X-ray Metrology for Quality Assurance

A. Noble, R. Hartley, J. Mundy and J. Farley
GE Corporate Research & Development
Schenectady, NY 12301

Abstract

There is considerable current interest in deriving accurate dimensional measurements of the internal geometry of complex manufactured parts, particularly castings. This paper describes an approach to the reconstruction of 3D part geometry from multiple digital X-ray images. A novel approach to radiographic stereo is decribed which takes into account the special imaging geometry of the digital X-ray sensor which is modeled by a linear moving array, or pushbroom, camera. The 3D construction algorithm employs a nominal geometric model which is perturbed by X-ray image constraints. Manufacturing applications are discussed and illustrated by experimental results on actual casting images.

1 Introduction

Investment casting designs, particularly airfoils for aircraft engines and gas turbines, are rapidly evolving in complexity. In order to reduce the development cycle for a new design it is necessary to monitor and control the critical dimensions of the casting and associated cores and molds. In addition, detailed knowledge of the casting geometry is necessary to plan the drilling of cooling holes during airfoil manufaturing. A new approach to dimensional control of castings casting machining operations based on X-ray metrology will be described. Precise measurements of casting dimensions and hole geometry can be acheived using a new algorithm for radiographic stereo.

1.1 Related X-ray Work

The most common form of X-ray image is a 2D digital radiographic (DR) image which is formed as the projection of rays from an x-ray point source on to a linear array of detectors, figure **??**. The 3D material density is projected to a 2D image as a line integral. Intensity can be directly related to material thickness assuming that factors such as the point spread function of the imaging system and beam-hardening correction are known and can be corrected for. However, in practice these parameters are not known and to-date industrial inspection from single DR images has largely been restricted to defect detection and part screening applications where the focus is on locating (usually local) abnormalities in a part [1, 2].

A widely used method for internal dimensional measurement is 2D computed tomography (CT) reconstruction from X-ray sources which provides a slice-by-slice view of the internal geometry of a part [3, 4, 5]. CT slices can be stacked on top of each other to provide a volumetric representation of a part (typically with interpolation between the slices to give a smoother object appearance). However, from a practical standpoint volumetric reconstruction remains a thing of the future due to the large amounts of data that have to be analyzed and the fact that data acquisition is slow. Further in many applications, a full volumetric (voxel) analysis of an object (industrial or human) is not necessarily the desired final output. This is particularly true when the goal is to perform dimensional analysis and/or control where analysis typically only involves boundaries of the object.

One can, in theory, measure 3D object geometry from a limited number of views of the object using assumptions about the geometry of the features being reconstructed, X-ray imaging distortions, and feature-based stereo reconstruction techniques. In the medical domain, this approach has been applied to estimate artery structure from biplane angiograms (2 views taken at 90 degrees apart). Here the focus to date has been on evaluating the precision of boundary extraction techniques from 2D X-ray images to provide the features for matching and reconstruction [10, 8, 7] modelling arteries (including bifurcations) and, recovering the 3D medial axes of arterial structures [6, 9].

In this paper we present a novel approach to 3D reconstruction from multiple views based on a linear pushbroom camera which is a simplified version of the pushbroom camera often used in photogrammetry to analyze satellite imagery. In most previous multiple view x-ray reconstruction work, a parallel (affine) pro-

Figure 1. *Scanning geometry.*

Figure 2. *Feature projection.*

jection geometry has been assumed which is only a valid apporximation if the source-to-object distance is much larger than the size of the object. A linear pushbroom camera, however, generates an image that can be considered as a projective image in one direction and an orthographic image in the other. This more accurately captures the imaging geometry of a real X-ray system than an affine or perspective camera model.

2 Approach

2.1 X-ray Imaging Scanning Geometry

The scanning geometry is depicted in Fig 1. A source of X-rays projects X-rays through the part onto a linear X-ray. The plane defined by the X-ray source and the sensor array is known as the sensor plane. A complete image is captured by moving the part by a series of step motions in a direction perpendicular to the sensor plane and capturing a new line of image data at each step. Subsequent images are captured by in the same way after rotating the part through known angles about an axis perpendicular to the sensor plane.

2.2 Feature/Intensity Modeling

See figure 2.

