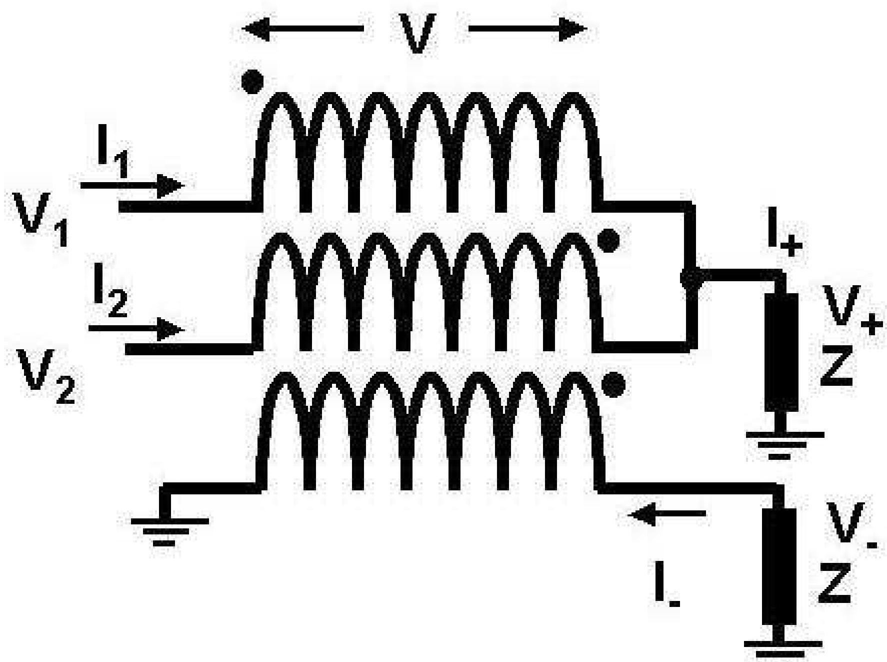
ELECTROMAGNETISM SUMMARY

- Transmission line transformers
- Skin depth
- Matching circuits
- Noise
- Link Budgets

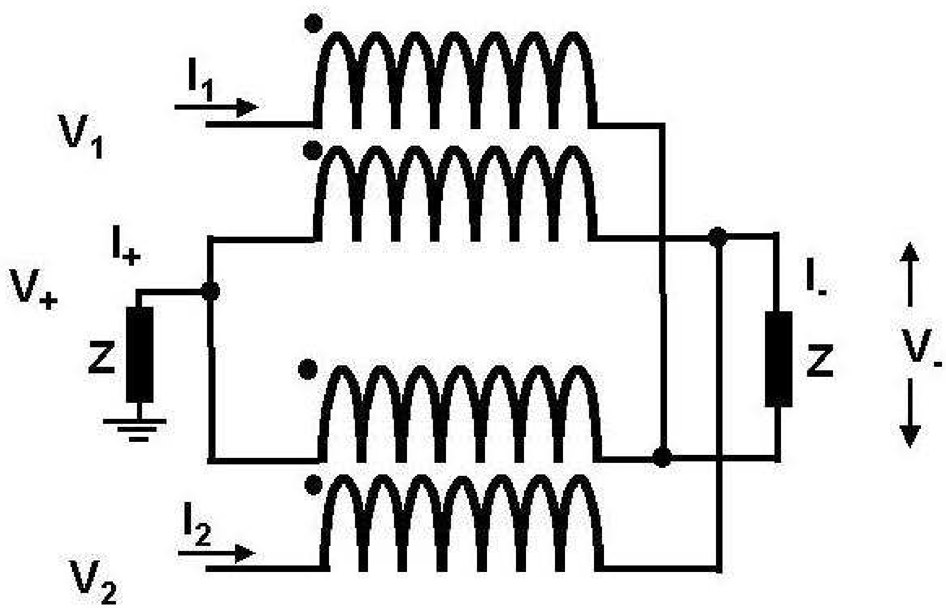
Rules for Transmission Line Transformers

- Always wind the windings in multifilar fashion.
- Can use either toroidal or linear or whatever shaped ferrites. Toroidal ferrites are usually the best.
- The dot on the transformer diagram points to one end of the wires at one end of the transformer.
- The voltage drop across all windings must be same. **WHY?**
- The currents in the same direction in the windings must sum to zero. **WHY?**
- Respects phase delays along the transmission line when doing its sums?

