Overview of Wireless Communications

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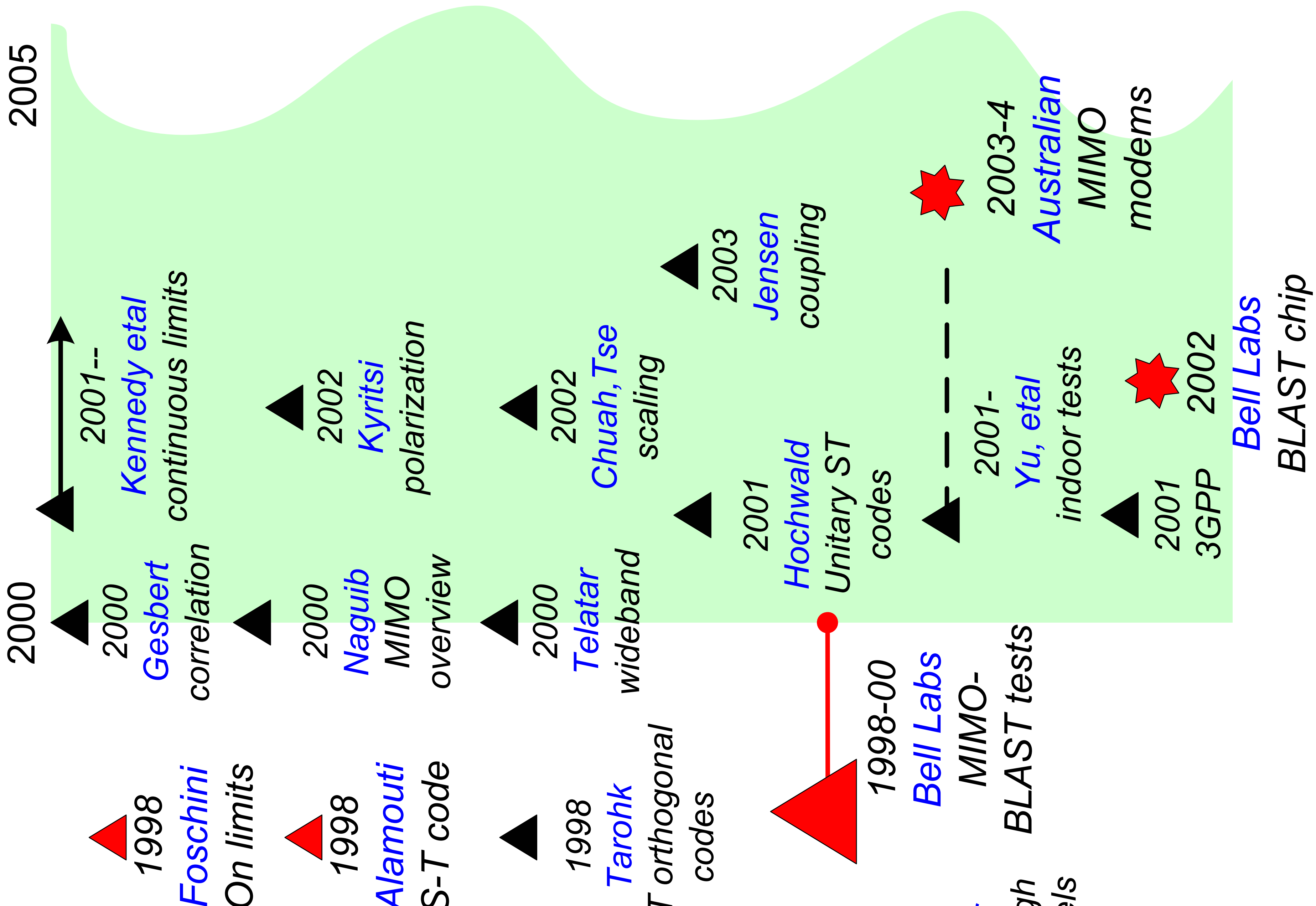
MIMO Overview

- Day 1
  - Introduction - Motivation and Background details
  - History
  - Beamforming
  - Multipath environments
  - Multiple-input, multiple-output concept
  - Information Theory for matrix channels
- Day 2
  - Extensions
  - Non-ideal environments
  - Channel models
  - Coding
  - Implementation

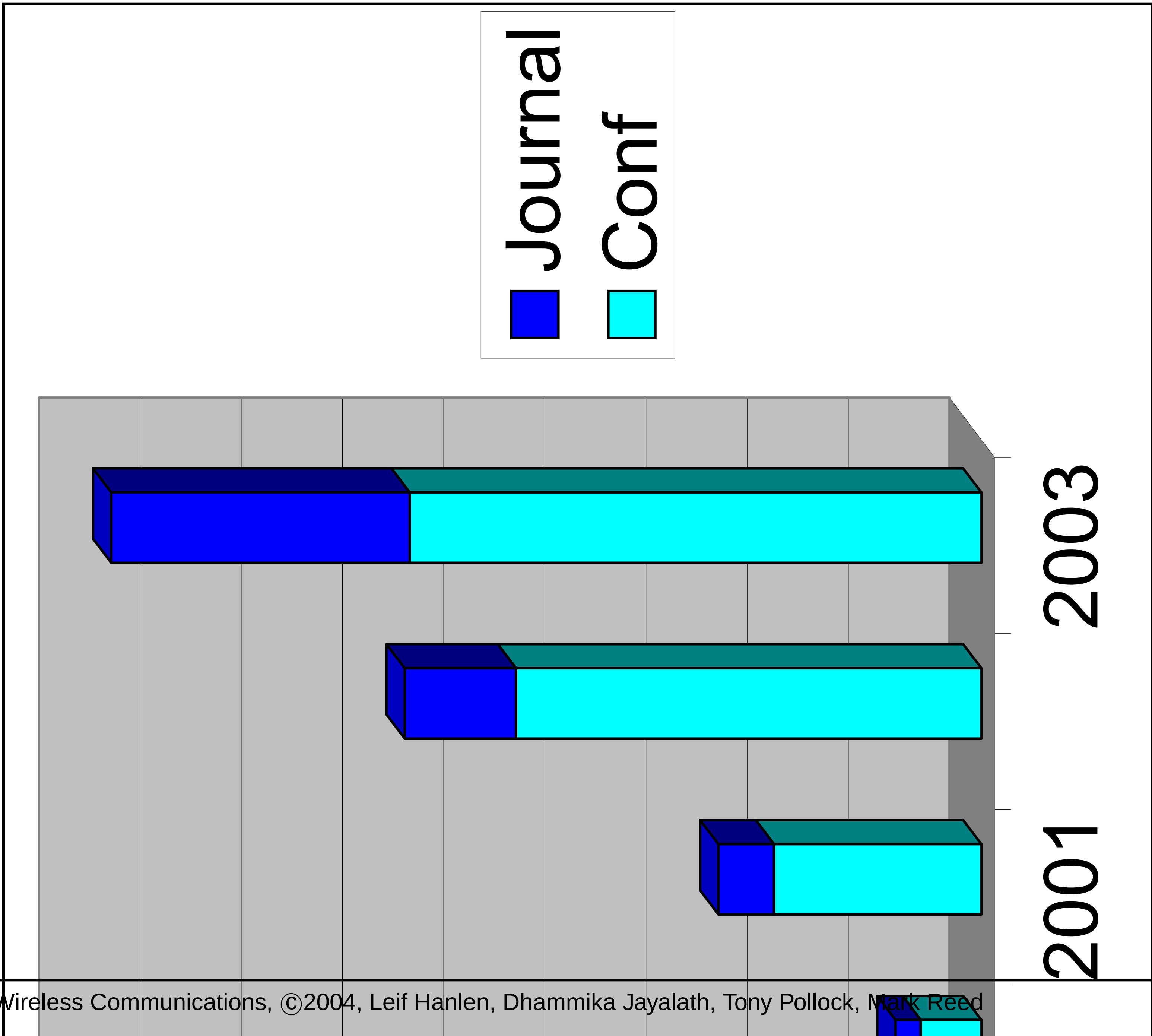
MIMO Day 1

- Introduction - Motivation and Background details
- Multiple-input, multiple-output concept
- By the end of today, you should
  - Understand beamforming terminology, eg. “steering vectors”
  - Understand what multipath is, and how it is mitigated
  - Know what MIMO is
  - Be able to derive basic MIMO information theoretic results

World-wide interest in MIMO


2005
2000
1998 Foschini On limits
1998 Alamouti S-T code
1998 Tarohk S-T orthogonal codes
1998-00 Bell Labs MIMO-BLAST tests
2000 Gesbert correlation
2000 Naguib MIMO overview
2000 Telatar wideband
2001-- Kennedy etal continuous limits
2002 Kyritsi polarization
2002 Chuah, Tse scaling
2001 Hochwald Unitary ST codes
2003 Jensen coupling
2001- Yu, etal indoor tests
2001 3GPP
2002 Bell Labs BLAST chip
2003-4 Australian MIMO modems

World-wide interest in MIMO (cont)

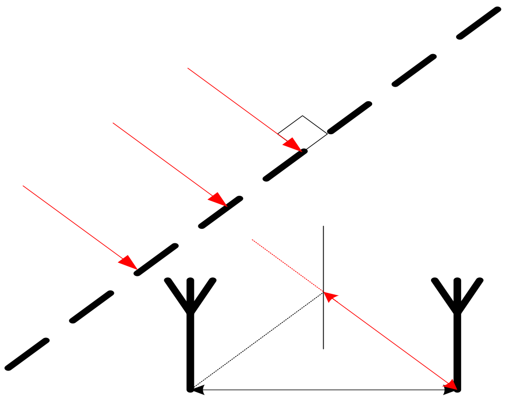

Journal
Conf
2003
2001

MIMO publications since 1996[1]

[1] Search on IEEE-Xplore, using selected keywords, in Comms conferences & journals.

Historical Background: Beamforming

- Beamforming arose from RADAR technology in 1960's
  - used in sonar, acoustics, EM broad-band and narrow-band
  - "Beam" used to focus transmission/reception of signals according to location.

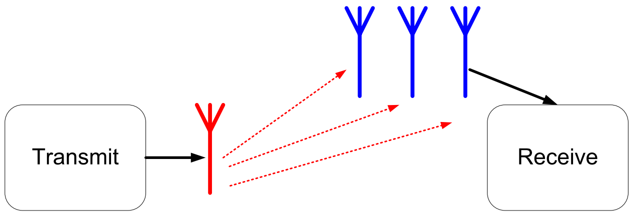


[KV96] H. Krim and M. Viberg. Two decades of array signal processing. *IEEE Signal Processing Mag.*, pages 67–94, July 1996.

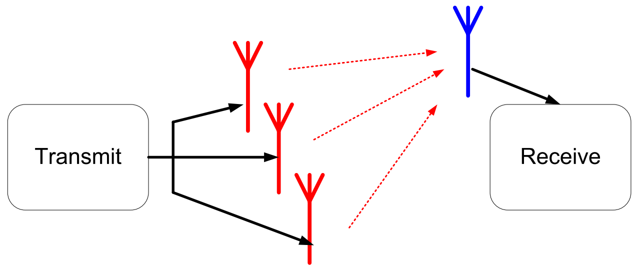
[VVB88] B. D. Van Veen and K. M. Buckley. Beamforming: a versatile approach to spatial filtering. *IEEE ASSP Mag.*, 5(2):4 – 24, April 1988.

[PP97] A. J. Paulraj and C. B. Papadias. Space-time processing for wireless communications, improving capacity, coverage, and quality in wireless networks by exploiting the spatial dimension. *IEEE Signal Processing Mag.*, pages 49–83, November 1997.

