received power due to mutual coupling [20].

III. SPATIAL CHANNEL MODEL

The SCM is a detailed system level model for simulating urban micro-cell, urban macro-cell and suburban macro-cell fading environments [16]. It considers N cluster of scatterers. Each cluster corresponds to a resolvable path. Within a resolvable path (cluster), there are M unresolvable subpaths. In this paper, we consider a downlink system where a Base Station (BS) transmits to a Mobile Station (MS). A simplified plot of the model is shown in Fig. 1.

For a N_T element linear BS array and a N_R element linear MS array, the channel coefficients of one of the N multipath components are given by a $N_R \times N_T$ matrix of complex amplitudes. Assuming omnidirectional antenna elements are employed at the BS and MS and neglecting pathloss and shadowing, the channel impulse response for the lth path between the sth transmit and uth receive antenna can be written as [16]

$$h_{u,s,n}(t) = \sqrt{\frac{P_n}{M}} \sum_{m=1}^{M} \left\{ \exp[j(kd_s \sin(\theta_{n,m,AOD}) + \phi_{n,m})] \times \exp[jkd_u \sin(\theta_{n,m,AOA})] \times \exp[jk\|\mathbf{v}\| \cos(\theta_{n,m,AOA} - \theta_v)t] \right\}$$
(10)

where $j=\sqrt{-1}$, k is the wave number $2\pi/\lambda$, λ is the carrier wavelength in meters, P_n is the power of the nth path, M is the number of subpaths per-path, d_s is the distance in meters from BS antenna element s to the reference (s=1) antenna, d_u is the distance in meters from MS antenna element u to the reference (u=1) antenna, $\|\mathbf{v}\|$ is the magnitude of the

TABLE I

MAIN SIMULATION PARAMETERS FOR SCM CHANNEL MODEL

Aspect	Parameters	Value or Description	
Carrier frequency f_c		2 GHz	
	BS antenna spacing d_T	0.5λ	
Global	MS antenna spacing d_R	0.5λ	
	SNR ρ	3, 10, 20 dB	
	No. of simulation runs	10,000	
	No. of Paths	1	
	No. of sub-paths	20	
	Mean AS at BS	$E(\sigma_{AS}) = 5^{\circ}$	
	$r_{AS} = \sigma_{AoD}/\sigma_{AS}$	1.2	
Suburban	Per-path AS at BS (Fixed)	2°	
Macro	BS per-path AOD Distribution	$\eta(0,\sigma_{AoD}^2)$	
	Mean AS at MS	$E(\sigma_{AS,MS}) = 68^{\circ}$	
	Per-path AS at MS (Fixed)	35°	
	MS Per-path AOA Distribution	$\frac{\eta(0, \sigma_{AoA}^2)}{1}$	
	No. of Paths	1	
	No. of sub-paths	20	
	Mean AS at BS	$E(\sigma_{AS}) = 19^{\circ}$	
Urban	Per-path AS at BS (Fixed)	5° (LOS and NLOS)	
Micro	BS per-path AOD Distribution	$\mathcal{N}(-40^{\circ}, 40^{\circ})$	
	Mean AS at MS	$E(\sigma_{AS,MS}) = 68^{\circ}$	
	Per-path AS at MS (Fixed)	35°	
	MS Per-path AOA Distribution	$\eta(0, \sigma_{AoA}^2)$	

MS velocity vector, $\theta_{n,m,AOA}$ is the Angle of Arrival (AOA) for the mth subpath of the nth path with respect to the MS broadside and $\theta_{n,m,AOD}$ is the Angle of Departure (AOD) for the mth subpath of the nth path with respect to the BS broadside. The details of the generation of relevant parameters are given in [16]. The values of important parameter used to generate the results in this paper are summarized in Table I, where $E(\cdot)$ is the expectation operator, $\mathcal{N}(a,b)$ denotes a uniform distribution over (a,b) and $\eta(0,\sigma_{AoD}^2)$ denotes a zero-mean Gaussian distribution with variance σ_{AoD}^2 .

IV. RESULTS

In this section, we analyse the MIMO capacity using the SCM channel model. We consider the two cases of urban micro-cell and suburban macro-cell scenarios here. This is because the simulated capacity of the suburban macro-cell and urban macro-cell environments was found to be close to each other. This may be due to the fact that path loss and shadowing are not considered in (10). The BS antenna inter-element spacing is set to 0.5λ while the MS antenna inter-element spacing is varied over the range $0.01\lambda \leq d_R \leq 1\lambda$. The figure of merit used is the mean capacity which is obtained by averaging over 10,000 independent channel realizations. For comparison, the MIMO capacity results using the well known i.i.d. [3] and one-ring [21] channel models are also shown. The main values of the parameters used in the simulations are taken from Table I [16].

