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A study on the digital moiré technique with circular and radial gratings

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Abstract

A digital moiré technique with circular and radial gratings is proposed to measure in-plane displacement and strain distributions of soft materials in a polar coordinate system. By introducing the coordinate-transform technique, the concentric circles and radial lines in the Cartesian coordinate system are converted to cross gratings in a polar coordinate system. Then, the digital moiré fringes are generated using logical and filter operations on the one-bit binary images. Finally, the in-plane displacement distributions are obtained by means of the four step phase-shifting procedure. Also, the measurement principles and the basic procedures of the new digital moiré technique are explained in detail. An application for rubber sheet with a single-edge crack is studied, and the experimental results show that the presented method is feasible.

Keywords: Digital moiré; Phase shifting; Circular and radial gratings; Coordinate transform; Large deformation

1. Introduction

The moiré method is a noncontact, full-field and convenient measuring technique of in-plane displacement and strain [1]. Moiré fringes, which are contour maps of the displacement components in the direction perpendicular to the grating lines, are typically generated by the superposition of two periodic gratings: a reference grating and a deformed (specimen) grating. Using logical operations for generating moiré fringes, Asundi et al. [2–4] developed the digital moiré method and adopted phase-shifting technique for quantification of the patterns. According to this method, logical operations of computer binary gratings provided an alternative to simulate the aliasing phenomenon generated in traditional moiré patterns. Subsequently, there have been lots of efforts in the application for topographic contouring and deformation measurement using the phase-shifting and logical moiré (PSALM) technique [5–8]. Since the two gratings handled in digital moiré method are made up of binary numbers, the computer-generated reference grating can be precisely shifted without any phase shifters used in traditional moiré method. With the development of nano-meter technology, Xie et al. [9–11] presented the digital nano-moiré method and its phase-shifting technique, a new method in which the nano-meter deformation of an object is measured with high-resolution microscopes.

For several kinds of solid mechanics problems, it is more convenient to analyze the in-plane displacement and strain fields in a polar coordinate system than in the Cartesian coordinate system. Accordingly, moiré fringes created by circular and radial gratings are more adaptable for the purpose of specific applications in the polar coordinate system. As such, some studies on circular and radial gratings have been carried out for the measurement of rigid-body displacements or small angle displacements. Park and Kim [12,13] proposed a moiré technique of concentric-circle gratings to measure the radial motion of a spindle and the relative two-dimensional planar displacement. Circular or radial moiré gratings used for rotation measurement [14,15] and angular displacement were also investigated [16]. Moreover, an elongated circular (EC) grating, instead of the conventional circular grating, was chosen to determine both the rigid-body displacement and its direction [17,18]. Besides, the circular and radial

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gratings were also employed in vision identification with optical measuring systems [19–21]. In order to improve the automatic analysis of moiré fringes, Zhang et al. [22] also suggested an optical method which combined the phase-shifting technique with circular grating moiré method.

To the authors' knowledge, however, there is no report on deformation measurement using circular and radial gratings. In this paper, a new digital moiré technique with circular and radial gratings for measuring in-plane displacement and strain fields in large deformation materials are presented. Using the coordinate-transform method [23] and the digital phase-shifting technique [3–9], the digital images of deformed gratings in the polar coordinate system are processed automatically. An experiment of rubber sheet with a single-edge crack is shown, and the distributions of the displacements u_r and u_θ near the crack tip are directly given.

2. Digital moiré technique with circular and radial gratings (DMTCR)

2.1. Principles of the in-plane displacement measurement using DMTCR

In the moiré technique the deformed grating is used as the carrier of displacement information and the spatial frequency of the carrier has been modulated by the applied displacement. Considering a circular grating (the radial grating can be treated analogously), the transmission function T_r is given by:

$$T_r(r,\theta) = a_0 + a_1 \cos\frac{2\pi}{p} r, \tag{1}$$

where p is the pitch of reference grating, a_0 and a_1 are constants.

For the grating deformed by a displacement $u(r,\theta)$, the transmission function T_d can be expressed as

$$T_\mathrm{d}(r,\theta) = a_0 + a_1 \cos\frac{2\pi}{p}[r - u(r,\theta)], \tag{2}$$

where $u(r,\theta)$ represents displacement in the r direction.