Beamforming, signal benefits



- Single-input, multiple-output
  - Each rx-antenna has independent noise, equal variance (i.i.d.)
  - Each rx-antenna r receives signal power $P_r \approx P_t \frac{1}{D^2} g_r g_t \cdot h_{1 \to r}$
  - ⋆ Total signal power $P \propto R \cdot P_t$
  - Noise power only σ^2 (independent)
  - SNR $\propto RP_t/\sigma^2$
  - Rx can "listen" to one point in space

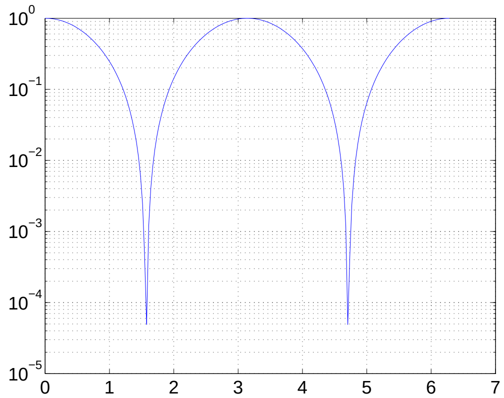


- Multiple-input, single-output
  - Each tx-antenna t sends signal power P_t/t
  - ⋆ Total signal power $P \propto TP_t$ *if coherent*
  - Noise power σ^2
  - SNR $\leq TP_t/\sigma^2$
  - Tx can focus on one point in space

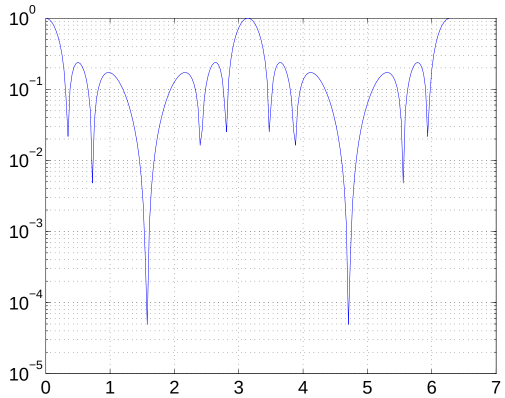
Potential diversity of $T \cdot R$

Beamforming;

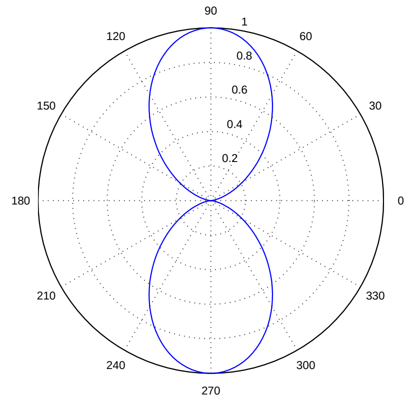
- Linear array is simplest beamforming system: concepts carry across easily to other array geometries.
- Element spacing gives phase offsets across array
- Increasing number of elements gives sharper beam
- “Natural” beam shape shown



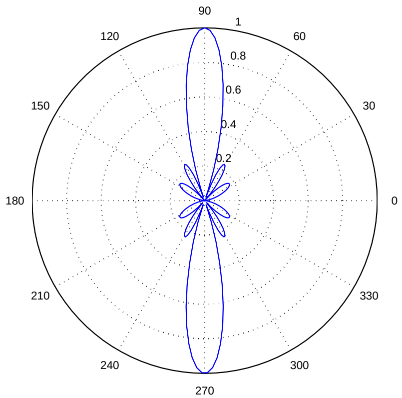
Array magnitude response, vs angle, 2elem array



Array magnitude response, vs angle, 6elem array



Array magnitude response, vs angle, 2elem array



Array magnitude response, vs angle, 6elem array

Beamforming; vector channel

- Each element of beamformer has (desired) signal plus noise. $\mathbf{r}_m = \mathbf{s}_m + \mathbf{z}_m$
- Output of beamformer is weighted sum $y = \sum_{m=1}^{M} \mathbf{w}_m \left(\mathbf{s}_m + \mathbf{z}_m\right)$
- Vector notation: $y = \mathbf{w}\left(\mathbf{s} + \mathbf{z}\right)$
- What is SNR?

$$E\left\{\mathrm{P}_{\mathrm{Rec}}\right\} = E\left\{y\bar{y}\right\} == E\left\{\mathbf{w}\mathbf{s}\mathbf{s}^{\dagger}\mathbf{w}^{\dagger} + \mathbf{w}\mathbf{z}\mathbf{z}^{\dagger}\mathbf{w}^{\dagger}\right\}$$

$$\mathrm{SNR} = \frac{\mathbf{w}\mathbf{S}\mathbf{w}^{\dagger}}{\mathbf{w}\mathbf{R}\mathbf{w}^{\dagger}}$$

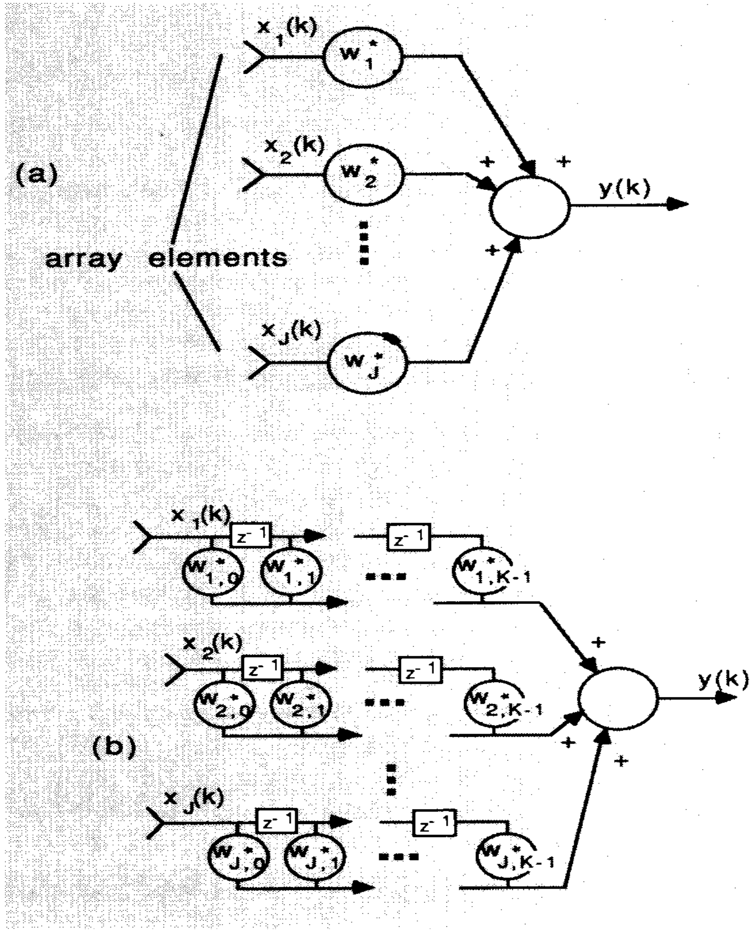
- Optimal MMSE beamformer chooses $\mathbf{w}$ to *minimise* SNR.

By selecting the weight vector $\mathbf{w}$ we can alter the “shape” of the beam, to a particular desired response. This response may be simple eg. “steering the beam” to simulate physical rotation of the array, or more complex. In each case, $\mathbf{w}$ is referred to as a “steering vector.”

Beamforming; extensions

- Can extend from single-freq to broadband
  - must consider beamformer as multiple FIR filters.

- Choice of “steering vectors”
  - chase desired signal
  - steer nulls at noise
  - adaptive



Single-freq and Broadband beamforming [VVB88]

Multipath

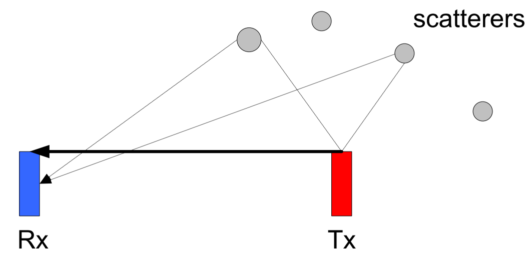
- Signal $s(t)$ arrives at multiple times (echoes)[Pro89] with varying amplitudes.

$$r(t) = \sum_{l=1}^{L} \alpha_l(t) s\left(t - \tau_n(t)\right)$$

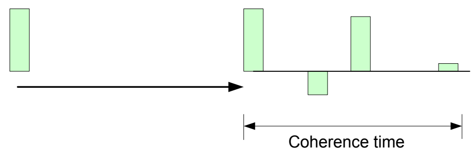
- Simpler: consider unmodulated narrow-band carrier.

$$r(t) = \sum_{l=1}^{L} \alpha_l(t) e^{-\jmath 2\pi f \tau_l(t)}$$

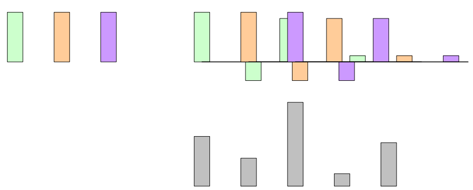
$$r(t) = \int_L \alpha(t,l) e^{-\jmath 2\pi f \tau(t,l)}\, dl$$



General multipath



Single narrow-band signal



Inter-symbol interference

[Pro89] J. G. Proakis. *Digital Communications*. Computer Science Series. McGraw-Hill, New York, USA, 2nd edition, 1989.

Multipath; models

- Frequency selective
  - Signals arrive at different times
  - tap-delay line

$$y_k = \sum_{l=1}^{L} \alpha_l x_{k-\Delta} + z_k$$

- Frequency flat
  - Signals arrive together
  - single scalar gain

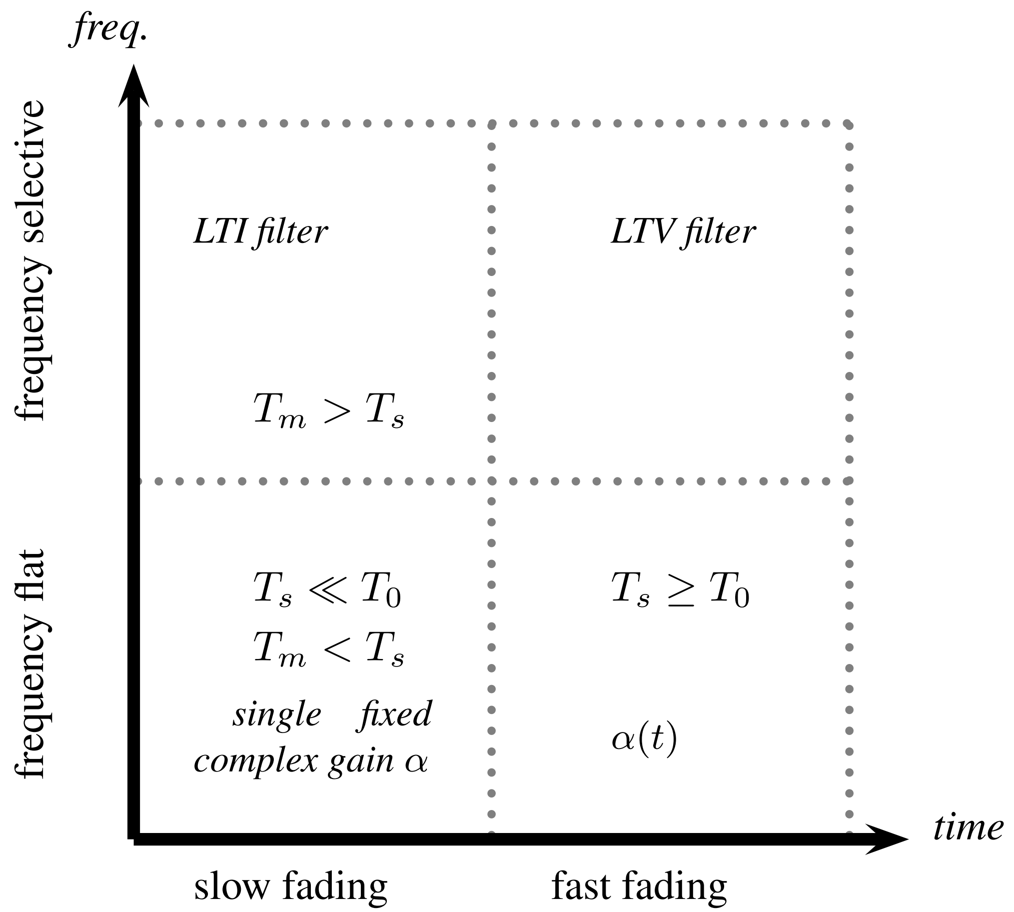
$$y_k = \alpha_k x_{k-\Delta} + z_k$$

Models choose coefficients α

- Fast/Slow fading – speed at which α change.
- Geometric: *multipath caused by "little dots"*
- Stochastic: *random variable, according to a distribution*
- Measurement-based: *Some given data, matched to channel*

We shall consider *slow-fading, frequency-flat* models

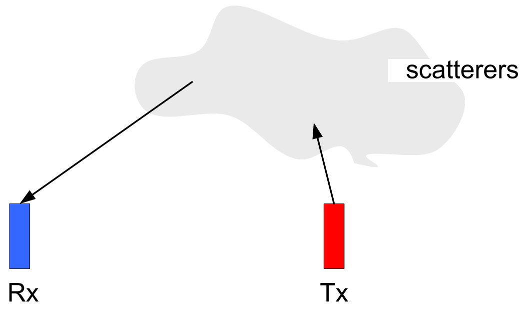
Multipath; SISO model map



With permission from [Han03]

Stochastic Multipath; Rayleigh model

Simplest stochastic model.
Arises from ionospheric measurements

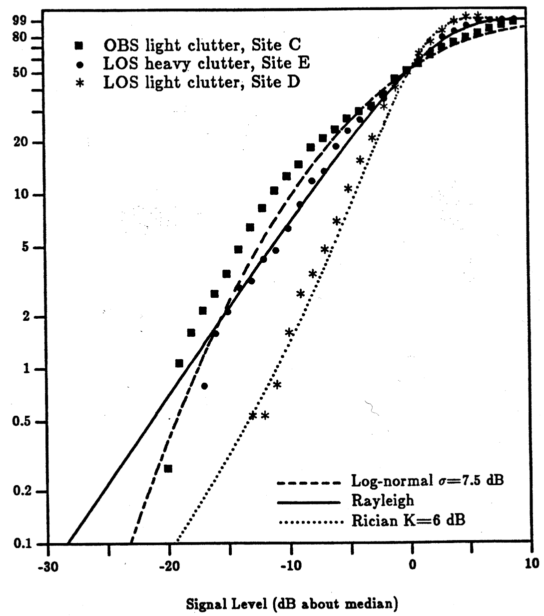


- Receive signal

$$r = \alpha = g e^{\jmath\theta} = \sum (a_i + \jmath b_i) \approx a + \jmath b$$

- a and b are zero-mean, Gaussian r.v.'s (law-large-numbers)

- $|\alpha|$ is *Rayleigh distributed*



From [Rap02]

Other options include *Ricean* (non-zero mean), *Log-normal* and many others. Each model is tailored for the environment of interest.