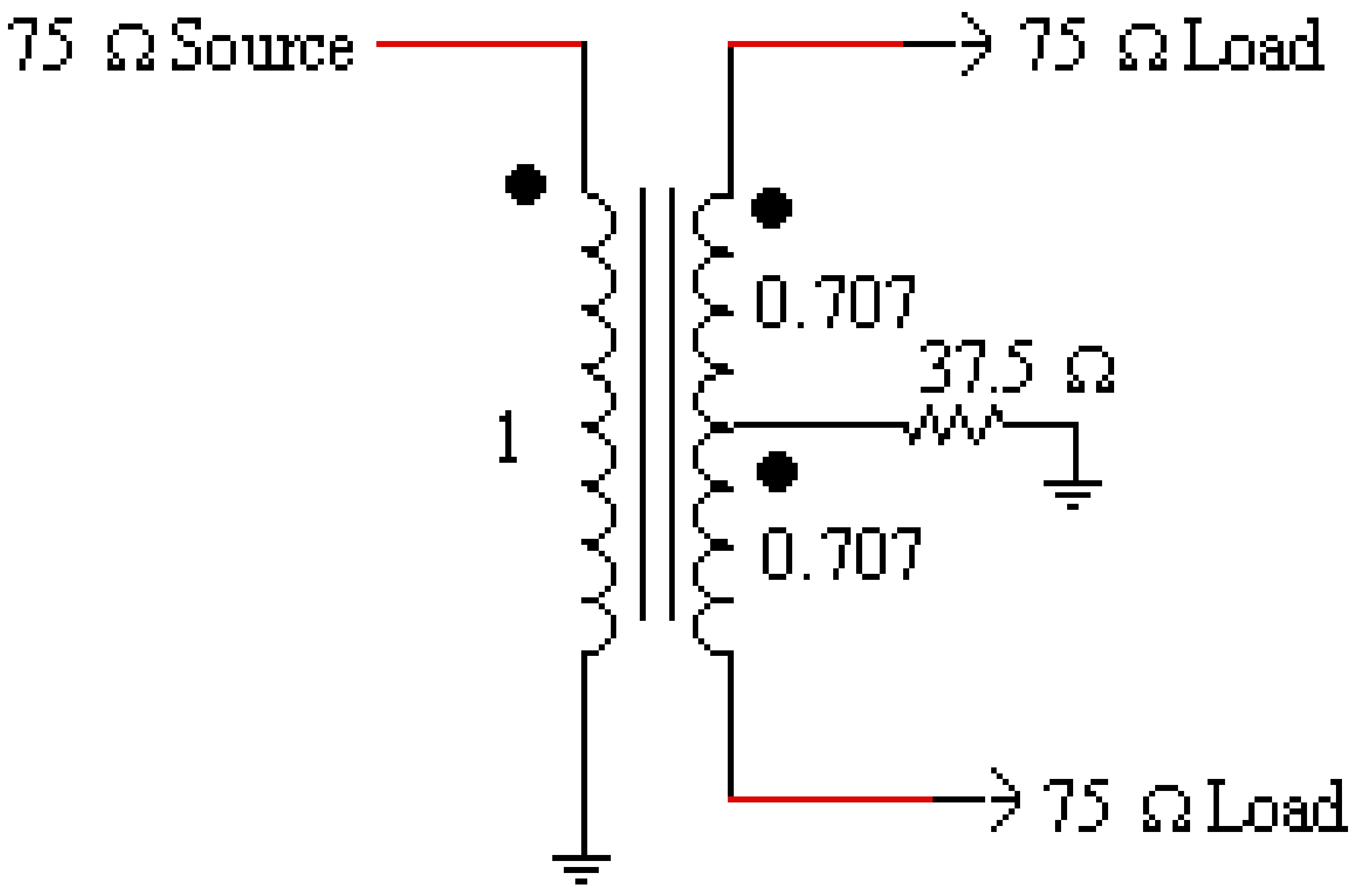
Transmission Line Transformer 180^O Hybrid



The Linear Phase Shift Combiner.