By superposing reference and deformed gratings the moiré pattern is obtained, and the transmission function for the moiré pattern is

$$\begin{aligned} T(r,\theta) = T_r T_\mathrm{d} &= a_0^2 + a_0 a_1 \cos\frac{2\pi}{p}[r - u(r,\theta)] + a_0 a_1 \cos\frac{2\pi}{p} r \\ &+ \frac{a_1^2}{2}\cos\frac{2\pi}{p}[2r - u(r,\theta)] + \frac{a_1^2}{2}\cos\frac{2\pi}{p}u(r,\theta), \end{aligned} \tag{3}$$

where the first term is the background contribution; the second and third terms are corresponding to the original gratings; the fourth term is corresponding to double frequency of the specimen grating; the fifth term represents moiré fringes. Usually the high frequencies are filtered out, and the intensity of moiré pattern can be simplified as

$$T(r,\theta) \approx \frac{a_1^2}{2}\cos\frac{2\pi}{p}u(r,\theta). \tag{4}$$

Then the phase of fringes is calculated according to formula

$$\phi(r,\theta) = \frac{2\pi}{p}u(r,\theta). \tag{5}$$

Obviously, the displacement $u(r,\theta)$ at each point in the moiré pattern varies linearly with its phase $\phi(r,\theta)$. In other words, once the phase of each point is solved, the displacement distribution is readily obtained.

2.2. Coordinate-transform technique

In this method, moiré patterns are obtained by a logical operation of specimen grating and computer-generated reference grating. After getting the logical moiré patterns of circular and radial gratings, filter operation is needed to leach the noise and create the digital moiré patterns which are identical to the conventional moiré patterns. However, it is difficult to carry out the filter operation and phase-shifting technique in the Cartesian coordinate system. So we introduce the coordinate-transformation method to solve the problem in this paper.

Because a pair of circular and radial gratings used for in-plane deformation measurement can also be expressed by a set of parameters for a polar coordinate system, we transform the coordinate of a point (x, y) in the Cartesian coordinate system into that of the polar coordinate system (r,θ):

$$\begin{cases} x = X_0 + r\cos\theta, \\ y = Y_0 + r\sin\theta, \end{cases} \tag{6}$$

where $O'(X_0, Y_0)$, which is the center of the circular gratings (which is also the intersection point of the radial gratings), is chosen to be the origin of the polar coordinate system. The coordinate-transform method is schematically illustrated in Fig. 1. With the help of the coordinate transform, the circular and radial grating is converted to a cross grating. Then the moiré fringes from the circular and radial gratings can be analyzed by the conventional moiré method. In the end, the obtained displacement and strain distributions will be transformed back to the Cartesian coordinate system with the inverse coordinate transformation, and the distributions corresponding to the original moiré pattern can be recovered.

2.3. Phase-shifting technique and deformation analysis of DMTCR

As an automatic fringe analysis method, phase-shifting technique can highly improve the accuracy of the measurement. In the digital moiré method, the ability of precisely shifting the reference grating generated by

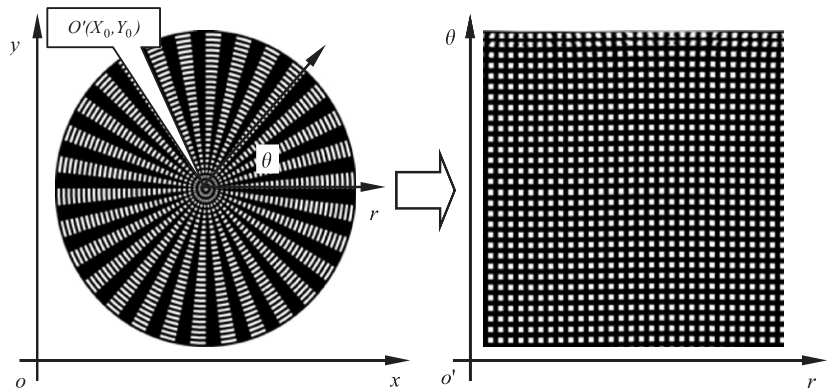

Fig. 1. Coordinate transform from $x-y$ space to $r-\theta$ space.