Multipath; mitigation

- Most works (pre-1995) focus on removing effects of multipath[Skl97]
- Multipath mitigation generates many channel models
- Fading is *time-varying multipath*

- *Estimate multipath channel*
- *Consider initial signal "good" everything else unwanted noise*
- *Use temporal signal char's to remove ISI*
- *Use spatial signal char's to remove additional*
- *Iterate if desired*

[Skl97] B. Sklar. Rayleigh fading channels in mobile digital communication systems part II: Mitigation. *IEEE Commun. Mag.*, pages 105–112, July 1997.

Multipath mitigation; diversity

- Diversity: if some part of the channel is bad (sometimes) then use ensemble

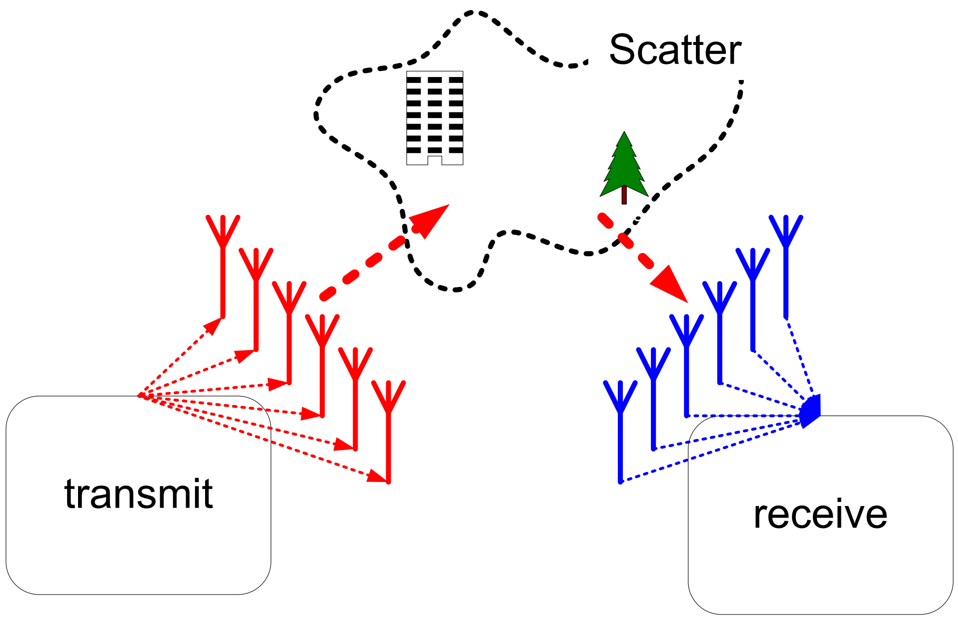
- Time-diversity:
  - Channel may fade (drop-out)
  - spread bits over time (recall previous lectures!)

- Frequency-diversity:
  - doppler, ISI, frequency-selective fades
  - spread frequency of signal (eg. CDMA, UWB)

- Spatial-diversity:
  - multipath comes from different angles
  - steer a beam toward the good angles
  - average over all angles to “stabilize” channel.

MIMO Concept

All that multipath contained signals... why not use it?



We will shortly delve into matrix channels....

"Unfortunately, no-one can be told what the channel is, you have to see it for yourself."

[WSG92] J. H. Winters, J. Salz, and R. D. Gitlin. The capacity of wireless communication systems can be substantially increased by the use of antenna diversity. In *1st International Conference on Universal Personal Communications, (ICUPC '92) Proceedings.*, pages 02.01/1 – 02.01/5, 29 September –1 October 1992.

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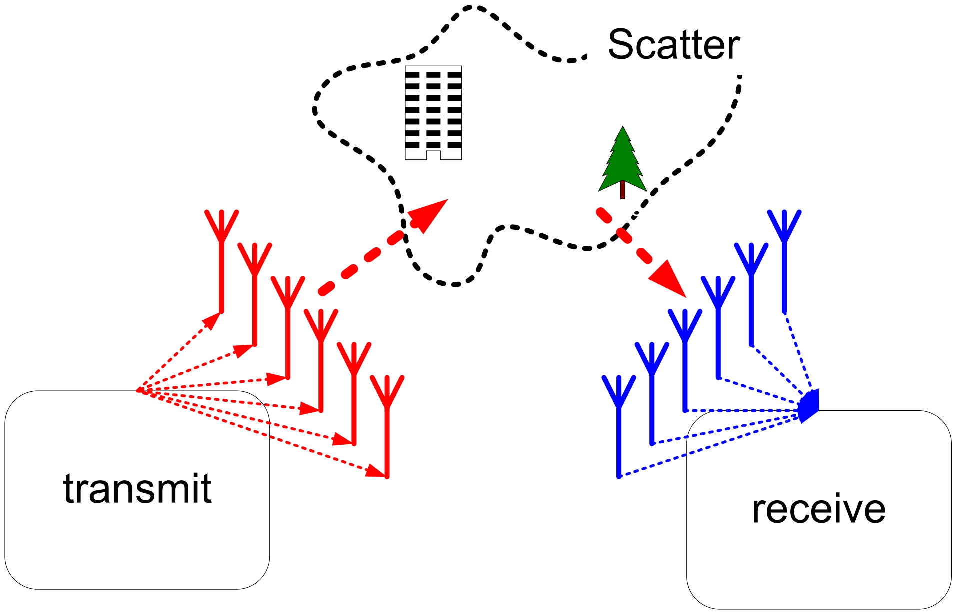
MIMO Big claims



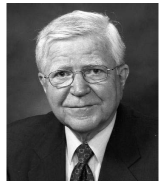
Jack H. Winters

- Capacity of channel increases linearly with number of elements.
- Coding is possible, and not *too* complex
  - Some early results showed huge improvements without coding

[WG94] J. H. Winters and M. J. Gans. The range increase of adaptive versus phased arrays in mobile radio systems. In *Twenty-Eighth Asilomar Conference on Signals, Systems and Computers*, volume 1, pages 109 – 115, 31 October –2 November 1994.

[Win94] J. H. Winters. The diversity gain of transmit diversity in wireless systems with rayleigh fading. In *IEEE Intl. Conf. on Commun., ICC'94 and SUPERCOMM'94 Serving Humanity Through Communications.*, volume 2, pages 1121 – 1125, May 1–5 1994.

Parallel, Additive White Gaussian, Channels[Gal68]



Robert G. Gallager

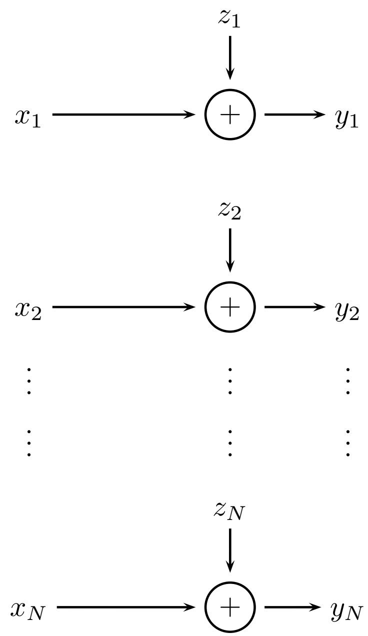
- Consider N channels, which are independent, discrete, parallel, AWGN.

  We have N inputs, power limited, so that $\sum_{n=1}^{N} |x_n|^2 \leq P$. Noise in channel n is *white, Gaussian* with variance (power) $E\left\{|z_n|^2\right\} = \sigma_n^2$

- Recall:

  Capacity of single channel ($N = 1$) with transmit power P and σ^2 noise

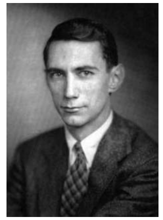
$$\mathcal{I}\left(X_1; Y_1\right) \leq C \leq \log\left(1 + \frac{P}{\sigma^2}\right)$$



[Gal68] R. Gallager. *Information Theory and Reliable Communication*. John Wiley & Sons, New York, USA, 1968.

Parallel, Additive White Gaussian, Channels

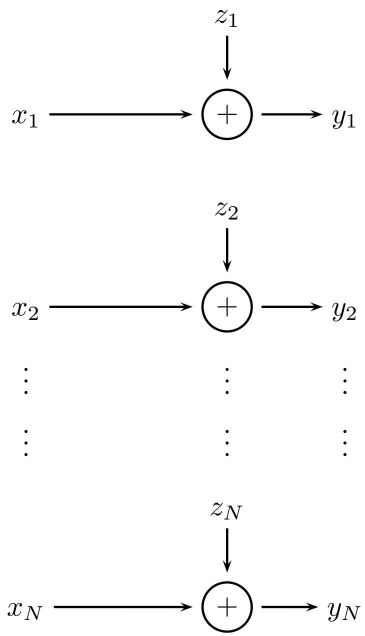
- We can consider the entire ensemble of inputs $\mathbf{X}^N = \{X_1, \ldots, X_N\}$ and entire ensemble of outputs $\mathbf{Y}^N = \{Y_1, \ldots, Y_N\}$



Claude E. Shannon

$$\mathcal{I}\left(\mathbf{X}^N; \mathbf{Y}^N\right) \leq \sum_{n=1}^{N} \mathcal{I}\left(X_n; Y_n\right) \leq \sum_{n=1}^{N} \log\left(1 + \frac{P_n}{\sigma_n^2}\right)$$

$$\sum_{n=1}^{N} P_n \leq P$$



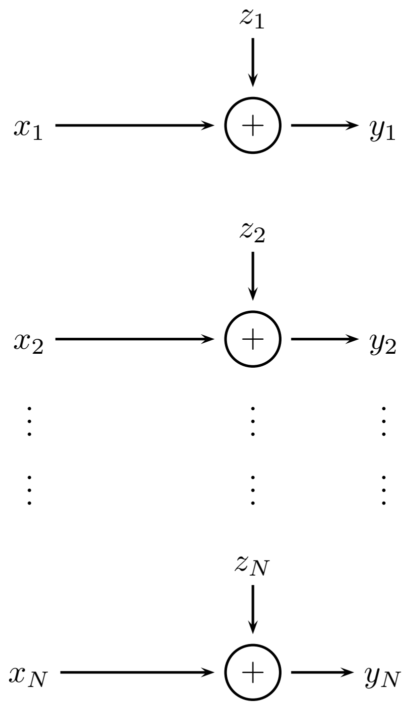
- Equality iff x_n & z_n are independent Gaussian[Sha48]. Gallager uses 1/2, this corresponds to real r.v.'s
- ***Question:*** Can we simplify this?

[Sha48] C. E. Shannon. A mathematical theory of communication. *Bell System Tech. J.*, 27:379–423, 623–656, July 1948.

Parallel, Additive White Gaussian, Channels

- What about equal power noise?

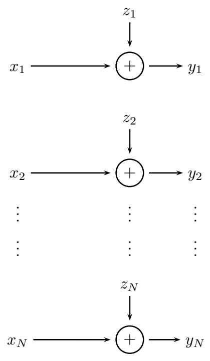
$$\sigma_n^2 = \sigma^2 \quad \forall n$$



Does equal power noise mean we get identical noise samples?