2.3 Object Reconstruction

The imaging geometry is shown in Fig 3. We define a Euclidean coordinate frame as follows. The source is

Figure 3. *Imaging geometry.*

located at the origin of the reference frame. The y and z axes lie in the sensor plane, the z-axis (or *principal axis*) being perpendicular to the sensor array, and the y axis parallel to it. The x axis is perpendicular to the sensor plane, completing a right-handed coordinate system.

A mathematical model has been developed for this sort of imaging geometry, the linear-pushbroom model ([11]). Let $(x, y, z)^T$ be the coordinates of a point in the part at time $t = 0$, the time when the zeroth image line is captured. The coordinates of the corresponding image point are $(u, v) = (u', v'/w')$ where

$$\begin{pmatrix} u' \\ v' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & f & p_v \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/V_x & 0 & 0 \\ -V_y/V_x & 1 & 0 \\ -V_z/V_x & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{1}$$

1. f is the focal length, that is the distance from origin to the sensor array along the z axis.

2. p_v is the coordinate of the principal point.

3. (V_x, V_y, V_z) is the motion vector of the part between lines of the image. Components V_z and V_y will nominally equal zero, corresponding to motion perpendicular to the sensor plane.

Separate Views For each separate image the part is rotated about an axis nominally perpendicular to the sensor plane. This rotation may be parametrized by the following parameters :

1. The rotation angle θ_x;

2. The coordinates (t_y, t_z) of the point where the rotation axis meets the sensor plane;

3. Two angles ϕ_y and ϕ_z determining the orientation of the rotation axis with respect to the perpendicular to the view plane. Nominally, these two angles are zero.

In terms of these parameters, it is possible to compute a 4×4 matrix R such that $(x', y', z', 1)^\top = R(x, y, z, 1)$, where $(x', y', z')^\top$ are the coordinates of a point in the rotated part, and $(x, y, z)^\top$ are coodinates of the same point prior to rotation. Putting this together 1, we find that

$$(u, vw, v) = M(x, y, z, 1)^\top$$

where $M = M_{f,p_v,V_x,V_y,V_z,\theta_x,t_y,t_z,\phi_y,\phi_z}$ is a 3×4 matrix depending on all the parameters.

Constraints : Though parameters V_y, V_z, ϕ_y and ϕ_z are nominally zero, there may be slight inaccuracies which cause them to deviate from their ideal values. In modelling the imaging process we model these parameters as gaussian random variables with zero mean and a small variance. The other model parameters have known values. They will similarly be modelled as random variables with approriately chosen variances reflecting the degree of confidence in their nominal values. All model parameters except the rotation angle θ_x may be assumed to take the same values for all the images.

3 Estimation of Point Positions

Suppose we know the coordinates $\mathbf{u}^i$ of the image of a point $\mathbf{x}$ in a part being inspected, as seen as seen in several views. As long as the camera modelling parameters are known, point $\mathbf{x}$ may in principle be computed as the intersection of the rays corresponding to all the image points $\mathbf{u}^i$. If there are errors in the measurements of the $\mathbf{u}^i$, then the rays will not intersect exactly, and it will be necessary to compute a best fit to the ray intersection. Commonly, however, there may also be uncertainties in the modelling parameters. In this case, to find the point $\mathbf{x}$ it is necessary to weigh the uncertainties in the modelling parameters against the uncertainties in the image coordinate measurements to estimate the most likely point position.

As seen in the section 2.3, the mapping from 3D points $\mathbf{x}$ to image points $\mathbf{u}$ may me expressed as a function $F_{p_1 p_2 \ldots p_N}$ from R^3 (the 3D object space) to R^2 (the image), parametrized by a set of *model parameters* $p_1, p_2, \ldots, p_N$. Suppose we are given measured matching points $\mathbf{u}^i_j$, each of which is the image of an unknown point $\mathbf{x}_j$ as seen in image number i. We are required to estimate the point coordinates $\mathbf{x}_j$ and the parameters p^i_k of each of the views so as to minimize a certain penalty function. Here p^i_k is the $k-th$ parameter of the $i-th$ view. Let $\hat{\mathbf{u}}^i_j = F_{p^i_1 p^i_2 \ldots p^i_N}(\mathbf{x}_j)$, which is the image of the point $\mathbf{x}_j$ as seen in the i-th view with the given calibration. Furthermore, let $\hat{p}^i_k$ be *a priori* estimations of the values of the modelling parameters. The penalty function to be minimized is

$$\sum_{i,j} w^i_j ||\mathbf{u}^i_j - \hat{\mathbf{u}}^i_j||^2 + \sum_{i,k} v^i_k (p^i_k - \hat{p}^i_k)^2 \qquad (2)$$

where $(w^i_j)^{-1}$ is the variance in the measurement of $\mathbf{u}^i_j$ and $(v^i_k)^{-1}$ is the variance associated with the a priori estimates of the parameters p^i_k.