computer enables the quantification of moiré patterns using the phase-shifting technique. The principle of phase-shifting method resets on setting a file of phase shift values $\delta = \delta_i$, $i = 1, 2, \ldots, N$ and fitting a cosine function to the corresponding set of intensity values I_i at a single point. The intensity distribution of moiré fringe can be expressed as

$$I_i = a(x,y)\{1 + b(x,y)\cos[\phi(x,y) + \delta_i]\}, \tag{7}$$

where $a(x,y)$ is background contribution, $b(x,y)$ is communication contribution, $\phi(x,y)$ is the function of fringe phase. In this paper, the four step phase-shifting method is chosen. By moving three steps, four images can be obtained and the phase difference between subsequent images is $\pi/2$. According to Eq. (7), the intensity distributions of four shifted fringe patterns can be expressed as

$$I_1(x,y) = a(x,y)\{1 + b(x,y)\cos[\phi(x,y)]\}, \tag{8}$$

$$I_2(x,y) = a(x,y)\{1 + b(x,y)\cos[\phi(x,y) + \pi/2]\}, \tag{9}$$

$$I_3(x,y) = a(x,y)\{1 + b(x,y)\cos[\phi(x,y) + \pi]\}, \tag{10}$$

$$I_4(x,y) = a(x,y)\{1 + b(x,y)\cos[\phi(x,y) + 3\pi/2]\}. \tag{11}$$

Then, the phase of observation point can be determined by

$$\phi(x,y) = \arctan\frac{I_4 - I_2}{I_1 - I_3}. \tag{12}$$

From Eq. (1), it can be seen that when the reference grating moves (dilates) in the r direction by the steps of $p/4$, $p/2$, $3p/4$, the corresponding phase shift values are $\pi/2$, π, $3\pi/2$, respectively. Substituting Eq. (12) to Eq. (5) produces the displacement distribution in the polar coordinate system.

What is more, being contour maps of the displacement components, moiré fringes indicate the displacement distributions in a vivid and graphic way, even though they are far from as accurate as the phases of moiré patterns do. The displacement components u_r and u_θ in the r and θ directions can be, respectively, expressed as

$$\begin{cases} u_r = Np, & N = 0, 1, 2, \ldots, \\ u_\theta = N'p', & N' = 0, 1, 2, \ldots, \end{cases} \tag{13}$$

where N and N' are fringe orders, p and p' are pitches of reference gratings in the polar coordinate system.

According to Eqs. (5) and (13), the relation between the fringe order and the fringe phase can be expressed as

$$N = \phi(r,\theta)/2\pi. \tag{14}$$

3. Procedures of deformation analysis using DMTCR

In this section, we explain in detail the procedures of analysis by exemplifying a simulation model shown in Figs. 2 and 3. First, the experimental images of deformed gratings are captured by a CCD camera and saved in the computer for further analysis. In order to eliminate the interaction effect, the circular grating and the radial grating are extracted from the image using morphological image processing [24]. Second, the center of the circular gratings is selected as the origin of the polar coordinate system. And the image is transformed from the Cartesian coordinate system into a polar coordinate system from Eq. (6). Third, a reference grating is generated by computer, and logical operations (AND, OR, XOR) of two binary gratings produce an identical result to the superposition of two circular gratings in conventional moiré. Before application of the phase-shifting equations, the logical moiré pattern should be filtered to simulate the aliasing feature. In this paper, the filter operation with a series of filter windows is carried out to average the logical moiré pattern over the pitch of the reference grating by MATLAB. According to the proposed phase-shifting technique, four digital moiré patterns are obtained and shown in Fig. 3. Therefore, the phase difference between subsequent images is $\pi/2$. Finally, using the four step phase-shifting algorithm, the phase of the digital moiré pattern can be determined. Then, a phase unwrapping technique which is the process of resolution of

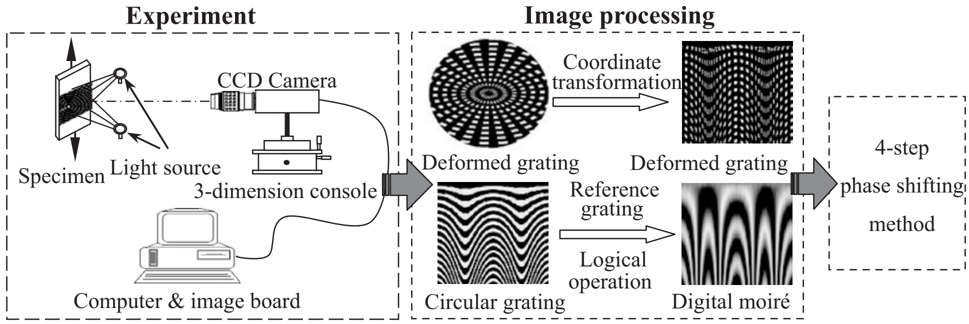

Fig. 2. Scheme of the analysis using image processing.