Parallel, ... Waterfilling

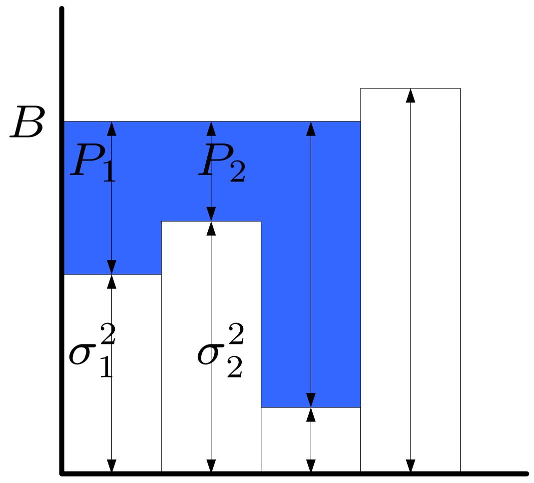
- Assume transmitter knows channel. Assume noise has non-equal powers.
  - Assign more power to "better" channels.
  - Must satisfy power constraint
- Solution: Waterfill. Choose x_n as independent Gaussians, with variance (power) given by P_n such that:



$$P_n = \begin{cases} B - \sigma_n^2; & \sigma_n^2 < B \\ 0; & \text{otherwise} \end{cases}$$

$$C = \sum_{n=1}^{N} \log_2 \left(1 + \frac{P_n}{\sigma_n^2}\right) \quad \text{bit/s/Hz}$$

$$= \sum_{n:\sigma_n^2 \le B}^{N} \log_2 \left(\frac{B}{\sigma_n^2}\right)$$



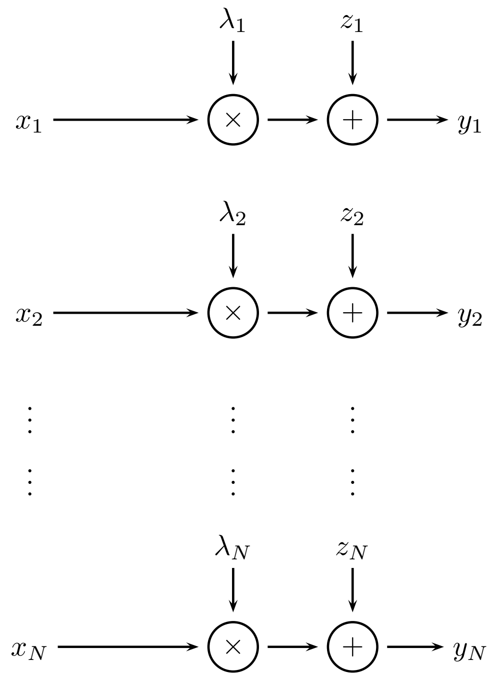
Parallel AWGN channels with gains

- Assume transmitter knows channel.
- Assume noise has non-equal powers.
- Assume each channel also has a gain λ_n
  - Assign more power to “better” channels.
  - Must satisfy power constraint

$$\boxed{C \leq \sum_{n=1}^{N} \log\left(1 + P_n \frac{\lambda_n^2}{\sigma_n^2}\right) \qquad (1)}$$

- Solution?

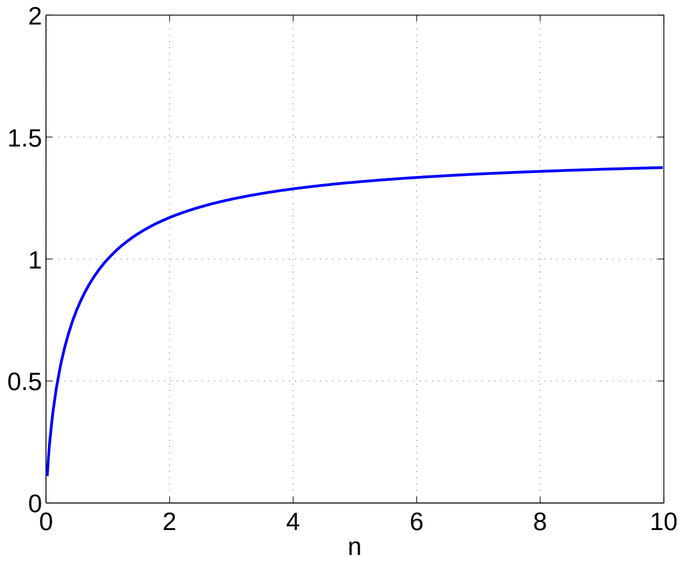
Is this such an amazing result? It was known in 1968.

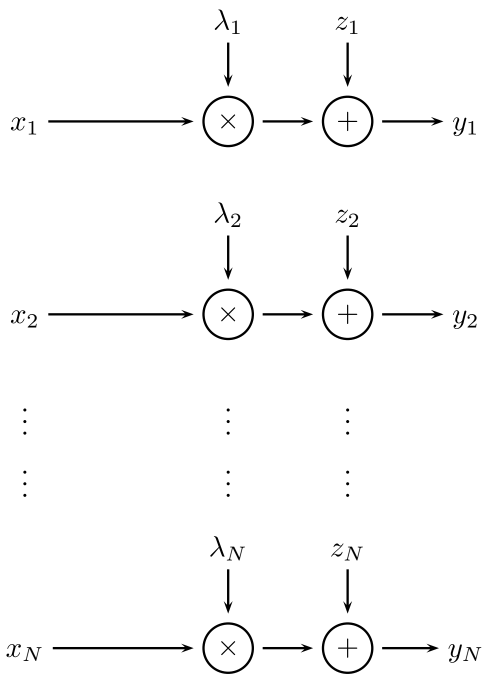


- Assume transmitter knows channel. $\sigma_n^2 = 1$. $\lambda_n = 1$

$$C \leq \sum_{n=1}^{N} \log\left(1 + P_n \frac{\lambda_n}{\sigma_n^2}\right) \qquad P_n = \frac{P}{N}, \sigma_n^2 = 1, \lambda_n = 1$$

$$= N \log\left(1 + \frac{P}{N}\right) \leq P \log_2 e \quad \text{bit/s/Hz}$$





The world pre-MIMO

- Single channel.
  - Capacity proportional to log of power.

$$\boxed{C = \log(1 + P) \qquad (2)}$$

  - lower SNR reduces capacity – equivalent to less power.
- Parallel channels.
  - Waterfilling gives capacity
  - larger gains improve capacity, noise reduces capacity
  - Best capacity is equal gain, equal noise, gives

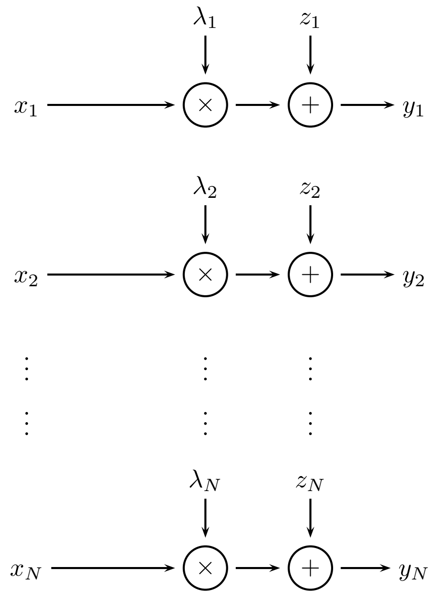
$$\boxed{C \leq N \log\left(1 + \frac{P}{N}\right) \leq P \log e \qquad (3)}$$

MIMO preliminaries

- How can we arrange the parallel channels neatly?
  - collect all inputs and outputs into vectors.

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_N \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_N \end{bmatrix}$$

$$\mathbf{y} = \Lambda \mathbf{x} + \mathbf{z} \tag{4}$$



  - Read off each row to get parallel channels
  - Power limit becomes

$$E\left\{ \mathrm{Tr}\left(\mathbf{x}\mathbf{x}^\dagger \right) \right\} \leq P$$

  - Matrix $\mathbf{Q} = E\left\{ \mathbf{x}\mathbf{x}^\dagger \right\}$ called “covariance” of input signal.

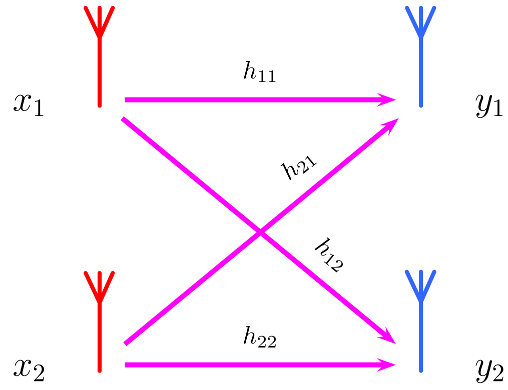
MIMO prelim. toward real systems

- Each receiver (blue) detects signals from *all* transmitters (red) plus iid noise

$$y_r = \sum_{t=1}^{T} h_{tr} x_t + z_r$$

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z} \quad \mathbf{H} = \begin{bmatrix} h_{11} & h_{21} & \cdots & h_{t1} \\ h_{12} & h_{22} & \cdots & h_{t2} \\ \vdots & \vdots & \ddots & \vdots \\ h_{1r} & h_{2r} & \cdots & h_{tr} \end{bmatrix} \tag{5}$$

T transmit, R receive.

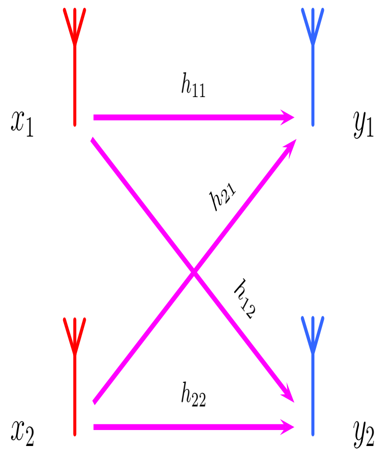


- Note similarity to previous "vector" channel.
  - h_{ij} gives *complex* gain from transmit i to receiver j. Be careful of notation!
  - We will assume receiver has full channel knowledge. Is this reasonable?

MIMO prelim. Full tx knowledge

- Transmitter "diagonalises" channel $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z}$
  - transmit signals in certain way, to ensure no cross-talk.
  - Basic rule: capacity determined by SNR, need white noise. Invariant transforms....
- Unitary transforms are invariant for Gaussian processes
  - SVD of channel, $\mathbf{H} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^{\dagger}$. $\mathbf{\Lambda}$ contains *singular* values, not eigenvalues.
  - Receiver applies "filter" $\hat{\mathbf{y}} = \mathbf{U}^{\dagger}\mathbf{y}$
  - Transmitter modifies distribution $\hat{\mathbf{x}} = \mathbf{V}\mathbf{x}$
  - "New channel" $\hat{\mathbf{y}} = \mathbf{\Lambda}\hat{\mathbf{x}} + \hat{\mathbf{z}}$

$$C = \sum_{m=1:\frac{\sigma_m^2}{\lambda_m^2} \leq B}^{M=\min(R,nt)} \log\left(B\frac{\lambda_m^2}{\sigma_m^2}\right) \tag{6}$$



MIMO prelim. Full tx knowledge [Tel99]

- $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z}$
  - Assume unitary noise $\sigma^2 = 1$. (We adjust P arbitrarily)
  - Covariance of received signal $E\left\{\mathbf{y}\mathbf{y}^\dagger\right\} = \mathbf{H}\mathbf{Q}\mathbf{H}^\dagger + \mathbf{I}_\mathbf{r}$
  - mutual info $\mathcal{I}\left(\mathbf{x};\mathbf{y}\right) = \sum_n \log\left(1 + P_n\lambda_n\right)$
- BUT.
  - $\sum_j \log(a_j) = \log\left(\prod_j a_j\right)$
  - $\det(X) = \prod_j \lambda_j$ where λ is eigenvalues of X.

$$\mathcal{I}\left(\mathbf{x};\mathbf{y}\right) = \log\det\left(I + \mathbf{H}\mathbf{Q}\mathbf{H}^\dagger\right)$$

  - Transmitter gets to choose $\mathbf{Q}$.

$$C = \sup_{\mathbf{x}:\mathrm{Tr}(\mathbf{Q})\leq P} \mathcal{I}\left(\mathbf{x};\mathbf{y}\right) = \sup_{\mathbf{Q}:\mathrm{Tr}(\mathbf{Q})\leq P} \log\det\left(I + \mathbf{H}\mathbf{Q}\mathbf{H}^\dagger\right) \qquad (7)$$

I. Emre Telatar

Jerry G. Foschini

Thomas Marzetta

[Tel99] I. E. Telatar. Capacity of multi-antenna gaussian channels. *Euro. Trans. Telecomm.*, 10(6):585–595, November 1999.

MIMO for real now...

- What if transmitter doesn't know channel? (Is this important?)
  - Receiver still knows $\mathbf{H}$, so we can calculate mutual information
  - $\mathcal{I}(\mathbf{x};(\mathbf{y},\mathbf{H})) = E_{\mathbf{H}}\{\mathcal{I}(\mathbf{x};(\mathbf{y}|\mathbf{H}=H))\}$

$$\boxed{C = E_{\mathbf{H}}\left\{\log\det\left(I + \frac{P}{T}\mathbf{H}\mathbf{H}^{\dagger}\right)\right\}} \tag{8}$$

  - We can't take expectation inside $\log\det$.
  - P/T is the "equal power, transmit white"
- Eqn (8) is HIGHLY abused in literature. *It only applies to* $\mathbf{H}$ *being circularly symmetric Gaussian.*
  - Can ask similar (though not equivalent) question: "what capacity do we get if the input is equal-power Gaussian" for arbitrary channels.
  - Question not necessarily well posed.

[FG98] G. J. Foschini and M. J. Gans. On limits of wireless communications in a fading environment when using multiple antennas. *Wireless Personal Communications*, 6:311–335, 1998.

[MH99] T. L. Marzetta and B. M. Hochwald. Capacity of a mobile multiple-antenna communication link in rayleigh flat fading. *IEEE Trans. Inform. Theory*, 45(1):139–157, January 1999.

