Asymptotics for 0−1 Matrices With Prescribed Line Sums

Brendan D. McKay

Australian National University
Canberra, Australia

ABSTRACT

Let $M(n, k)$ be the number of $n \times n$ 0−1 matrices with each line sum equal to k. Let $0 < \epsilon < \frac{1}{6}$. Then

$$M(n, k) = \frac{(nk)!}{(k!)^{2n}} \exp\left(-\frac{(k-1)^2}{2} + 0\left(\frac{k^3}{n}\right)\right)$$

uniformly for $1 \le k \le \epsilon n$. This is generalised to rectangular 0−1 matrices with arbitrary (possibly non-equal) line sums. A limited set of compulsory zeroes can also be specified.

1. Introduction.

Let $g = g(n) = (g_1, g_2, \ldots, g_n)$ and $g' = g'(n') = (g_{1'}, g_{2'}, \ldots, g_{n'})$ be sequences of non-negative integers, and define $M(g, g')$ to be the set of all 0−1 matrices of order $n \times n'$ whose i-th row sum is g_i $(1 \le i \le n)$ and whose i-th column sum is $g_{i'}$ $(1 \le i \le n')$. For obvious reasons, we will always assume that $\sum_{i=1}^n g_i = \sum_{i=1}^{n'} g_{i'}$. We will be concerned with the asymptotic properties of $M(g, g')$ as $n, n' \to \infty$. In particular, let $N(g, g') = |M(g, g')|$. The value of $N(g, g')$ has been the object of much study over the past 20 years, so we start by summarizing the previous results.

Define $I = \{1, 2, \ldots, n\}$, $I' = \{1', 2', \ldots, n'\}$, $I^* = I \cup I'$, $g_{\max} = \max_{i \in I^*} g_i$, and $\alpha = 2(\sum_{i=1}^n \binom{g_i}{2}) (\sum_{i=1}^{n'} \binom{g_{i'}}{2}) e(G)^{-2}$, where $e(G) = \sum_{i \in I} g_i$. Define $P(g, g')$ by

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