A coupled estimation and control analysis for attitude stabilisation of mini aerial vehicles.

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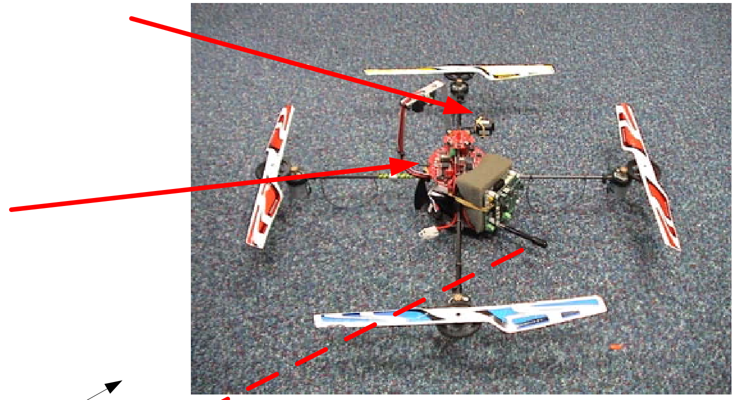
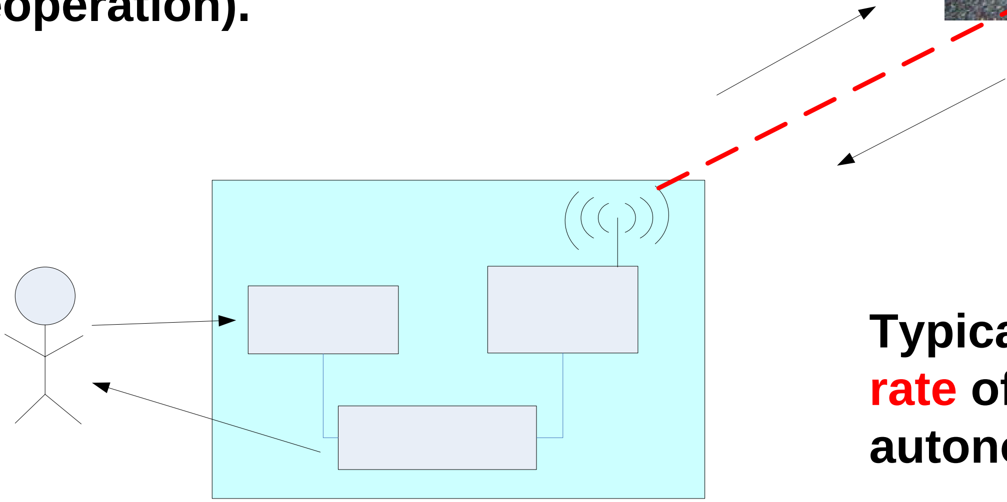
A typical university lab UAV research testbed

- Lightweight **low-cost** airframe
- Low-quality **low-cost** inertial sensor unit.
- Low grade **low-cost** camera system with off board processing.
- Low data-rate **low-cost** uplink (lag in teleoperation).