The Magic-T (Wilkinson)



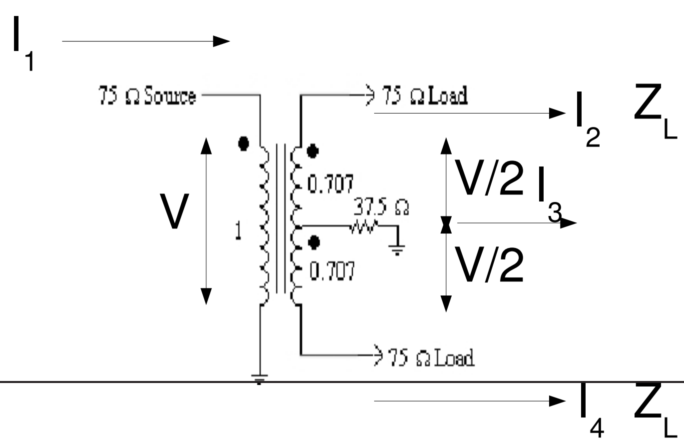
The Magic-T Analysis

$$I_2 + I_4 + I_3 = 0 but I_3 = 0$$

$$I_1 + I_4 = I_2 = -I_4 => I_4 = -I_1/2$$

$$V/2 = I_2 Z_L => \frac{V}{I_2} = 2Z_L => \frac{V}{I_1} = Z_L = 75\Omega$$

$$V/2 = -I_4 Z_L => \frac{V}{I_4} = -2Z_L => \frac{V}{I_2} = 2Z_L$$



Skin Depth

- **Electromagnetic waves, j, E, B, ... only penetrate a distance δ into a metal. Check the magnitude of δ in lab and web exercises.**
- The wave equation for metals simplifies to...

$$\frac{\partial^2 E_y(z)}{\partial z^2} = j\omega\sigma\mu_0 \, E_y(z)$$

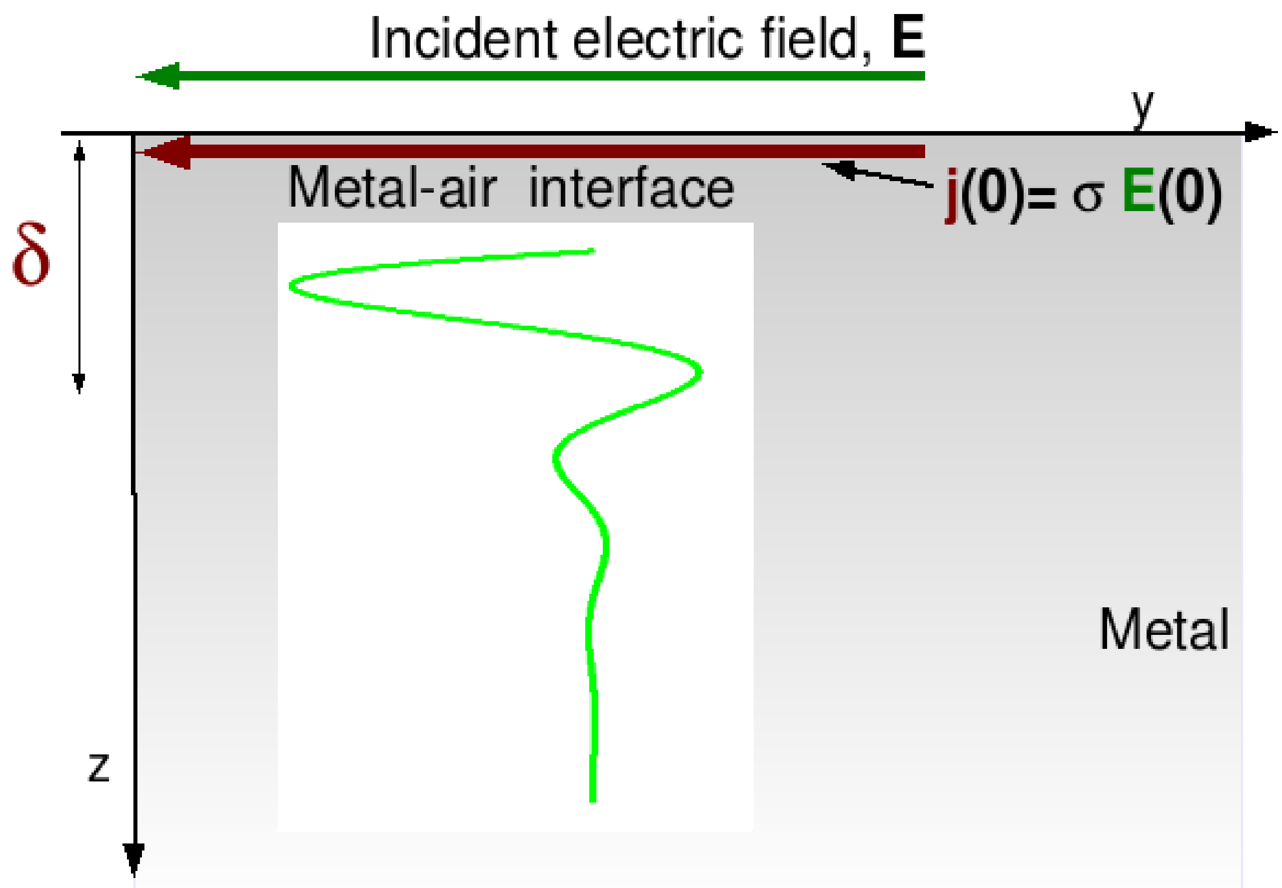
- The solution...