This estimation problem is solved using the Levenberg-Marquardt ([**?**]) parameter estimation program. In this method, an initial guess at the values of the parameters is refined by iteration to reach a final least-squares estimate optimizing (2). At each step, of the iteration, an adjustment to the values of the active variables is computed under an assumption of local linearity. If the modelling parameters are known with moderate accuracy, as is the case with the turbine-blade imaging setup, then the convergence is rapid from any initial estimate of the points $\mathbf{x}_j$.

4 Registering two 3D point sets

Given two point sets $\{\mathbf{x}_j\}$ and $\{\mathbf{x}'_j\}$ in R^3 we address here the problem of registering these two point sets. In particular, we assume that the points are related by an unknown 3D similarity transformation, T, that is, the composition of a rotation, translation and isotropic scaling. The goal is to compute T. Since in the presence of noise one can not expect an exact fit, one seeks instead an optimal least-squares solution. In particular, we seek a similarity transformation T that minimizes the error

$$\sum_j ||\mathbf{x}_j - T\mathbf{x}'_j||^2 \qquad (3)$$

An efficient algorithm for computing the T that minimizes this term was given by Horn ([**?**]). This algorithm uses quaternions to represent the unknown rotation, leading to a non-iterative rapid solution.

5 Results

Three x-ray images from the sequence used in the experiments are shown in figure 4.

5.1 Synthetic Phantom

Experiment to determine the accuracy of the linear pushbroom camera.

Figure 4. *Images from the sequence.*

5.2 Example Reconstruction for Blade 1

5.3 Varying the number of blade views

5.4 Statistics for a suite of blades

5.5 Varying the camera model

Perspective vs. Affine vs Linear Pushbroom.

6 Future Work

- Use of statistics -¿ tolerancing.
- improved intensity model - Accomodate realistic point spread function and non-Gaussian (Poisson) noise model.
- Nonlinear optimization - Combined intensity and geometry approach to refine geometry-based 3D reconstruction.
- Other imaging geometries such as conebeam geometry (area detectors).

References

[1] H.Boerner, and H. Strecker. Automated X-Ray Inspection of Aluminum Castings. PAMI, vol 10, no. 1, pages 79-91, 1988.

[2] A. Noble, V.D. Nguyen, C. Marinos, et al. Template Guided Visual Inspection. ECCV, Santa Margherita, Italy, pages 893-901, 1992.

[3] H.J. Scudder. Introduction of Computer Aided Tomography. Proc. IEEE, vol. 66, pages 628-637, 1978.

[4] A. Kak. Computerized Tomography with X-rays, Emission, and Ultrasound Sources. Proc. IEEE, vol. 67, pages 1245-1272, 1979.

[5] A.C. Kak and M. Slaney. Principles of Computerized Tomographic Imaging. IEEE Press, New York NY, 1988.

[6] K. Kitamura, J. Tobis, and J. Sklansky. Estimating the 3D Skeletons and Transverse Areas of Coronary Arteries from Biplane Angiograms. IEEE Trans. Med. Imaging, vol 7, no 3, pages 173-187, 1988.

[7] L.V. Tran and R.C. Bahn and J. Sklansky. Reconstructing the Cross Sections of Coronary Arteries from Biplane Angiograms. IEEE Trans. Med. Imaging, vol. 11, no. 4, pages 517-529, 1992.

[8] T.N. Pappas and J.S. Lim. A New Method for Estimation of Coronary Artery Dimensions in Angiograms. IEEE Trans. Acoust. Sig., vol. 36, no. 9, pages 840-858, 1988.

[9] Y. Bresler and J.A. Fessler and A. Macovski. A Bayesian Approach to Reconstruction from Incomplete Projections of a Multiple Object 3D Domain. IEEE PAMI, vol. 11, no. 8, pages 840-858, 1989.

[10] K. Shmueli, W.R. Brody, A. Macovski. Estimation of Blood Vessel Boundaries from X-ray Images. Opt. Eng., vol. 22, no. 1, pages 110-116, 1983.

[11] R. Hartley and R. Gupta. Linear Pushbroom Cameras. Submitted for publication.