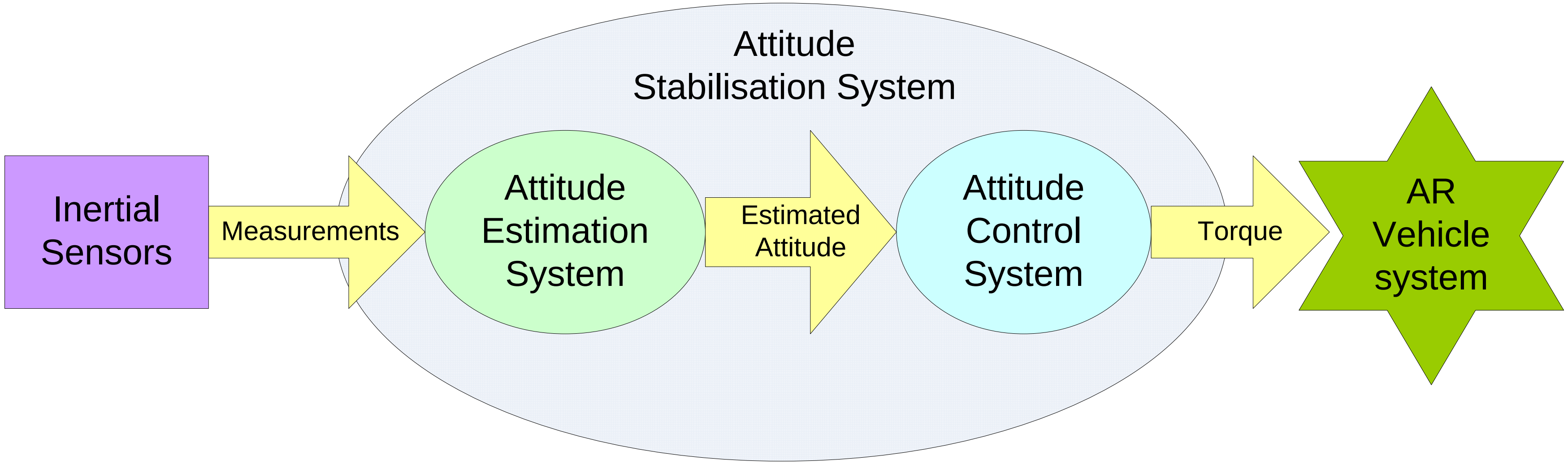


Typical outcome is a low-rate of success in autonomous flight.

Contribution of this paper

- To develop an autonomous attitude stabilisation algorithm for a hovering UAV that:
  - Deals with low quality inertial measurement units.
    - High levels of noise, particularly in accelerometer readings.
    - Slowly time varying gyro biases.
    - Unreliable magnetometer output.
  - Is simple and computationally straightforward to implement.
  - Is highly reliable and very robust.

Coupled estimation and control (dual control)

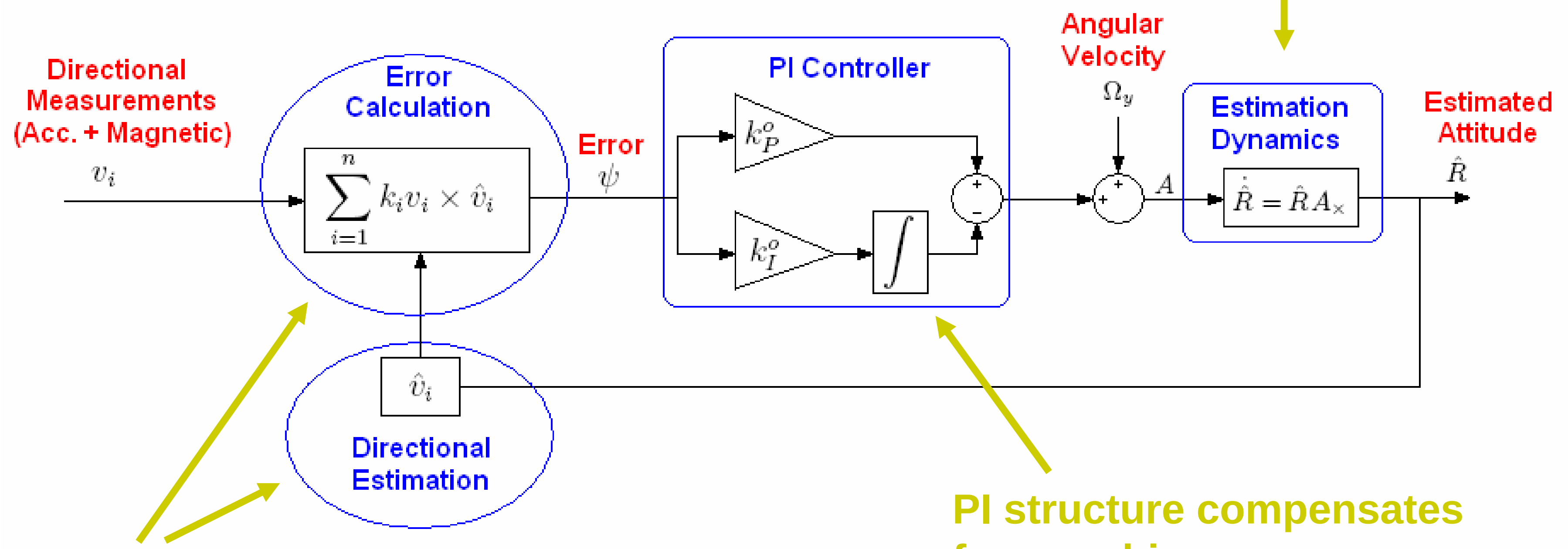


- **Estimating the state (airframe attitude) is more important than clever control.**
- **The overall closed-loop control algorithm must be robust to noise and to measurement bias.**
- **The algorithm must be computationally efficient (for implementation on embedded avionic system)**

Non-linear complementary filter

- Global stability proved in prior work.
- Quaternion implementation is computationally efficient

Fully non-linear model of system kinematics



Driving error term expressed in terms of IMU measurements.
No algebraic reconstruction of attitude.

