ELECTROMAGNETISM SUMMARY

- Magnetostatics
- Ampere’s law and Faraday’s law
- Maxwell’s equations
- Waves

Magnetostatics: The static magnetic field

- Gauss's law for the magnetic field:

$$\oint_A \mathbf{B}.\mathbf{dA} = 0$$

- There is no static sink or source of the magnetic field. Also generally true.
- However **current** is the source of a static magnetic field,

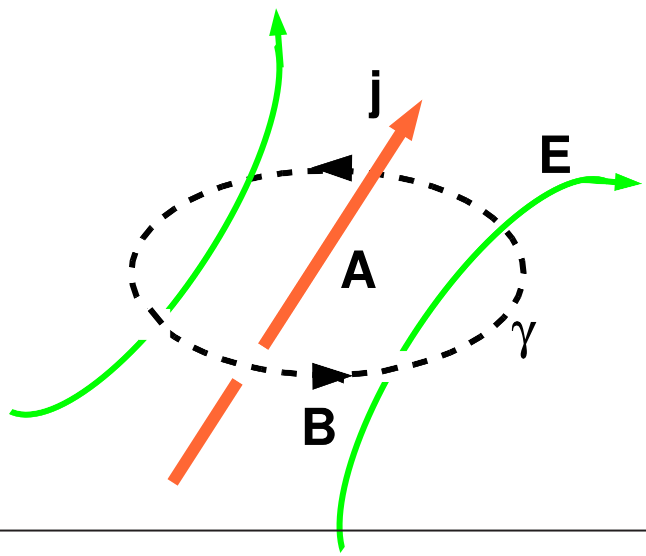
$$\oint_\gamma \mathbf{B}.\mathbf{dl} = \mu_0 I$$

The line integral of **B** around a closed circuit γ bounding a surface **A** is equal to the current flowing across **A**. **Ampere's law**

Important Correction to Ampere's Law

- **A time varying Electric field** is also a source of the time varying magnetic field,

$$\oint_{\gamma} \mathbf{B}.\mathbf{dl} = \int_{A} \mu_0 \left(\mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right) .\mathbf{dA}$$

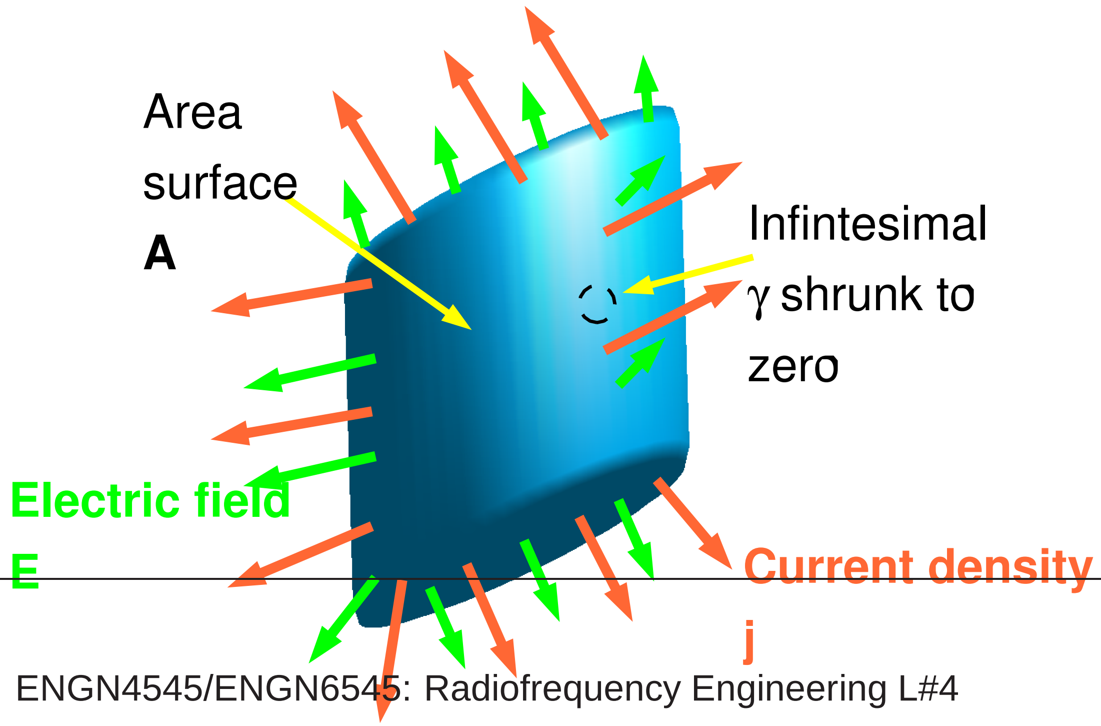


Why this Correction to Ampere’s Law?

