Preface

H_∞ control originated in an effort to codify classical control methods where one shapes frequency response functions to meet certain objectives. These techniques have dominated industrial design and commonly involved trial and error. H_∞ control underwent a tremendous development in the 1980's and arguably this made a considerable step towards systematizing classical control. The next major issue, how this extends to nonlinear systems, is what this book addresses. What we present is an elegant general theory and corresponding formulas.

At the core of nonlinear control theory lie two partial differential equations. One is a first order evolution equation called the *information state* equation, and as we shall see, it constitutes the dynamics of the controller. One can view the information state equation as a *nonlinear dynamical system* and much of this book is concerned with properties of this system, such as the nature of trajectories, stability, and most importantly, how it leads to a general solution of the nonlinear H_∞ control problem.

In addition to the information state PDE discussed above there is a second Partial Differential Inequality **PDI** which is defined on the space of possible information states (which is an infinite dimensional space). While the information state PDE determines the dynamics of the controller this second PDI determines the output of the controller. As it happens, this is a new type of PDI (and associated PDE), which is now being studied for its own sake. In this book we explore the system theoretic significance of this equation, present its gross structure (which is reasonably complete for smooth solutions), and ways to actually solve it in particular circumstances.

Many challenges are encountered, such as dealing with *singular* information states (functions which may assume the value $-\infty$ on nontrivial sets). These occur naturally, especially in linear systems, and though many technical issues concerning them are not resolved, they offer an enormous practical benefit. Namely, it is often possible to vastly reduce the dimension of the space on which the information state PDE must be solved, and thus vastly reduce the (online) computation required to implement the information state controller. The paradigm problem of classical control, which in the H_∞ context is called the mixed sensitivity problem, is one example where singular states are fruitful. This is because it is very common that the system to be controlled has a small number of unstable modes and this number is the dimension of the (reduced) space on which the singular information state PDE must be solved. While it is far from being proven in great generality, if a solution to the control problem exists, one based on singular solutions exists, and this controller not only solves the H_∞ problem, but the online part could in some cases be implemented.

The book presents a general structure, examples, and proofs at various levels of generality, and in various stages of completeness. Thus we guide the reader to an area of vigourous research.

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How to Read this Book

This book is divided into two parts, following the Introduction. Part I contains the basic problem definition and information state solution. The key formulas and results are presented. We have attempted to minimize the technical complexity of these first chapters. Part II contains further results, and in particular, some of the more detailed technical results. A number of appendices are provided for the reader's convenience. For readers interested primarily in using the results, we recommend studying the Introduction, and Part I, in particular Chapters 2, 3 and 4. For readers wishing to learn the full details and to contribute to the subject, we recommend reading Part II after the main ideas of Part I have been digested.

	Chapter 1	**Introduction.** A quick, light, presentation of the main ideas in this book, plus sections on history and classical control.
Part I	Chapter 2	**The H_∞ Control Problem.** The problem is specified and terminology is introduced.
	Chapter 3	**Information States.** Problem is expressed as minimax game, and information state is defined and studied.
	Chapter 4	**Information State Control.** Information state controllers are defined and used to solve the H_∞ problem using the dynamic programming PDE and dissipation PDIs.
	Chapter 5	**State Feedback H_∞ Control.** Ideas from the theory of dissipative systems, such as the bounded real lemma, stability and the state feedback case are reviewed.
	Chapter 6	**Storage Functions.** The closed-loop plant and information state controller system is in the context of storage functions on the product space.
	Chapter 7	**Special Cases.** Bilinear and linear systems, and the certainty equivalence cases.
	Chapter 8	**Factorization.** A general theory of factorization is developed and applied to the H_∞ problem.
	Chapter 9	**The Mixed Sensitivity Problem.** This paradigm control problem is solved.
Part II	Chapter 10	**Singular Information States.** These are important for both practical and theoretical reasons, and this chapter presents some detailed results.
	Chapter 11	**Stability of the Information State System.** The information state defines a nonlinear infinite dimensional dynamical system and this chapter analyzes its stability behavior.
	Appendix A	**Differential Equations and Stability.** A brief review of some basic facts.
	Appendix B	**Nonlinear PDE and Riccati Equations.** A summary of some key ideas.
	Appendix C	**Max-Plus Convergence.** Self-contained background information.