Proof outline

1. Start from Eqn (7) pp. 31
2. We can only choose $\mathbf{Q}$.
3. Distribution of $\mathbf{H}$ unchanged by unitary matrices: *not true in general, true for Gaussian ensembles!*
4. So, $\mathbf{Q} = \mathrm{diag}\{q_{11}, q_{22}, \ldots\}$ is a general choice.
5. Apply trace rule, $\mathrm{Tr}(\mathbf{Q}) \leq P$ implies $\sum_i q_{ii} \leq P$
6. Maximum with equality.

It's nice mathematics...

- IF each entry for $\mathbf{H}$ corresponds to an independent, flat-fading Rayleigh channel, then

$$h_{ji} = \mathcal{N}\left(0, 1/\sqrt{2}\right) + \jmath\mathcal{N}\left(0, 1/\sqrt{2}\right)$$

- Law-large-numbers: $\frac{1}{N}\sum_n^N |a|^2 \to 1$ for a complex Gaussian.

$$\boxed{\lim_{T\to\infty} C = R\log\left(1+P\right) \qquad (9)}$$

- Capacity growth is linear with respect to minimum number of transmit/receive elements.
- recall, we have already shown for T independent parallel channels, and P power constraint, the capacity is $C \le T\log\left(1+P/T\right) \le P\log e$ if the transmitter knows the channel.
- Ummm... but there are only T inputs to this channel, and R outputs... how can we get better growth, *when the transmitter doesn't know the channel?*

Main concepts

- MIMO exploits multipath, not mitigation...
- MIMO has linear growth wrt. $m = \min(R, T)$
  - Need full, random matrix
  - flat Rayleigh fading, with well spaced antenna elements, is a good approximation for this
  - Rich Scattering Environment
  - What is “rich?” What happens if environment is not rich?
  - a little counter-intuitive...
- Transmitter knowledge of channel not needed for linear growth:
  transmit equal power, independent signals from all elements.
- Linear growth in capacity means
  - Capacity increases without increasing power – just need more elements.
  - Many interesting theoretical results based on random matrix theory
  - Equal capacity over longer distances
  - More channels for fixed capacity (think telephone lines)

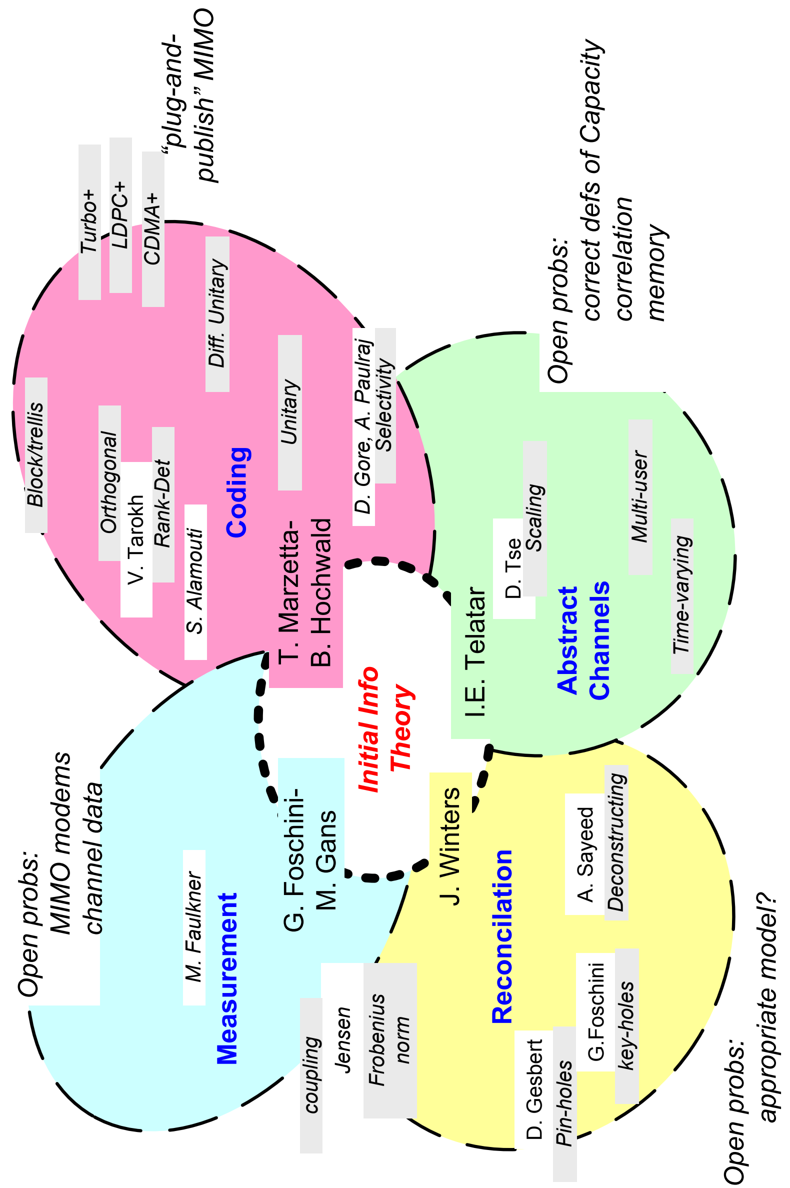
[Mül02] R. R. Müller. A random matrix model of communication via antenna arrays. *IEEE Trans. Inform. Theory*, 48(9):2495–2506, September 2002.

[CTKV02] C.-N. Chuah, D. N. C. Tse, J. M. Kahn, and R. A. Valenzuela. Capacity scaling in MIMO systems under correlated fading. *IEEE Trans. Inform. Theory*, 48(3):637–650, March 2002.

Day 2

- Extensions
- Non-ideal environments
- Coding
- Implementation

- By the end of today you should
  - Want to kill anyone who ever mentions MIMO again.


Initial Info Theory
Coding
T. Marzetta- B. Hochwald
D. Gore, A. Paulraj
Selectivity
Unitary
Diff. Unitary
S. Alamouti
V. Tarokh
Orthogonal
Rank-Det
Block/trellis
Turbo+
LDPC+
CDMA+
"plug-and-publish" MIMO
Abstract Channels
I.E. Telatar
D. Tse
Scaling
Multi-user
Time-varying
Open probs:
correct defs of Capacity
correlation
memory
Reconcilation
J. Winters
A. Sayeed
Deconstructing
G.Foschini
key-holes
D. Gesbert
Pin-holes
Open probs:
appropriate model?
Measurement
G. Foschini- M. Gans
M. Faulkner
Frobenius norm
Jensen
coupling
Open probs:
MIMO modems
channel data

Extensions

- Channel must be ergodic

  DEFINITION 1 (ERGODIC). *A source (channel) is ergodic if* ***every*** *measurable, invariant set of sequences has* ***either*** *probability one* ***or*** *probability zero.*

  Ergodicity allow us to use law-of-large-numbers, and may be interpreted as saying "the average of sampled outputs equals the ensemble average" or "all possible events will (eventually) occur."

- Why is this important? What does it imply about the physical channel?

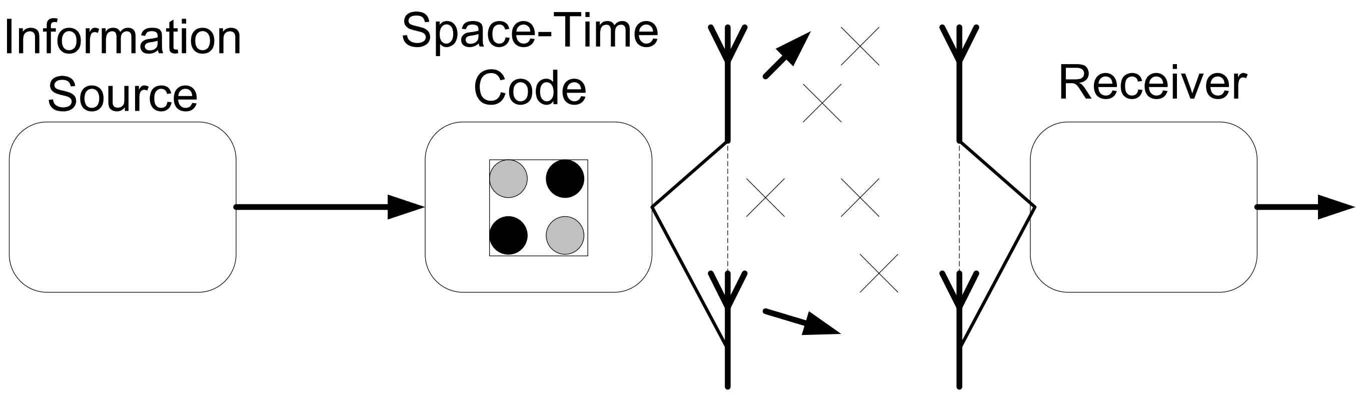
Extensions

- Capacity is a Random Variable.
  Recall, $C = E\left\{\log\det\left(I + (P/T)\mathbf{H}\mathbf{H}^{\dagger}\right)\right\}$
- Expectation is *over all possible channels*
- Random capacity, means there is probability that capacity is not achieved.

DEFINITION 2 (OUTAGE CAPACITY). *Probability that instantaneous capacity is below mean (ergodic) capacity.*

$$P_{outage} = \Pr\left[C_{instant} < E\left\{\log\det\left(I + \frac{P}{T}\mathbf{H}\mathbf{H}^{\dagger}\right)\right\}\right]$$

Using MIMO



Space-time encoding [NSC00]

- We will visit MIMO codes (briefly) later

[NSC00] A. F. Naguib, N. Seshadri, and A. R. Calderbank. Increasing data rate over wireless channels. *IEEE Signal Processing Mag.*, 17(3):76–92, May 2000.

How can we use this?

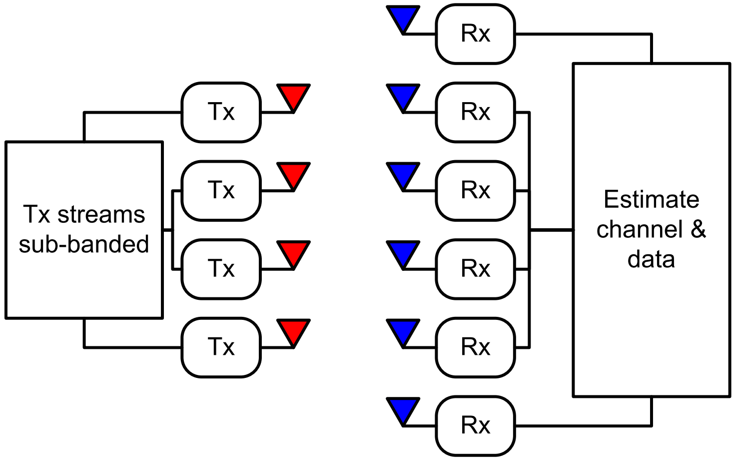
- Brute force: just send the same thing repeatedly!
- Some notation: we write $s_{t,k}$ as the symbol transmit from antenna t at symbol period k

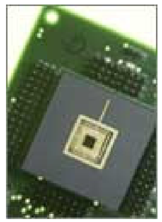
$$\left[\begin{bmatrix} s_{11} \\ s_{21} \\ \vdots \\ s_{T1} \end{bmatrix}^{k=1} \quad \begin{bmatrix} s_{12} \\ s_{22} \\ \vdots \\ s_{T2} \end{bmatrix}^{k=2} \quad \ldots \right]$$

- Every column shows the symbols transmit at time instant k. Rows are all symbols from a single transmitter, over time
  - Think of a transmitter which selects symbols of length T at each time interval.
  - Number of different "meta-codewords" is 2^T. This gives *significantly* lower probability of error.
- Channel assumed stationary

Using MIMO: The Blast experience

- Blast (and V-Blast) developed at Bell Labs to show MIMO capacity claims could be realised.
- Uncoded (simple vector data streams). $T = 8$, $R = 12$ Transmit power scaled $1/T$
- Train-and-estimate, then burst of data, then train ratio $20T_s : 100T_s$
- 30kHz bandwidth, achieved 780kbs indoors. 640kbs datarate
- Theory:
  - $T = 1$, $R = 12$, 9bit/s/Hz. conventional
  - $T = 8$, $R = 12$, 49bit/s/Hz. MIMO