A. Effect of Number of Antennas

Fig. 2 shows the mean capacity versus number of antennas $(N_T = N_R)$ for $\rho = 3$ dB, $d_T = d_R = 0.5\lambda$ assuming equal power (Eq. (2)) and waterfilling schemes (Eq. (3)). The SNR is set to 3 dB to accentuate the capacity difference between the two schemes. The figure shows that the i.i.d. channel

capacity increase linearly with the number of antennas and the one-ring channel model capacity is also very close to the i.i.d. channel model capacity. The SCM capacity, however, does not double by doubling the number of antennas e.g. in Fig. 2, when the number of antennas doubles from 2 to 4, the suburban macro-cell equal power capacity increases from 2.33 bps/Hz to 3.28 bps/Hz, i.e. an increase of about 40%, while the urban micro-cell capacity increases by about 51%. Similarly, doubling the number of antennas from 4 to 8, results in increase of capacities of about 47% and 65% for suburban macro-cell and urban micro-cell, respectively. The results show that the SCM model gives a modest increase in capacity due to a more realistic assumption of the signal propagation environment.

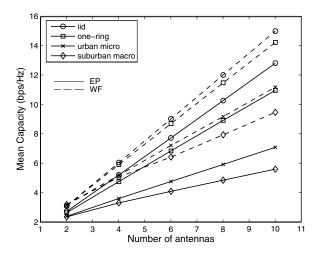
Fig. 2 also shows that the mean capacity of urban microcell scenario is greater that for suburban macro-cell scenario. This is explained as follows: the angle spread for the suburban macro-cell environment is lower than that for the urban microcell. Thus the multipath richness is greater in urban micro-cell, which leads to lower correlation and thus higher capacity.

Fig. 3 shows the mean capacity versus number of antennas $(N_T=N_R)$ for the same conditions as Fig. 2, but SNR $\rho=20$ dB. The figure shows that the difference between i.i.d. channel capacity and SCM channel capacity is greater at this higher SNR value. This difference becomes bigger with increasing number of antennas. The general trends identified in Fig. 2 are also present in Fig. 3. In addition, the performance of waterfilling and equal power allocation schemes is very close to each other.

B. Effect of Inter-element distance

Fig. 4 shows the mean capacity versus inter-element distance d_R for SNR = 20 dB, $N_T = N_R = 4$, $d_T = 0.5\lambda$, urban micro-cell scenario assuming equal power and waterfilling schemes. The figure reveals that both waterfilling and equal power results show the same trend. When inter-element distance is greater than about 0.4λ , the capacity results with and without mutual coupling are roughly the same. This indicates that the effect of mutual coupling can be neglected when inter-element distance is greater than 0.4λ . When the inter-element distance is less than 0.4λ , we expect in general for the capacity to decrease due to increased correlation. From Fig. 4, we see that effect of mutual coupling in closely spaced antennas can be beneficial or detrimental depending on the normalization methods outlined in Section II.

The first normalization in (8) leads to slight increase in capacity compared with no mutual coupling case while the second normalization in (9) leads to degradation in capacity compared with no mutual coupling case. This can be explained as follows. The first normalization removes the instantaneous received power variations due to mutual coupling by performing the normalization on the channel matrix as well as the coupling matrix. In this situation, mutual coupling decreases correlation between antenna elements by generating dissimilar antenna element pattern which leads to pattern diversity. Hence the capacity is improved. The second normalization takes



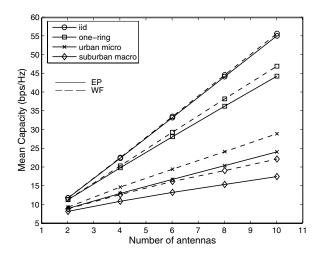


Fig. 2. Mean capacity vs. Number of antennas $(N_T=N_R)$ for SCM model, Fig. 3. Mean capacity vs. Number of antennas $(N_T=N_R)$ for SCM model, $\rho=3$ dB and $d_T=d_R=0.5\lambda$.