PI structure compensates for gyro bias seen as constant load disturbance in feed-forward term.

Non-linear passivity based control

⋆ Introduce a desired trajectory $(R_d(t), \Omega^d(t))$ such that

$$\dot{R}_d = R_d \Omega^d_\times \qquad \text{Kinematics}$$
$$I\dot{\Omega}^d = -\Omega^d \times I\Omega^d + \tau \qquad \text{Dynamics,}$$

⋆ Introduce a kinematic and velocity error

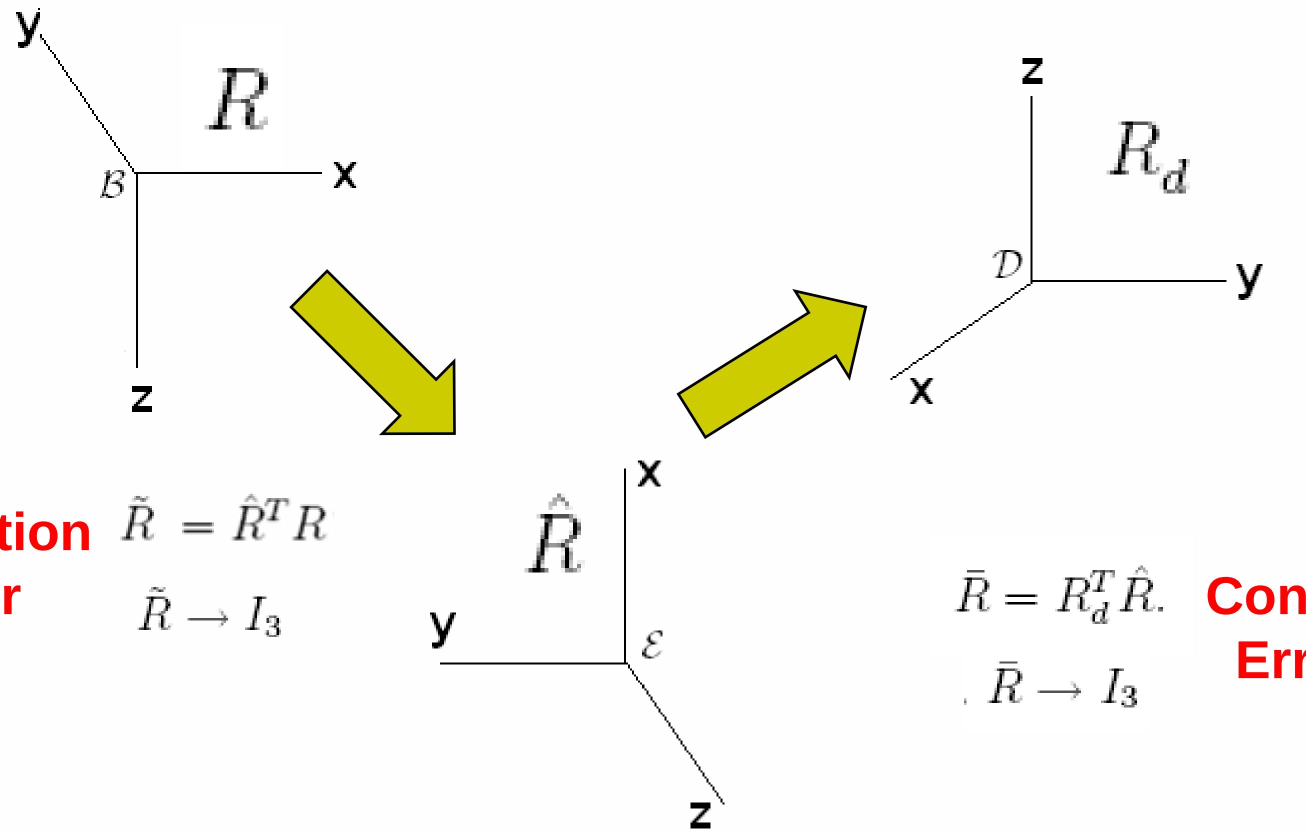
$$\bar{R} = R_d^T \hat{R}$$
$$\varepsilon = \Omega - \Omega^d + k_P^v \text{vex}(\mathbb{P}_a(\bar{R})),$$

where $k_P^v > 0$ denotes a constant proportional gain.