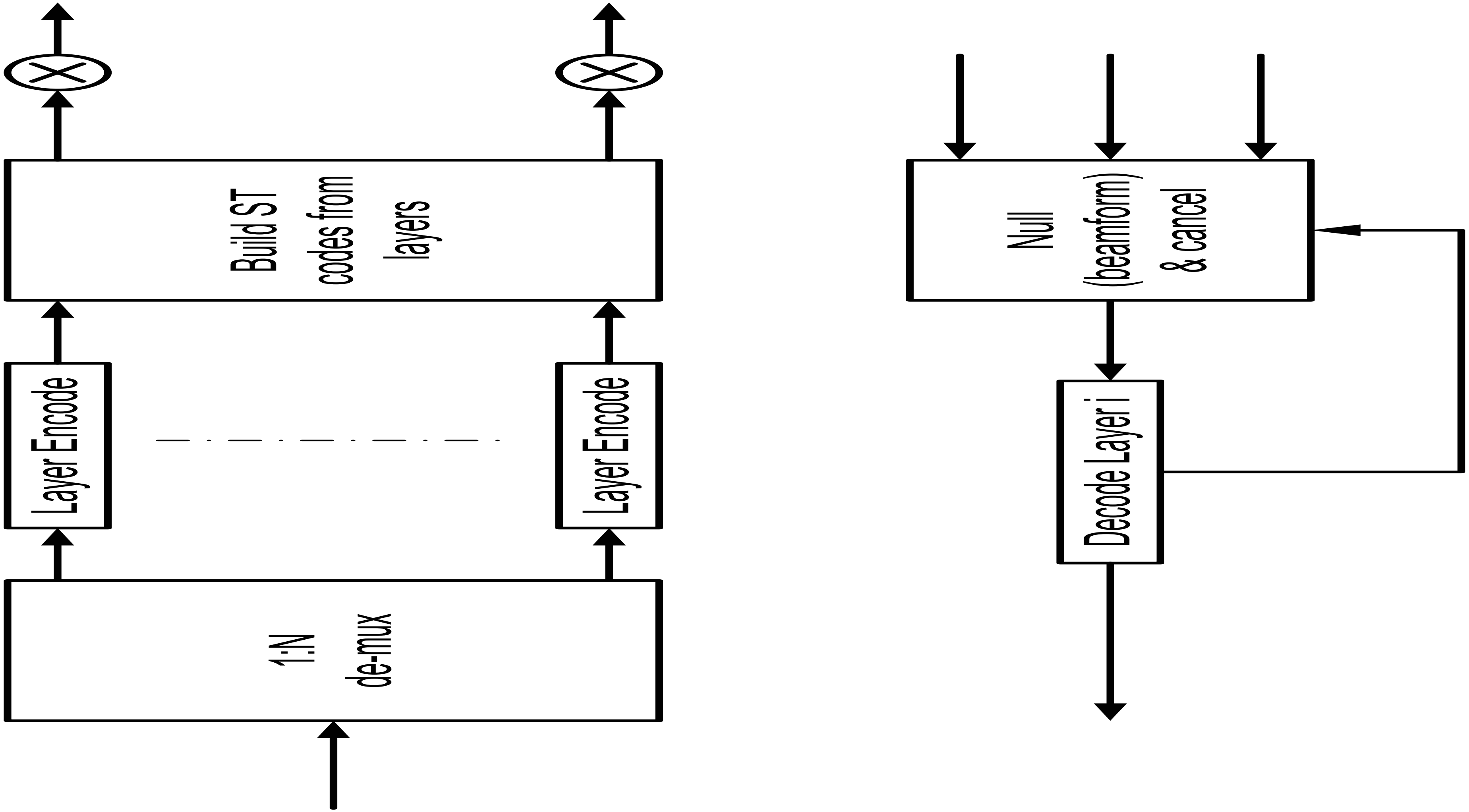




Current BLAST technology.
With perm. from Bell-Labs

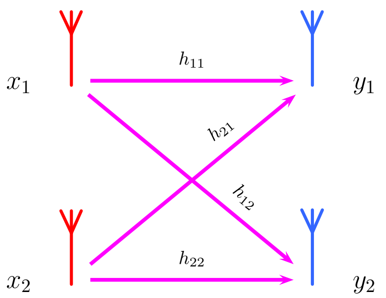
[WGFV98] P. W. Wolniansky, G. D. Golden, G. J. Foschini, and R. A. Valenzuela. V-BLAST: An architecture for realizing very high data rates over the rich-scattering wireless channel. In *URSI Int. Symp. on Signals, Systems, and Electronics, ISSSE-98*, pages 295–300, September 29 – October 2 1998.

Bell Labs lAyered Space Time

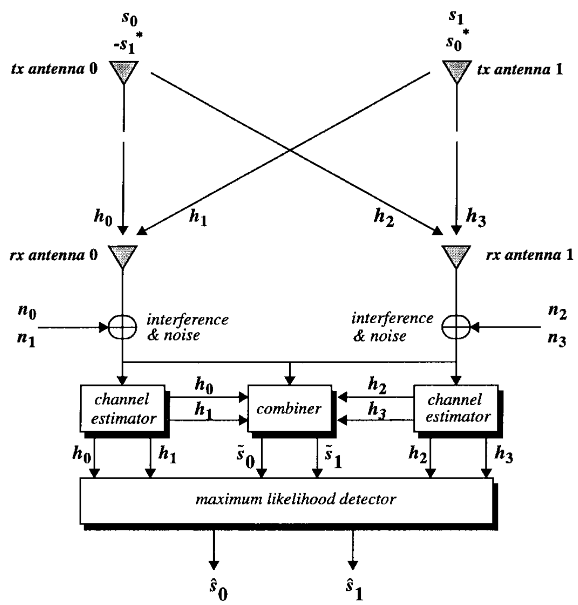


Initial steps to Space-Time Coding

- Whatever is transmitted on transmit element t appears on all receive elements $r = \{1, \ldots, R\}$. Receive signal has gain due to channel plus iid AWGN due to receiver. Initial codes for $T = R = 2$ case.



	Tx 0	Tx 1
time t	s_0	s_1
time $t + T_s$	-$\bar{s}_1$	$\bar{s}_0$



2banch diversity scheme [Ala98]

[Ala98] S. Alamouti. Space block coding: A simple transmit diversity technique for wireless communications. *IEEE J. Select. Areas Commun.*, 16:1451–1458, October 1998.

Space-Time Coding: Alamouti flavour

- Designed as simple way of generating diversity at transmitter
  - Can be used in any multi-dimensional setting *not just space-time*... although all the MIMO interest has clouded that... Not the first, but most famous.
- Received signals:

$$\left\{ \begin{bmatrix} r_0 \\ r_1 \end{bmatrix}^t \quad \begin{bmatrix} r_0 \\ r_1 \end{bmatrix}^{t+T_s} \right\} = \begin{bmatrix} h_{00} & h_{01} \\ h_{10} & h_{11} \end{bmatrix} \left\{ \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} \quad \begin{bmatrix} -s_1^\dagger \\ s_0^\dagger \end{bmatrix} \right\} + \left\{ \begin{bmatrix} z_0 \\ z_1 \end{bmatrix} \quad \begin{bmatrix} z_0 \\ z_1 \end{bmatrix} \right\}$$

$$\mathbf{R} = \mathbf{HS} + \mathbf{Z}$$

- Decoder then applies channel "inverse"

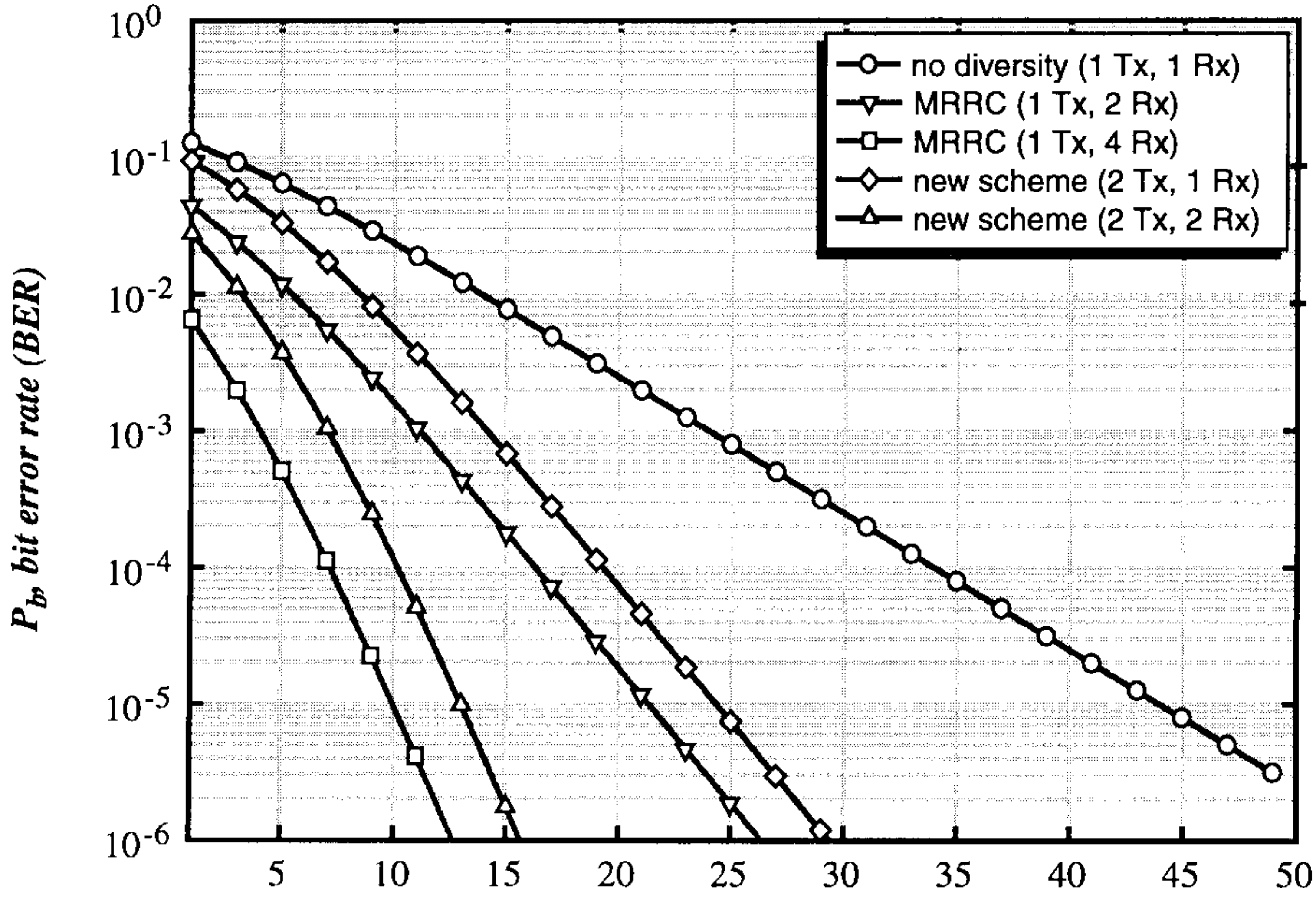
$$d_0 = \overline{h_{00}} r_{00} + h_{01} \overline{r_{01}} + \overline{h_{10}} r_{10} + h_{11} \overline{r_{11}} = s_0 \sum_{jk} |h_{jk}|^2 + w$$

$$d_1 = \overline{h_{01}} r_{00} - h_{00} \overline{r_{01}} + \overline{h_{11}} r_{10} - h_{10} \overline{r_{11}} = s_1 \sum_{jk} |h_{jk}|^2 + w$$

- How to choose correct signal? Pick the either s_1 or s_0 dependent on which is closest to d_i.

[SW93] N. Seshadri and J. H. Winters. Two signaling schemes for improving the error performance of frequency-division-duplex (FDD) transmission systems using transmitter antenna diversity. In *IEEE Vehicular Technol. Conf. (VTC 93)*, pages 508 – 511, May 18–20 1993.

Results



BER performance, coherent BPSK [Ala98, fig 4]

Alamouti problems

- Arbitrary code & decoder design
- What can we do with 3 antenna elements?
- Block codes are rarely as good as trellis codes

Tarokh, etal. Performance Criterions

- Assume random channel (nominally Rayleigh, but some potential for Ricean)
- Make use of simple trick:
  - Error probability given by ensemble average of code-word distance
  - Can be approximated by simple sum.

$$\Pr\left(\mathbf{c} \to \mathbf{e} | h_{ij}, i = 1, \ldots, T, j = 1, \ldots, R\right) \le \prod_{j=1}^{R} \exp\left\{ -H_{:,j} A(\mathbf{c}, \mathbf{e}) H^{\dagger}_{:,j} E_b / 4N_0 \right\}$$

  - $A(\mathbf{c}, \mathbf{e})$ is the (Euclidean) code-distance matrix. This is just the (non-weighted) distance between $\mathbf{c}$ and all errors $\mathbf{e}$
  - $-H_{:,j} A(\mathbf{c}, \mathbf{e}) H^{\dagger}_{:,j}$ is the same thing, *after* H.

Tarokh, etal. Performance Criterions

- How should we choose A?
- **Rank:** $B(\mathbf{c}, \mathbf{e}) = \sqrt{A}$ must be maximal rank, for all code words. Span all possible space. Diversity $\leq \mathrm{rank}\{B\} \min(T, R)$
- **Determinant:** min r roots, of sum of determinants of co-factors (pari-wise) for A bounds diversity. For maximum diversity, maximise the minimum roots of A.