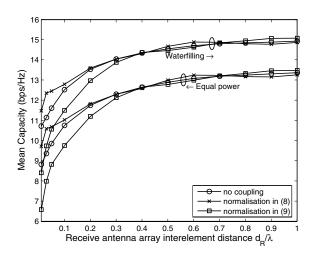


Fig. 4. Mean capacity vs. Inter-element distance for SCM urban micro-cell scenario, $\rho=20$ dB, $N_T=N_R=4$ and $d_T=0.5\lambda$.

into account the effect of mutual coupling on instantaneous received power. Thus the decorrelation effect of mutual coupling is not dominant and the capacity is slightly worse than no mutual coupling case, i.e. mutual coupling acts as a degradation factor. It is important to note that both of the above findings agree with those in [22]. They depend on the chosen normalization of the channel matrix.

V. COMPARISON TO PRIOR WORK

The MIMO capacity estimates obtained using the SCM model show good agreement with published measurement results. The comparison is summarised in Table II. We can see that the SCM capacity results differ from measured capacities in real environments by approximately 30%. Such a discrepancy has also been observed in the case of other channel models, as demonstrated in [11].

The conclusions regarding the effects of mutual coupling are also in line with published work, which indicate that for small inter-element distance the mutual coupling decorrelates the channel and reduces the received power, while for large inter-element distance the mutual coupling increases the channel correlation and slightly enhances the received power [5], [19], [20], [23].

VI. CONCLUSIONS

In this paper, we have investigated the capacity of the MIMO systems for different Spatial Channel Model [16] propagation scenarios. It has been shown that the SCM capacity does not increase linearly with the number of antennas. The capacity of the urban micro-cell was found to be higher than the capacity of the suburban macro-cell. This is because the increased angle spread in the urban micro-cell reduces correlation and increases capacity. In addition, mutual coupling is negligible for inter-element distances greater than about 0.4λ . For inter-element distance less than 0.4λ , mutual coupling can lead to an increase or decrease in capacity (compared to the no mutual coupling case) depending on the type of normalization used for the SNR. Finally, the comparison of simulation results with measurements shows that SCM capacity predictions are within 30% of measurement results. These findings should be of interest to the designers of future wireless systems, which will utilize the concept of MIMO.

REFERENCES

- [1] M. A. Jensen and J. W. Wallace, "A review of antennas and propagation for MIMO wireless systems," *IEEE Trans. Antennas Propagat.*, vol. 52, no. 11, pp. 2810–2824, Nov. 2004.
- [2] M. Bialkowski, "Research into multiple element antennas to enhance performance of wireless communication systems," in *Proc. XVI Intrnl. Conf. on Microwaves, Radar, Wireless Communications*, vol. 3, May 2006, pp. 1071–1082.
- [3] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Pers. Comm.*, vol. 6, no. 3, pp. 311–335, Mar. 1998.

TABLE II

COMPARISON OF SCM MIMO CAPACITY TO PRIOR WORK

No.	Reference	Scenario	Measured capacity (bps/Hz)	SCM capacity (bps/Hz)
1	[13]	Suburban	15	10.759
		$N_T = N_R = 4$	(68% Rayleigh i.i.d. capacity)	
		$d_T = d_R = 0.5\lambda$		
		SNR = 20 dB		
2	[14]	Urban	10.3	6.7634
		$N_T = N_R = 4$	(90% Rayleigh i.i.d. capacity)	
		$d_T = d_R = 0.5\lambda$		
		SNR = 10 dB		
3	[15]	Urban	17.05	12.625
		$N_T = N_R = 4$	(77% Rayleigh i.i.d. capacity)	
		$d_T = 0.4\lambda, d_R = 0.5\lambda$		
		SNR = 20 dB		
4	[7]	Cost 259 urban	7.4	9.9712
		$N_T = N_R = 4$	(1% outage capacity)	(1% outage capacity)
		$d_T = d_R = 0.5\lambda$		
		SNR = 20 dB		