$$E_y(z) = \exp\left(-\frac{1 + j}{\delta} \, z\right)$$

- where δ the **skin depth** is given by...

$$\delta = \sqrt{\frac{2}{\omega\sigma\mu_0}}$$

Skin Depth



Incident electric field, E
y
Metal-air interface
j(0)= σ E(0)
δ
Metal
z

Impedance per Square

- By integrating the formula for the electric field inside a metal,

$$E_y(z) = \exp\left(-\frac{1+j}{\delta}z\right)$$

to find the current per unit width I_s we defined the impedance per square as

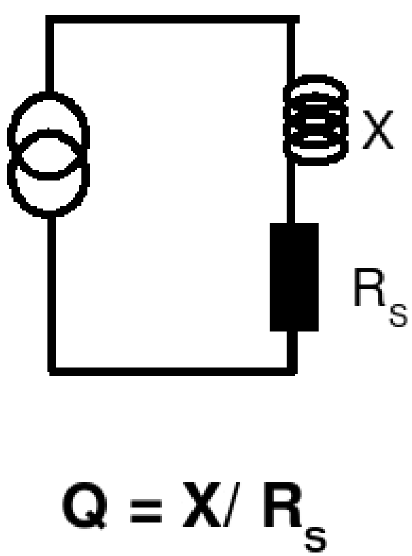
$$Z_s = E_y(0)/I_s = \frac{1+j}{\sigma\delta} = \sqrt{\frac{\pi\mu_0 f}{\sigma}}(1+j)$$

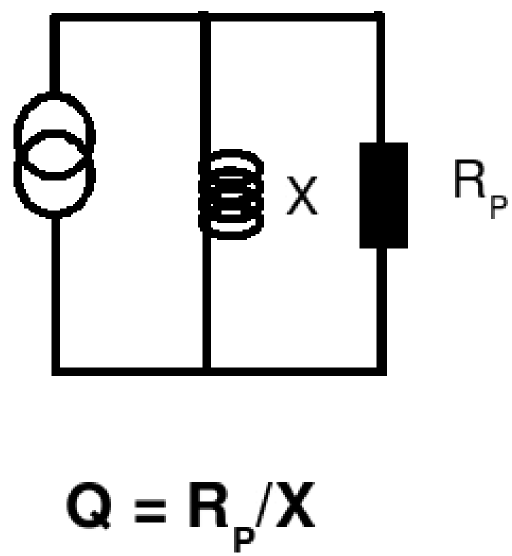
- For a wire of radius, a, length L and circumference $2\pi a$, we obtain

$$Z = \frac{L}{2\pi a}Z_s$$

Use Q to Design Matching Networks

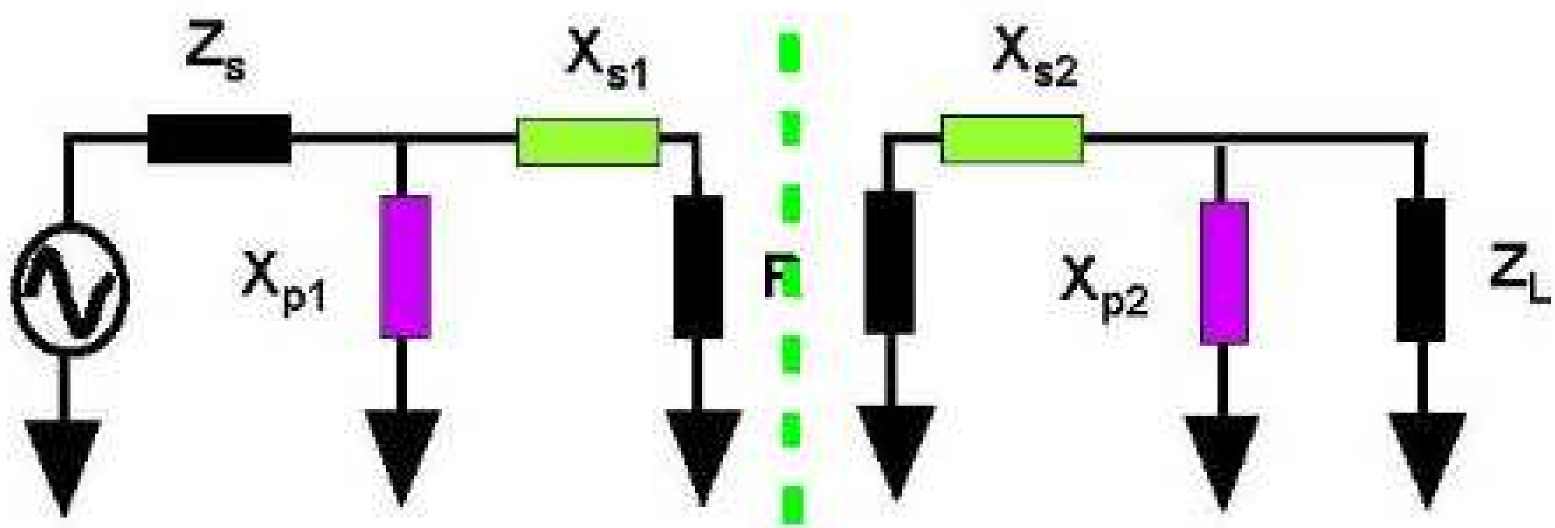
- The formula for Q depends on whether we imagine the R to be in series with or in parallel with the reactance. Just an issue of convenience.
- R in series with X, then $Q = X/R_s$
- R in parallel with X, then $Q = R_p/X$
- Notice that R_s is not the same as R_p but they are related (an exercise).