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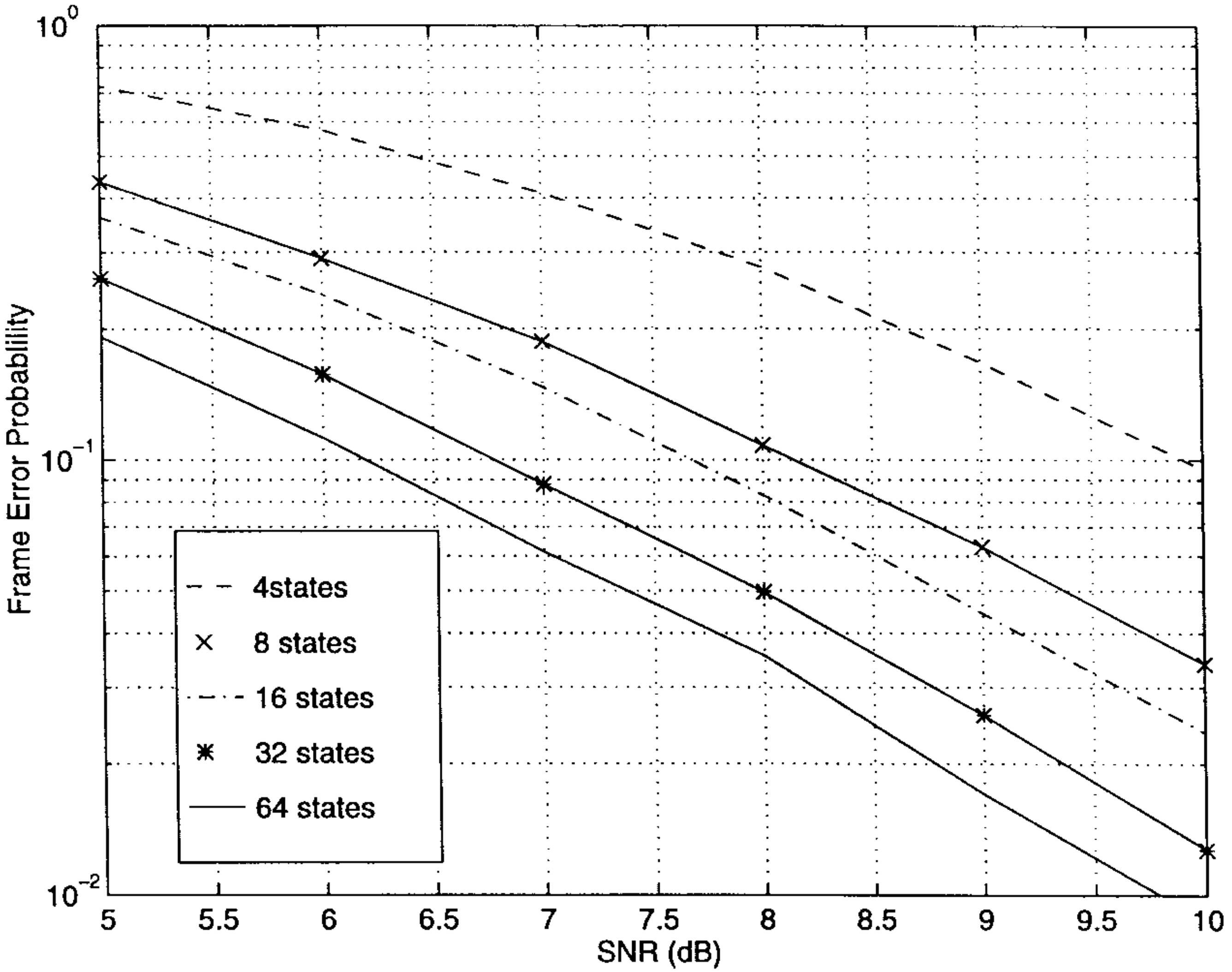
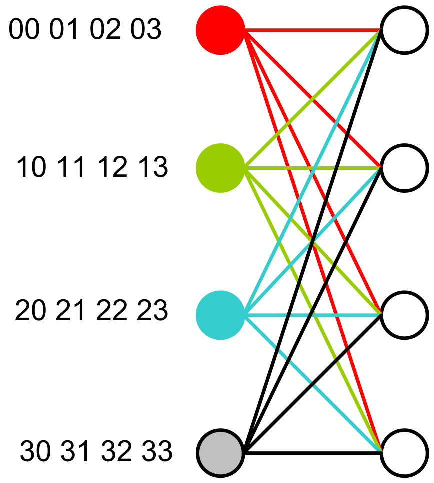
Tarokh, etal. Results

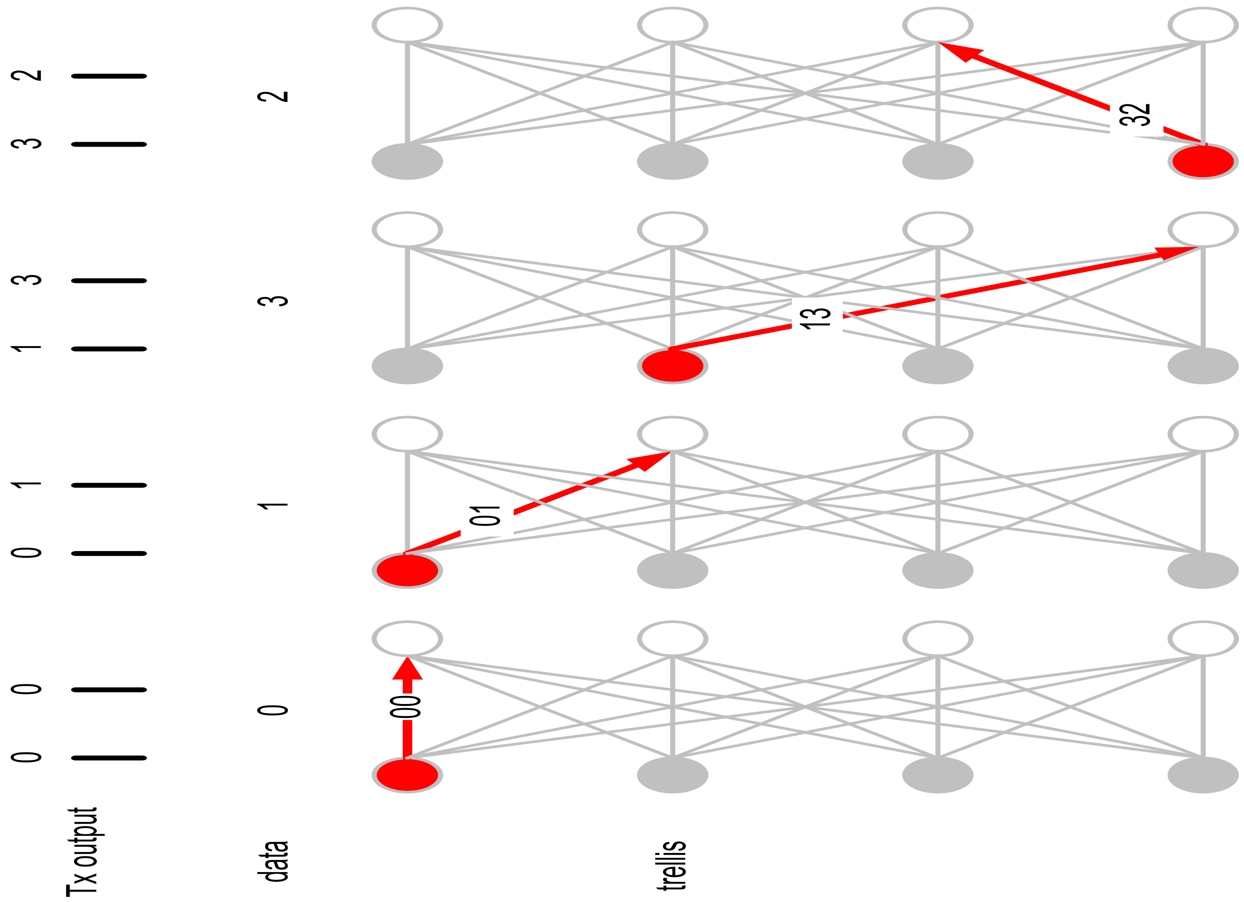


Codes for 4-PSK with rate 2 b/s/Hz that achieve diversity 4 with two receive and two transmit antennas [TSC98].
More states gives better code gain.

Example, encoding



2-transmit/receive,
4-PSK, 4state, 2b/s/Hz
[TSC98, fig 4]



[TSC98] V. Tarokh, N. Seshadri, and A. R. Calderbank. Space-time codes for high data rate wireless communication: Performance criterion and code construction. *IEEE Trans. Inform. Theory*, 44(2):744–765, March 1998.

Example, decoding

- Assumed perfect Channel state info (CSI) $h_{ij} \forall i, j$
- Receiver(s) have symbol $\{r_t^1 r_t^2 \cdots r_t^R\}$ at t
- Branch metric (for label $\{q_t^1 q_t^2 \cdots q_t^R\}$) given by

$$b = \sum_{j=1}^{R} \left| r_t^j - \sum_{i=1}^{T} h_{ij} q_t^i \right|^2$$

- Choose branch with smallest accumulated b.

Example, decoding

- Consider previous code with
  - $H = \begin{bmatrix} 1 & e^{\jmath\pi/2} \\ e^{-\jmath\pi/2} & -1 \end{bmatrix}$
  - Constellation: $0 : 1$, $1 : j$, $2 : -1$, $3 : -j$
- Received symbols:

$$\begin{bmatrix} 1+\jmath \\ -1-\jmath \end{bmatrix} \quad \begin{bmatrix} 0 \\ -2\jmath \end{bmatrix} \quad \begin{bmatrix} 2 \\ 1+\jmath \end{bmatrix} \quad \begin{bmatrix} -2\jmath \\ 0 \end{bmatrix}$$

Unknown channels: unitary

- So far all codes are coherent – need full CSI at receiver.
- Codes which can perform well *without* receiver training are valuable.
- Data model

$$\mathbf{Y} = \mathbf{H}\mathbf{C}_m + \mathbf{Z}$$

- Design:
  - Choose set of unitary matrices $\mathbf{C}_m$
  - and codebook $s_m \Leftrightarrow \mathbf{C}_m$
- Detection:

$$\hat{\mathbf{C}}_m = \arg\max_{\mathbf{C}_k} \mathrm{Tr}\left(\mathbf{Y}\mathbf{C}_m^\dagger \mathbf{C}_m \mathbf{Y}^\dagger\right)$$

[HMR+00] B. M. Hochwald, T. L. Marzetta, T. J. Richardson, W. Sweldens, and R. Urbanke. Systematic design of unitary space-time constellations. *IEEE Trans. Inform. Theory*, 46(6):1962–1973, September 2000.

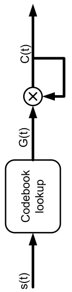
Unitary code design

- Pick a column vector with desired properties, then construct code matrix $\mathbf{S}$ by rotations in Fourier space

$$\mathbf{c} = \begin{bmatrix} c_1 \cdots c_m \end{bmatrix}$$

$$\mathbf{S} = \begin{bmatrix} \exp(\jmath 2\pi/q\mathbf{c}) & \exp(2\jmath 2\pi/q\mathbf{c}) & \exp(3\jmath 2\pi/q\mathbf{c}) & \cdots \end{bmatrix}^T$$

Unknown channels: differential



- Differential codes encode *difference* between successive symbols (NB: in MIMO this is matrix distance). Estimate data from differences, don't estimate channel.
- Received data matrix

$$\mathbf{Y}(t) = \mathbf{HC}(t) + \mathbf{Z}(t)$$

- $C(t)$ is transmit signal, $G(t)$ is a unitary code (see below)

[HS00] B. M. Hochwald and W. Sweldens. Differential unitary space-time modulation. *IEEE Trans. Commun.*, 48(12):2041 – 2052, December 2000.