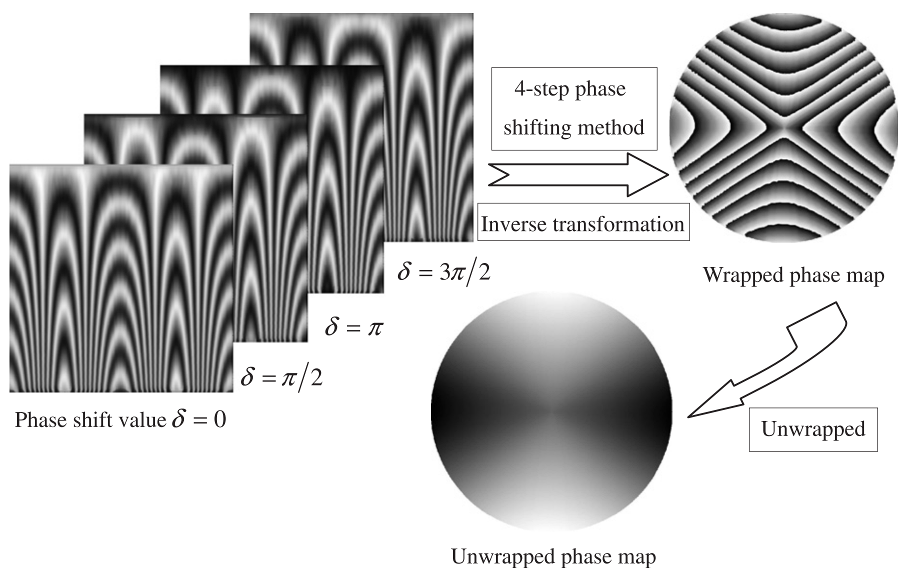

Fig. 3. Four step phase-shifting moiré fringes and phase unwrapped process.

the modulo 2π phase is performed, and the continuous phase of fringe is obtained. The four step phase-shifting moiré fringes and phase unwrapped process are shown in Fig. 3. The continuous fringe phase of the deformed radial gratings can be calculated in a similar way.

4. Experiment

In this study, rubber sheet with a single crack is selected as the test sample, and its deformation is analyzed using DMTCR. The deformed gratings are printed on the specimen before loading and can undergo large deformation without delaminating. The linear pitch of the circular grating is 1 mm/line and the angular pitch of the radial grating is 6°/line. The material used for the present analysis is polybutylene compound. The specimen geometry and dimensions are shown in Fig. 4. The thickness of the specimen is 1 mm and the length of the pre-crack is 10 mm. The specimen is clamped on a loading fixture and tensile load is applied to the specimen at a low rate. A CCD camera and a frame grabber are used to capture the images of deformed gratings after loading. Fig. 5 shows the images

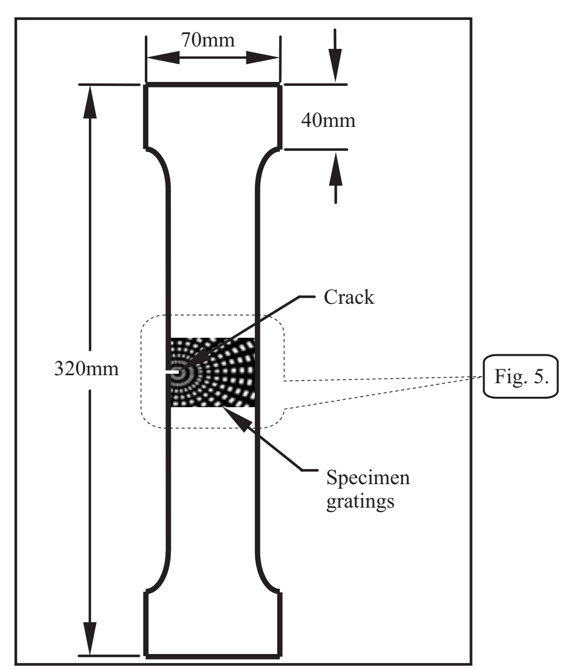

Fig. 4. Dimensions of the specimen.