- **Ampere’s law without E contradicts charge conservation**

$$\oint_{\gamma} \mathbf{B}.\mathbf{dl} = \int_{A} \mu_0 \left(\mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right) .\mathbf{dA}$$

- Consider the new Ampere’s law on a **close surface area, A**.



Why this Correction to Ampere's Law?

- If we shrink the closed contour γ on the left hand side to zero then we obtain

$$0 = \oint_A \mu_0 \left(\mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right) .\mathbf{dA}$$

- However the two terms on the right hand side are

$$\oint_A \mathbf{E}.\mathbf{dA} = \frac{q}{\epsilon_0}$$

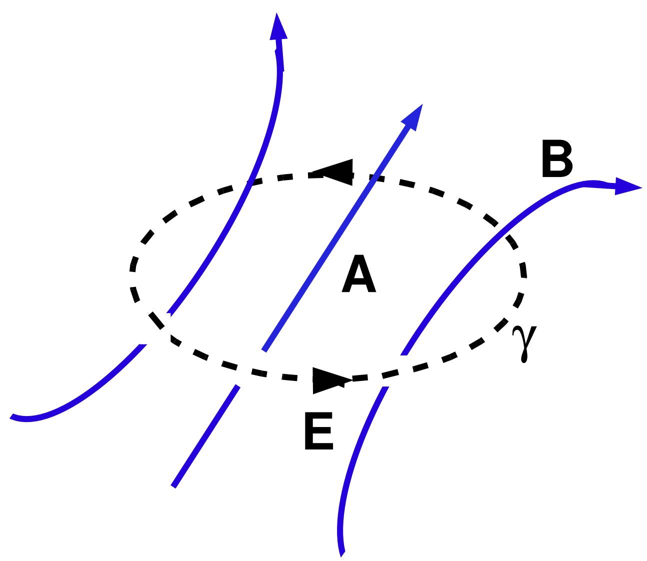
and

$$\oint_A \mathbf{j}.\mathbf{dA} = -\frac{\partial q}{\partial t}$$

Faraday’s Law

- The law of electromagnetic induction or Lenz’s law
- **A time varying magnetic field** is also the source of the electric field,

$$\oint_{\gamma} \mathbf{E}.\mathbf{dl} = -\int_{A} \frac{\partial \mathbf{B}}{\partial t}.\mathbf{dA}$$



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Maxwell's Equations: Integral Form

- **Gauss's law for the electric field.** Charge is the source of electric field:

$$\oint_A \mathbf{E}.d\mathbf{A} = \frac{q}{\epsilon_0}$$

- **Faraday's law.** A changing magnetic flux causes an electromotive force:

$$\oint_\gamma \mathbf{E}.d\mathbf{l} = -\int_A \frac{\partial \mathbf{B}}{\partial t}.d\mathbf{A}$$

- **Gauss's law for the magnetic field.** Magnetic fields are source free:

$$\oint_A \mathbf{B}.d\mathbf{A} = 0$$

- **Ampere's law:**

$$\oint_\gamma \mathbf{B}.d\mathbf{l} = \int_A \mu_0 \left(\mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right).d\mathbf{A}$$