- [4] D. Gesbert, M. Shafi, D.-S. Shiu, P. J. Smith, and A. Naguib, "From theory to practice: an overview of MIMO space-time coded wireless systems," *IEEE J. Select. Areas Commun.*, vol. 21, no. 3, pp. 281–302, Apr. 2003.
- [5] J. W. Wallace and M. A. Jensen, "Mutual coupling in MIMO wireless systems: A rigorous network theory analysis," *IEEE Trans. Wireless Commun.*, vol. 3, no. 4, pp. 1317–1325, July 2004.
- [6] C. Waldschmidt, S. Schulteis, and W. Wiesbeck, "Complete RF system model for analysis of compact MIMO arrays," *IEEE Trans. Veh. Technol.*, vol. 53, no. 3, pp. 579–586, May 2004.
- [7] M. Stege, M. Bronzel, and F. Fettweis, "MIMO-capacities for COST 259 scenarios," in *Proc. International Zurich Seminar on Broadband Communications*, 19-21 Feb. 2002, pp. 29–1–29–6.
- [8] T. Svantesson, "A physical MIMO radio channel model for multielement multi-polarized antenna systems," in *Proc. IEEE VTC'02*, vol. 2, 2001, pp. 1083–1087.
- [9] J. P. Kermoal, L. Schumacher, K. I. Pedersen, and P. E. M. F. Frederiksen, "A stochastic MIMO radio channel model with experimental validation," *IEEE J. Select. Areas Commun.*, vol. 20, no. 6, pp. 1211–1226, Aug. 2002.
- [10] B. K. Lau, S. M. S. Ow, G. Kristensson, and A. F. Molisch, "Capacity analysis for compact MIMO systems," in *Proc. IEEE VTC'05*, vol. 1, June 2005, pp. 165–170.
- [11] A. F. Molisch, M. Steinbauer, M. Toeltsch, E. Bonek, and R. Thoma, "Capacity of MIMO systems based on measured wireless channels," *IEEE J. Select. Areas Commun.*, vol. 20, no. 3, pp. 561–569, Apr. 2002.
- [12] V. Jungnickel, V. Pohl, and C. von Helmolt, "Capacity of MIMO systems with closely spaced antennas," *IEEE Commun. Lett.*, vol. 7, no. 8, pp. 361–363, Aug. 2003.
- [13] T. Sizer, D. Taylor, W. MacDonald, R. Storz, C. Tran, D. Mumma, M.Gans, N. Amitay, H. Xu, R. Valenzuela, and G. Rittenhouse, "Measurement of system capacity using BLAST for mobile applications," Lucent, Holmdel, NJ, Internal Tech. Memorandum., Tech. Rep.
- [14] D. Chizhik, J. Ling, P. W. Wolniansky, I. Reinaldo A. Valenzuela, Fellow, N. Costa, and K. Huber, "Multiple-Input-Multiple-Output measurements and modeling in Manhattan," *IEEE J. Select. Areas Commun.*, vol. 21, no. 3, pp. 321–331, 2003.

- [15] N. Skentos, A. G. Kanatas, G. Pantos, and P. Constantinou, "Capacity results from short range fixed MIMO measurements at 5.2 GHz in urban propagation environment," in *Proc. IEEE ICC'04*, vol. 5, no. 20-24, June 2004, pp. 3020–3024.
- [16] 3rd Generation Partnership Project (3GPP), "Spatial channel model for multiple input multiple output (MIMO) simulations (3gpp tr 25.996 version 6.1.0 release 6)," ETSI, Tech. Rep., 2003.
- [17] J. Salo, G. Del Galdo, J. Salmi, P. Kyosti, M. Milojevic, D. Laselva, and C. Schneider, "MATLAB implementation of the 3GPP Spatial Channel Model (3GPP TR 25.996)," On-line, Jan. 2005, http://www.tkk.fi/Units/Radio/scm/.
- [18] S. Durrani and M. E. Bialkowski, "Effect of mutual coupling on the interference rejection capabilities of linear and circular arrays in CDMA systems," *IEEE Trans. Antennas Propagat.*, vol. 52, no. 4, pp. 1130– 1134, Apr. 2004.
- [19] R. Janaswamy, "Effect of element mutual coupling on the capacity of fixed length linear arrays," *IEEE Antennas Wireless Propagat. Lett.*, vol. 1, pp. 157–160, Oct. 2002.
- [20] B. Clerckx, D. Vanhoenacker-Janvier, C. Oestges, and L. Vandendorpe, "Mutual coupling effects on the channel capacity and the space-time processing of MIMO communication systems," in *Proc. IEEE ICC' 03*, 2003.
- [21] T. Svantesson and A. Ranheim, "Mutual coupling effects on the capacity of multiple antenna systems," in *Proc. IEEE ICASSP'01*, vol. 4, May 2001, pp. 2485–2488.
- [22] M. K. Ozdemir, H. Arslan, and E. Arvas, "Mutual coupling effect in multi-antenna wireless communication systems," in *Proc. IEEE GLOBE-COM'03*, vol. 2, Dec. 2003, pp. 829–833.
- [23] S. Kruservac, P. B. Rapajic, and R. A. Kennedy, "Method for MIMO channel capacity estimation for electro-magetically coupled transmit antenna element," in *Proc. AusCTW'04*, Feb. 2004, pp. 122–126.