⋆ Introduce the energy based Lyapunov function

$$V = \frac{1}{2}\varepsilon^T I \varepsilon + \frac{1}{2} k_P^c \text{tr}(I_3 - \bar{R})$$

Dual error structure



Estimation Error $\tilde{R} = \hat{R}^T R$

$\tilde{R} \to I_3$

$\bar{R} = R_d^T \hat{R}.$ **Control Error**

$\bar{R} \to I_3$

Global stability follows from proving all error variables converge: $\tilde{R} \to I, \epsilon \to 0, \bar{R} \to I.$

Coupled stability analysis

Passivity based feedback control passes through a feed-forward control transformation

$$\tau_d = -k_D^c \varepsilon - k_P^c \text{vex}\mathbb{P}_a(\bar{R}))$$

Input transformation

$$\tau = I\dot{\Omega}^d + \left(\Omega^d \times I(\Omega_y - \hat{b})\right) - k_P^v \mathbb{P}_a(\bar{R}) I(\Omega_y - \hat{b}) - k_P^v I \text{vex}(\mathbb{P}_a(\dot{\bar{R}})\right) + \tau_d$$

System dynamics

$$\begin{cases} \dot{R} = R\Omega_\times \\ I\dot{\Omega} = -\Omega \times I\Omega + \tau \end{cases}$$

Estimator dynamics

$$\begin{cases} \dot{\hat{R}} = \hat{R}[\Omega_y - \hat{b} + k_P^o \omega]_\times \\ \omega = -\text{vex}\left(\sum_{i=1}^{n} k_i v_i \times \hat{v}_i\right) \\ \dot{\hat{b}} = -k_I^o \omega + k_I^o k_P^c \text{vex}(\mathbb{P}_a(\bar{R})) \end{cases}$$