Differential ST modulation

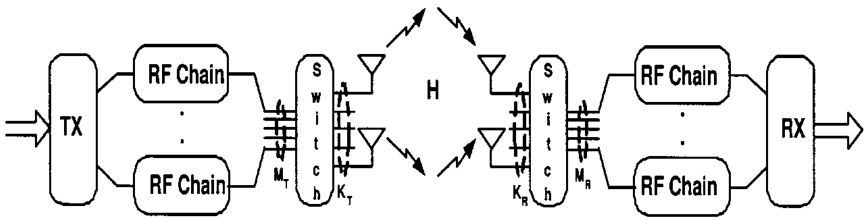
- Design:
  - Design matrix group $\mathcal{G} = \{\mathbf{G}_1, \ldots, \mathbf{G}_K\} : \mathbf{G}_i \mathbf{G}_i^\dagger = I$
  - K is number of possible symbols.
  - Choose mapping $s_k \Leftrightarrow G_k$, $k = 1, \ldots, K$
- Transmission:
  - Initial transmission is $C(0) = G_1$.
  - For each symbol $s(t)$, find the corresponding G from the codebook.
  - transmit

$$\mathbf{C}(t) = \mathbf{G}_m \mathbf{C}(t-1)$$

- Detection

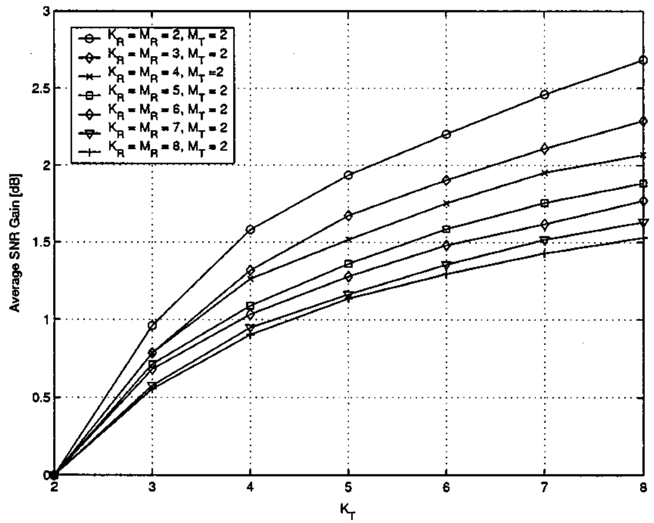
$$\hat{\mathbf{G}}_m = \arg\max_{\mathbf{G}_k} \Re\left\{ \mathrm{Tr}(\mathbf{G}_k)\, \mathbf{Y}(t)^\dagger \mathbf{Y}(t-1) \right\}$$

Problem channels: selectivity



Antenna selectivity [GP02]

- Diversity: if the channel is poor, then maximise a subset of the channels.
- Full knowledge: maximise Frobenius norm
- Stat. knowledge: maximise $\det(\Sigma)$ where $\Sigma = E\{HH^\dagger\}$

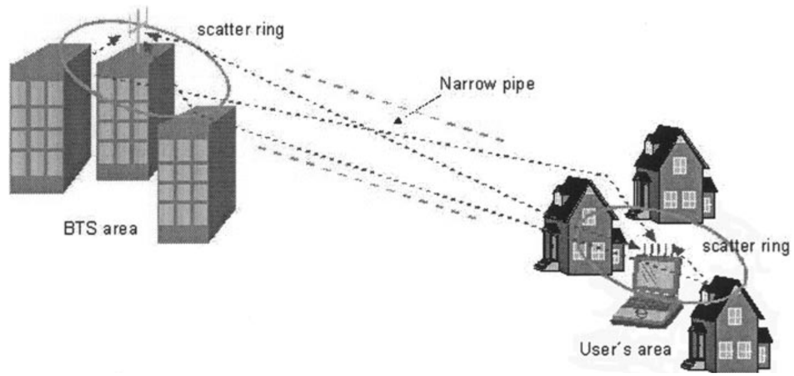


Average SNR gain with full channel knowledge, using Alamouti code [GP02]

[GP02] D. A. Gore and A. J. Paulraj. MIMO antenna subset selection with space-time coding. *IEEE Trans. Signal Processing*, 50(10):2580–2588, October 2002.

2000: Can this be real?

- Academic world essentially convinced that "linear growth" will work.
- Underlying assumption: local diversity is sufficient.
- **Gesbert etal.:** Possible to have full rank, random channel, with very low capacity. May occur when all signals pass through small region of space, even with local diversity.



Pin-hole channel [GBGP02]

Gesbert etal. triggered a rush of work relating to Kronecker models. Measurement campaigns [GAY$^+$01a] found evidence of "key-holes"

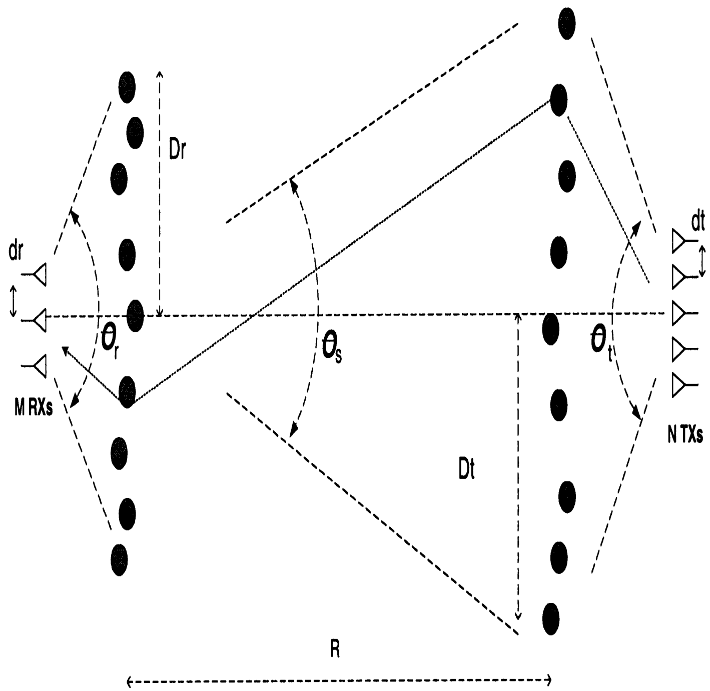
[GBGP00] D. Gesbert, H. Bölcskei, D. Gore, and A. Paulraj. MIMO wireless channels: Capacity and performance prediction. In *IEEE Global Telecommunications Conference, Globecom'00*, volume 2, pages 1083–1088, 2000.

Gesbert: scatterers as spots

- Assume equal power on all scatterers, gives a $1/\sqrt{N_s}$ normalisation, where N_s = number of scatterers.
- Arrays give correlation at each end of the link. Scatterers live on rings (defined abstractly, can be considered as power limit) and produce "virtual arrays"
- Gives overall correlation model:

$$H = \frac{1}{\sqrt{S}} R^{1/2}_{\text{rx}:\theta_r} X^{\text{iid}}_1 R^{1/2}_{\text{s}:2D/S} X^{\text{iid}}_2 R^{1/2}_{\text{rx}:\theta_r}$$

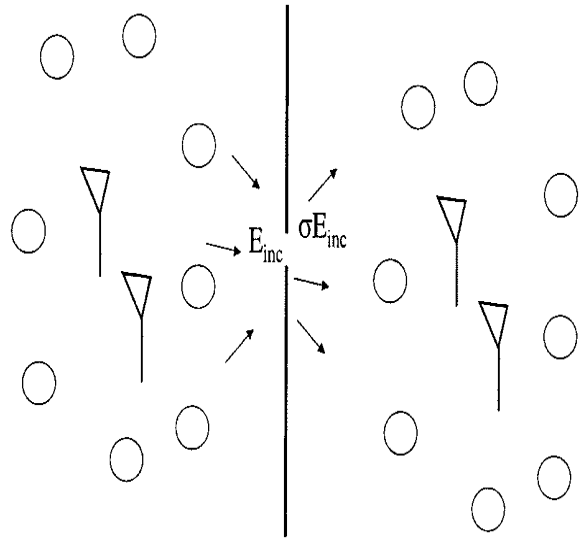
- centre correlation due to "focussing" of scattering rings. ***Weakness:*** virtual arrays.



Virtual scattering array [GBGP02]

Pin-holes vs key-holes.. debate rages

- **Key-holes and pin-holes are not the same!**
- Key-hole caused by inherent-low rank channel
- Pin-hole caused by focussing of scatter
- In the limit, key-hole = pin-hole.



Key-hole concept [CFGV02]

Key-holes & pin-holes both demonstrate local-diversity is not enough. Although some evidence of key-holes exists, they are hard to produce.

[CFGV02] D. Chizhik, G. Foschini, M. Gans, and R. Valenzuela. Keyholes, correlations, and capacities of multielement transmit and receive antennas. *IEEE Trans. Commun.*, 1(2):361–392, April 2002.

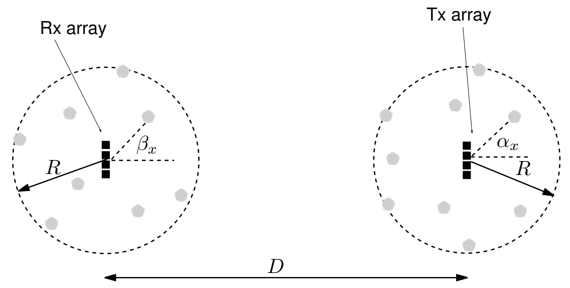
[GAY+01] M. Gans, N. Amitay, Y. Yeh, H. Xu, R. Valenzuela, T. Sizer, R. Storz, D. Taylor, W. MacDonald, C. Tran, and A. Adamiecki. BLAST system capacity measurements at 2.44GHz in suburban outdoor environment. In *VTC 2001 Spring, IEEE Vehicular Technol. Conf. (VTC 01)*, volume 1, pages 288–292, Rhodes, Greece, May 6–9 2001.

Local ring model

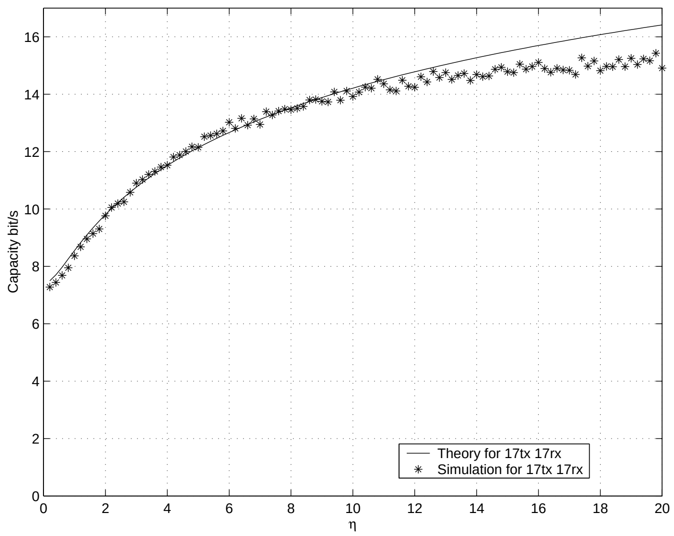
- Can extend Gesbert model [GBGP02] to allow for disks of scatterers (scatterers lie anywhere within rings
- pin-hole essentially due to ratio: $\epsilon = \frac{2\pi R}{\lambda D}$

$$C' \leq \log_2(1 + PR) + R\kappa \log_2\left(1 + P\epsilon^2\right)$$

κ constant.



Local scatter rings [HF02]

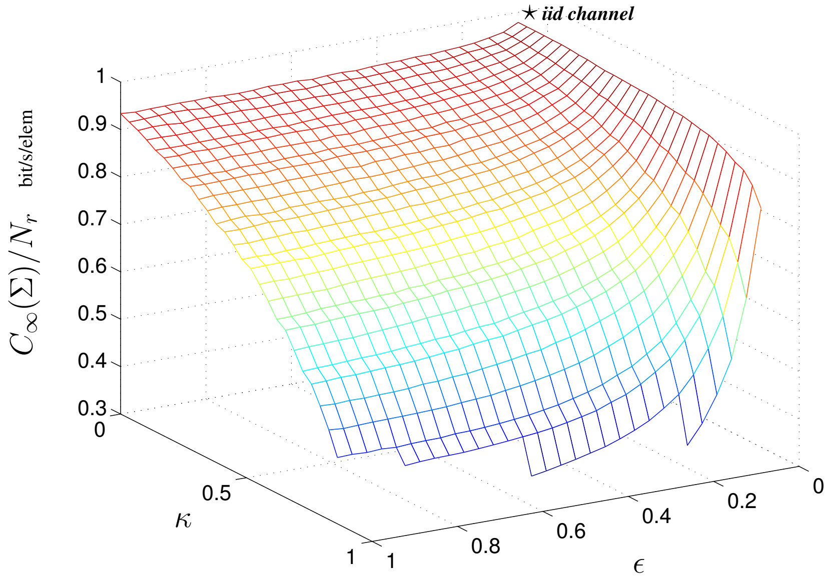


Local scatter rings [HF02]

[HF02] L. W. Hanlen and M. Fu. Multiple antenna wireless communication systems: Limits to capacity growth. In *IEEE Wireless Communication and Networking Conference, WCNC'02*, pages 172–176, Orlando, USA, March 17 – 21 2002.

Current state of the art (theory)

- Initial work sparked renewed interest in channel modelling.
  - Similarly to fading models, MIMO channel models come in many flavours, depending on the context.
- Basic result:
  - Linear growth always happens, just at different rates *if random matrix assumed*.
- Work is commencing on continuous space models



Capacity surface [HG03]

[YO02] K. Yu and B. Ottersten. Models for MIMO propagation channels: a review. *Wireless Communications and Mobile Computing*, 2:653–666, November 2002.

[CRT01] N. Chiurtu, B. Rimoldi, and E. Telatar. Dense multiple antenna systems. In *IEEE Inform. Theory Workshop*, pages 108–109, Cairns, Australia, September 2–7 2001.

Current state of the art (practical)

- Several modems either in test mode (eg. Bell Labs) or in production (eg. Uni Newcastle, CSIRO)
- Channel test results in indoor, outdoor-urban becoming cohesive
- Coding?
  - “plug-and-publish” MIMO currently big
  - New developments? Multi-user, iterative, enhanced channel-knowledge
- Channel models?
  - Lots!

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