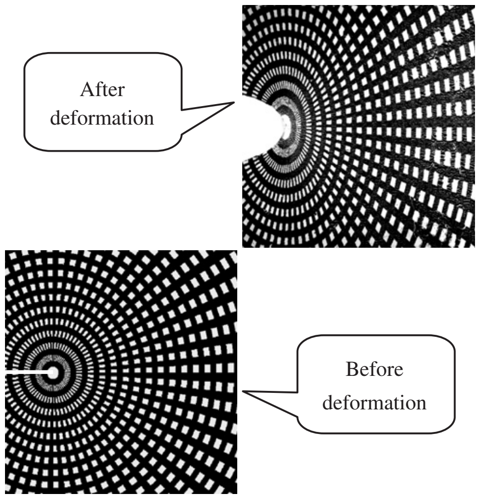

Fig. 5. Images of undeformed and deformed gratings.

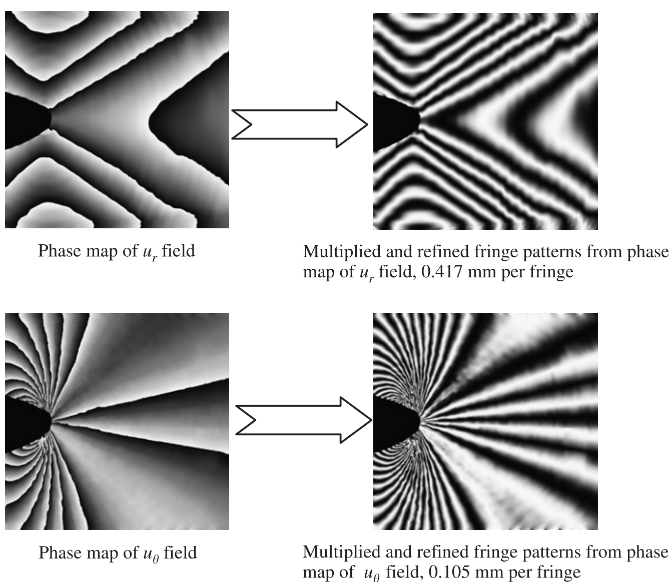

Fig. 6. Phase maps and the multiplied and refined fringe patterns.

of undeformed and deformed gratings. Following the procedures of the proposed method, the deformed gratings are transformed from the Cartesian coordinate system to a polar coordinate system. The deformed circular gratings and radial gratings are extracted from the image and analyzed separately. Logical operation and filter operation are carried out to produce the digital moiré patterns. The four digital moiré patterns are obtained and the four step phase-shifting is applied in the $r-\theta$ space. Finally, the unwrapped phase maps in the original $x-y$ space can be obtained by an inverse coordinate transformation, and the phase maps of the u_r and u_θ fields are shown in Fig. 6. By dividing the phase values of the original phase maps [25], the multiplied and refined fringe patterns of the u_r and u_θ fields can be evaluated and are shown in Fig. 6.

5. Conclusions

In this paper, the digital moiré technique with the circular and radial gratings (DMTCR) is proposed to measure the distributions of in-plane displacement and strain in the polar coordinate system directly. Only an image of deformed gratings is needed, the analysis is automatically carried out by the digital image processing.

The coordinate-transformation technique makes it possible to analyze the circular and radial moiré patterns with the digital phase-shifting method. The recovered phase maps are obtained by the inverse transformation. And multiplied and refined fringe patterns are evaluated from the unwrapped phase maps of the u_r and u_θ fields.

Compared with the conventional digital moiré method, the digital moiré method proposed in this paper is more convenient. Using a deformed grating, the distributions of in-plane displacement are measured and the distributions of strain can be calculated [26]. Using two gratings, the rigid-body displacements or angular rotation of the deformed body can also be obtained [13,15]. The results show that the present method is an efficient method for measurement of soft materials which can endure large deformation.

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