$$\varepsilon \to \mathbf{0}$$

$$\tilde{R} \to I_3$$

$$\bar{R} \to I_3$$

bounded error coupling

$\rightarrow$ Global Stability

Quaternion formulation of closed-loop system

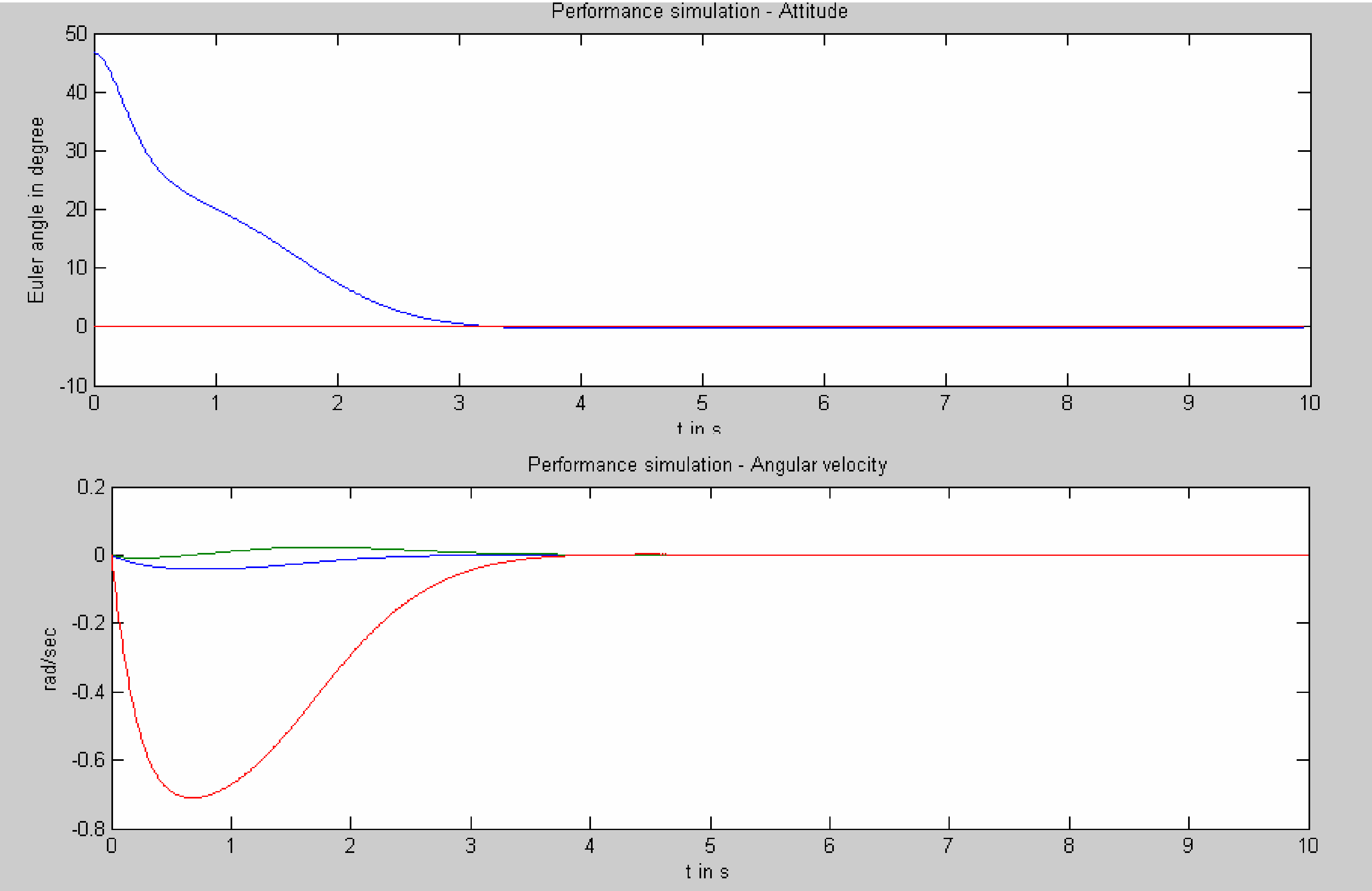
$$
\begin{aligned}
\dot{q} &= \frac{1}{2} q \otimes \mathbf{p}(\Omega) \\
I\dot{\Omega} &= -\Omega \times I\Omega + \tau \\
\tau &= I\dot{\Omega}^d + (\Omega^d \times I(\Omega_y - \hat{b})) - k_P^v(\bar{s}\bar{v} \times I(\Omega_y - \hat{b})) \\
&\qquad - k_P^v I(\dot{\bar{s}}\bar{v} + \bar{s}\dot{\bar{v}}) + \tau_d, \\
\tau_d &= -k_D^c \varepsilon_q - k_P^c \bar{s}\bar{v} \\
\varepsilon_q &= (\Omega_y - \hat{b}) - \Omega^d + k_P^v \bar{s}\bar{v}, \\
\dot{\hat{q}} &= \frac{1}{2}\hat{q} \otimes \mathbf{p}(\Omega_y - \hat{b} + \omega), \qquad \omega = -\mathrm{vex}\left(\sum_{i=1}^{n} k_i v_i \times \hat{v}_i\right) \\
\dot{\hat{b}} &= -k_I^o \psi_q + k_I^o k_P^c \bar{s}\bar{v}
\end{aligned}
$$

where $\{k_P^o, k_I^o, k_P^v, k_P^c, k_D^c\}$ are positive gains chosen such that $2k_P^v > k_P^c k_P^o$.

Simulations

Attitude response

Velocity response



Quadrotor stabilised in stable hover using complementary filter and high gain feedback

Quadrotor shown here was developed by Nicolas Guenard at the Centre d'Energie Atomique, Every, France.

Conclusions

- **Complementary Filter design is used for attitude estimation system.**
  - **Filter is robust and computationally efficient when implemented in quaternion formulation.**
- **Non-linear passivity based control is used for attitude control system.**
  - **Based on feed-forward input transformation and known trajectory.**

    [Perhaps trying to do too much…]
- **Global stability of the coupled scheme is proved.**
  - **Gain condition for global stability:** $2k^{v}_{P} > k^{c}_{P}\, k^{o}_{P}$

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