Dynamic Logic Semantics for UML Consistency

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The Big Picture

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- but its meaning is only approximate
- so we can not say if it is consistent

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- and we can automatically determine its consistency (undecidable, but most of the time)

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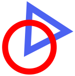
so we translate models into dynamic logic ...

Consistency (Logic 101)

In a situation, each statement is true or false.

	The circle is red.	*true*
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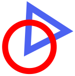

The circle is red. *true*

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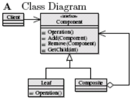
A statement is *consistent* if it is true in some situation.

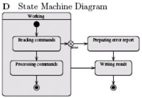
The circle is red.	*consistent*
The circle is square.	*inconsistent*

A model is a statement about systems

Given a system, each model is true or false.

situation/system | statement/model





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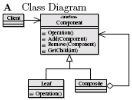

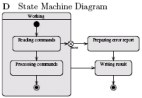

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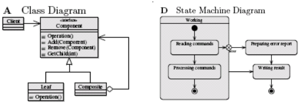

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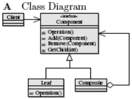

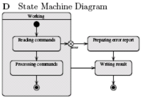

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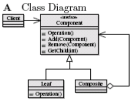

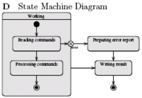

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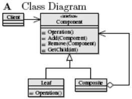

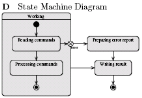

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To answer the questions automatically, we need

a procedure to search the (infinte) space of systems.

Mellor's Challenge

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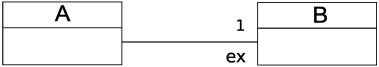

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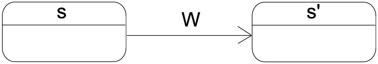

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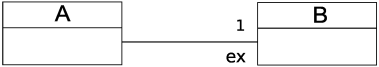

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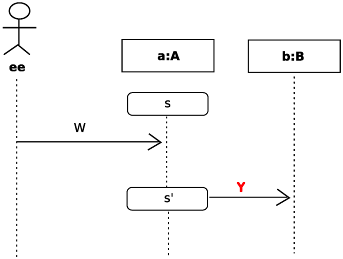

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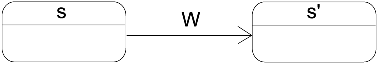

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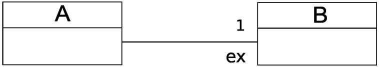

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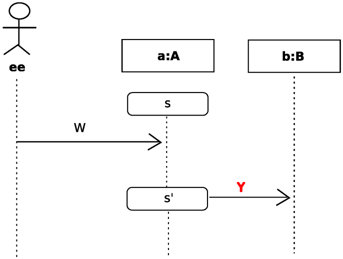

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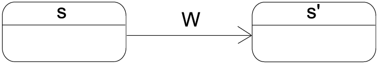

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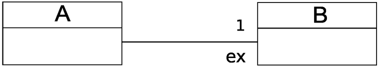

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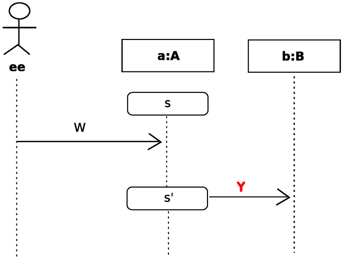

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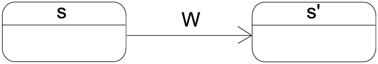

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- more syntax: $\rho ; \rho'$ $\rho \cup \rho'$ ρ^* $\varphi ?$ $[\rho]\varphi$

System States and Evolution

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$$\begin{aligned} sc(x, M, y) &\equiv x.class = ExternalEntity \\ &\vee (head(x.todo) = \mathtt{send}\ M\ \mathtt{to}\ y) \\ x.\mathtt{send}\ M\ \mathtt{to}\ y &\equiv y.intray := append(y.intray, M); \\ & \quad x.todo := tail(x.todo) \end{aligned}$$

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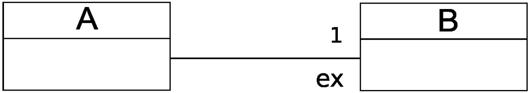

$$CD \equiv [\varepsilon](\forall x \bullet x.class = A \longrightarrow size(x.ex) = 1 \wedge (\forall y \bullet y \in x.ex \longrightarrow y.class = B))$$

State Machine Diagram

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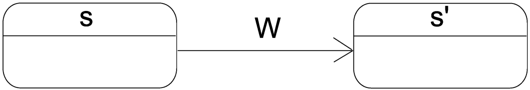

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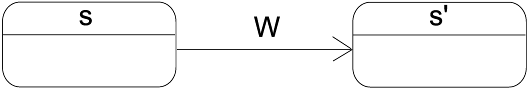

$$SM_s(x) \equiv [\varepsilon](x.state = s \lor x.state = s')$$

$$SM_t(x) \equiv [\varepsilon](x.state = s \land head(x.intray) = W \longrightarrow [x.\texttt{accept}]\, x.state = s')$$

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state machine - class join

Make objects of class A obey state machine formulae.

$$SM \equiv [\varepsilon](\forall x \bullet x.class = A \longrightarrow SM_s(x) \wedge SM_t(x) \wedge SM_p(x))$$

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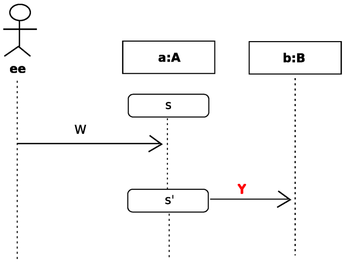

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- so next time we add $X.name =$ “X” etc. to our theory
- and then the UML model can be shown inconsistent

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- Why DL? Why not TLA+, Z, ASM's, OCL (?!), ...?
- With DL we have made *action outline statements*.
  - ignore irrelevant detail
  - raise the level of abstraction