Maxwell's Equations: Differential Form

- **Gauss's law for the electric field.**

$$\nabla . \mathbf{E} = \frac{\rho}{\epsilon_0}$$

- **Faraday's law.**

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

- **Gauss's law for the magnetic field.**

$$\nabla . \mathbf{B} = 0$$

- **Ampere's law:**

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

EM Waves

- **Start with Faraday's law and Ampere's law in VACUO.**

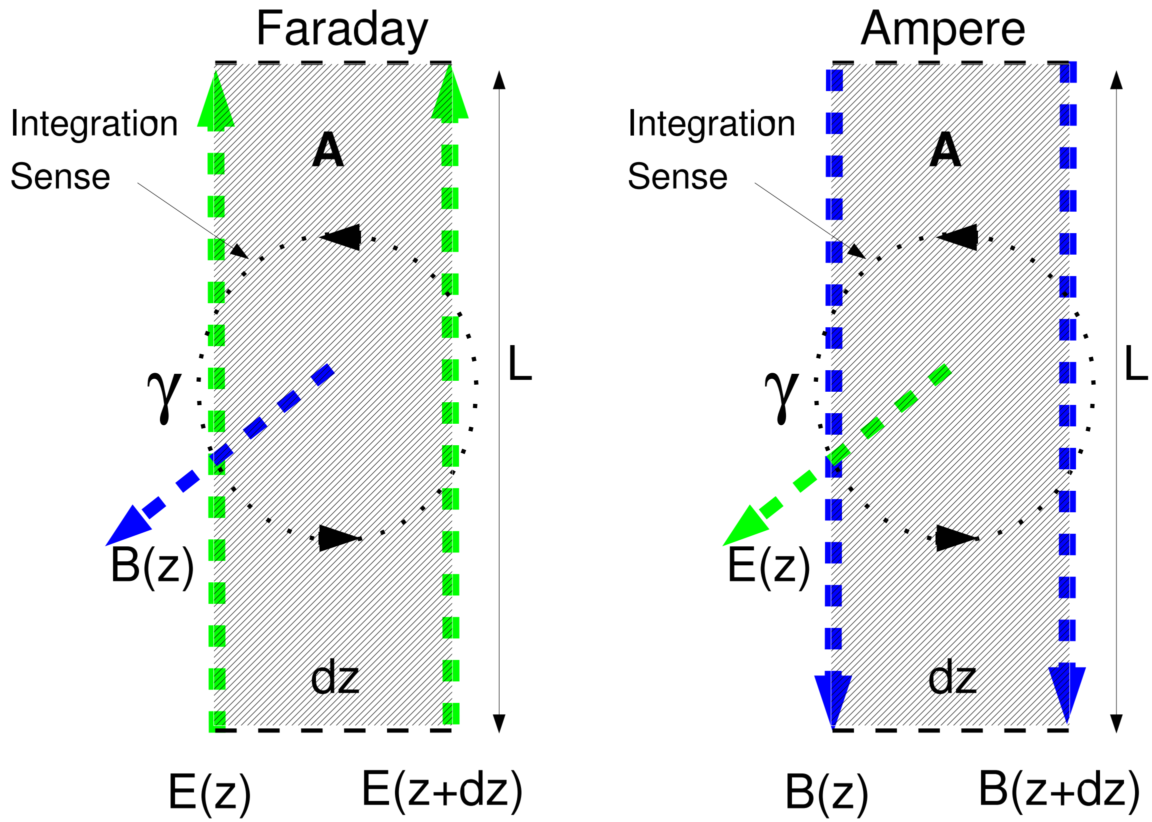
$$\oint_\gamma \mathbf{E}.\mathbf{dl} = -\int_A \frac{\partial \mathbf{B}}{\partial t}.\mathbf{dA}$$

$$\oint_\gamma \mathbf{B}.\mathbf{dl} = \int_A \mu_0\epsilon_0 \frac{\partial \mathbf{E}}{\partial t}.\mathbf{dA}$$

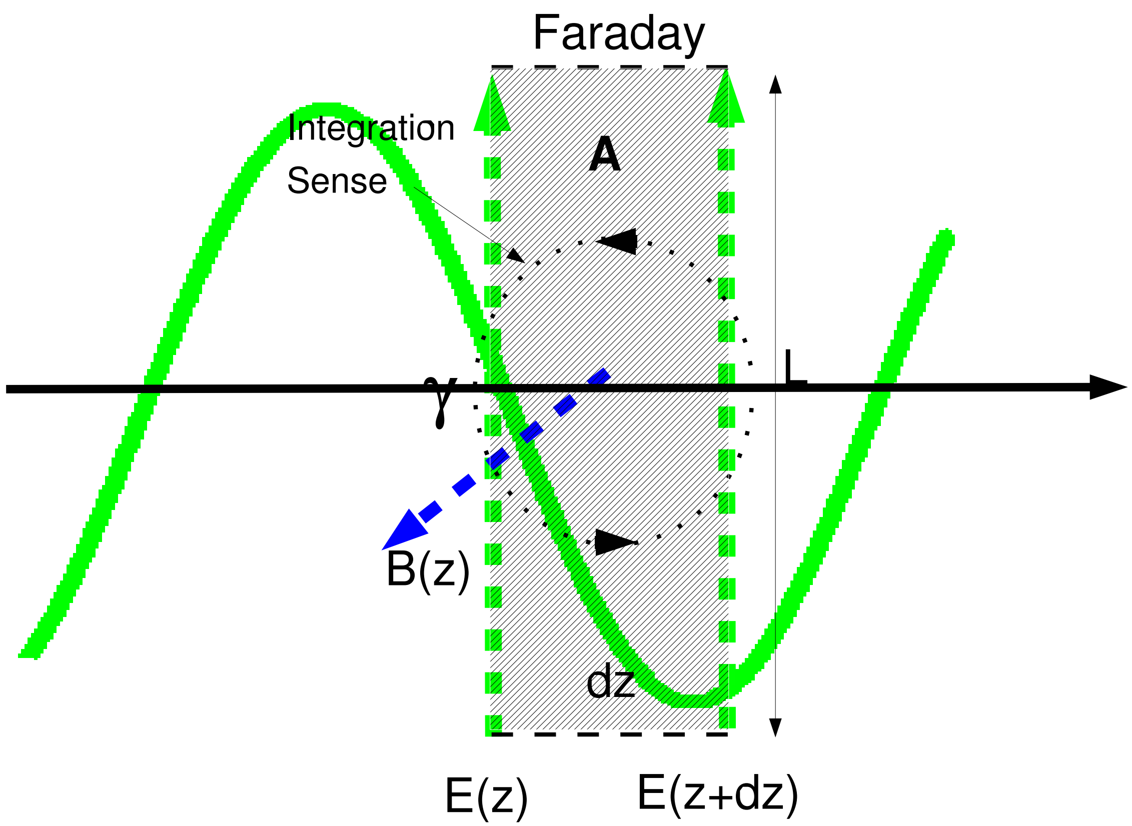
- Recall that the line integral along γ is on the perimeter of the surface $\mathbf{A}$
- Thus a one dimensional **E** and **B** must have **E.B** = 0.
- Moreover $\mathbf{E} \times \mathbf{B}$ points in the direction of propagation.

Monochromatic Waves 1

- Apply F and A to the diagrams below and note that **A** points in the direction of the right hand screw rule with respect to the direction of γ



Monochromatic Waves 2



Monochromatic Waves 3

- Integrating **E** around γ in the left figure...

$$[E(z+dz) - E(z)]\, L = -\frac{\partial B}{\partial t} L \Delta z$$

$$\frac{\partial E}{\partial z} L \Delta z = -\frac{\partial B}{\partial t} L \Delta z$$

- Integrating **B** around γ in the right figure...

$$[B(z+dz) - B(z)]\, L = -\epsilon_0 \mu_0 \frac{\partial E}{\partial t} L \Delta z$$

$$\frac{\partial B}{\partial z} L \Delta z = -\epsilon_0 \mu_0 \frac{\partial E}{\partial t} L \Delta z$$

Monochromatic Waves 4

- Two equations for **E** and **B**

$$\frac{\partial E}{\partial z} = -\frac{\partial B}{\partial t}$$

$$\frac{\partial B}{\partial z} = -\epsilon_0 \mu_0 \frac{\partial E}{\partial t}$$

- Simultaneous solution gives **the wave equation** for each field:

$$\frac{\partial^2 E}{\partial z^2} = \epsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 B}{\partial z^2} = \epsilon_0 \mu_0 \frac{\partial^2 B}{\partial t^2}$$

Solution of the Wave Equation 1

- Two equations for **E** and **B**

$$E = E(X); \qquad B = B(X)$$

where $X = z \pm v.t$

$$\frac{\partial^2 E}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 B}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 B}{\partial t^2}$$

- The solution of the wave equation is an arbitrary function of the displacement argument $X = z \pm v.t$

Solution of the Wave Equation 1

$$\frac{\partial E}{\partial z} = -\frac{\partial B}{\partial t} => E'(X) = \mp v B'(X)$$

$$\frac{\partial B}{\partial z} = -\epsilon_0 \mu_0 \frac{\partial E}{\partial t} => B'(X) = \mp v \epsilon_0 \mu_0 E'(X)$$

- Thus $E(X) = cB(X)$ and $v = c = \sqrt{\frac{1}{\epsilon_0 \mu_0}}$.
- The solution of the wave equation is an arbitrary function of the displacement argument $X = z \pm c.t$

Solution of the Wave Equation 2

- Solution as a function of z for four different times...