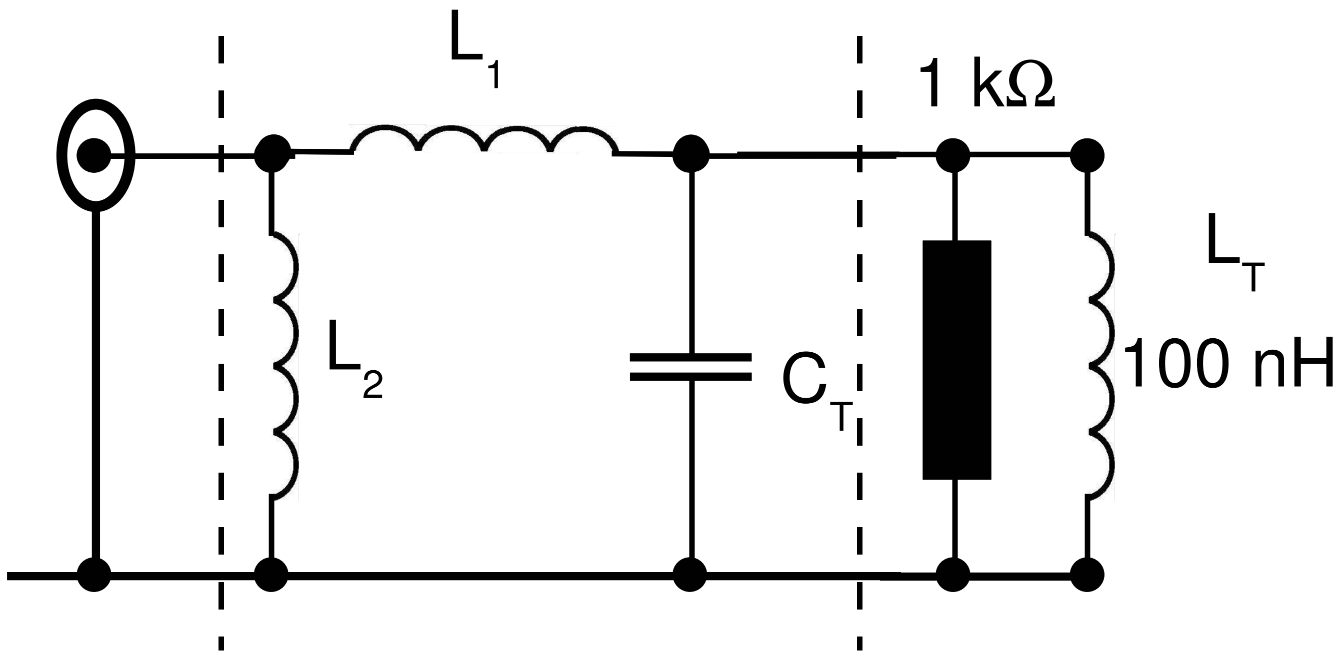
Analysis of T and Pi Networks

- Choose Q.
- Consider the T or Pi network to be a pair of back to back L networks.
- The virtual resistance in a Pi network must be smaller that those on the source and load.



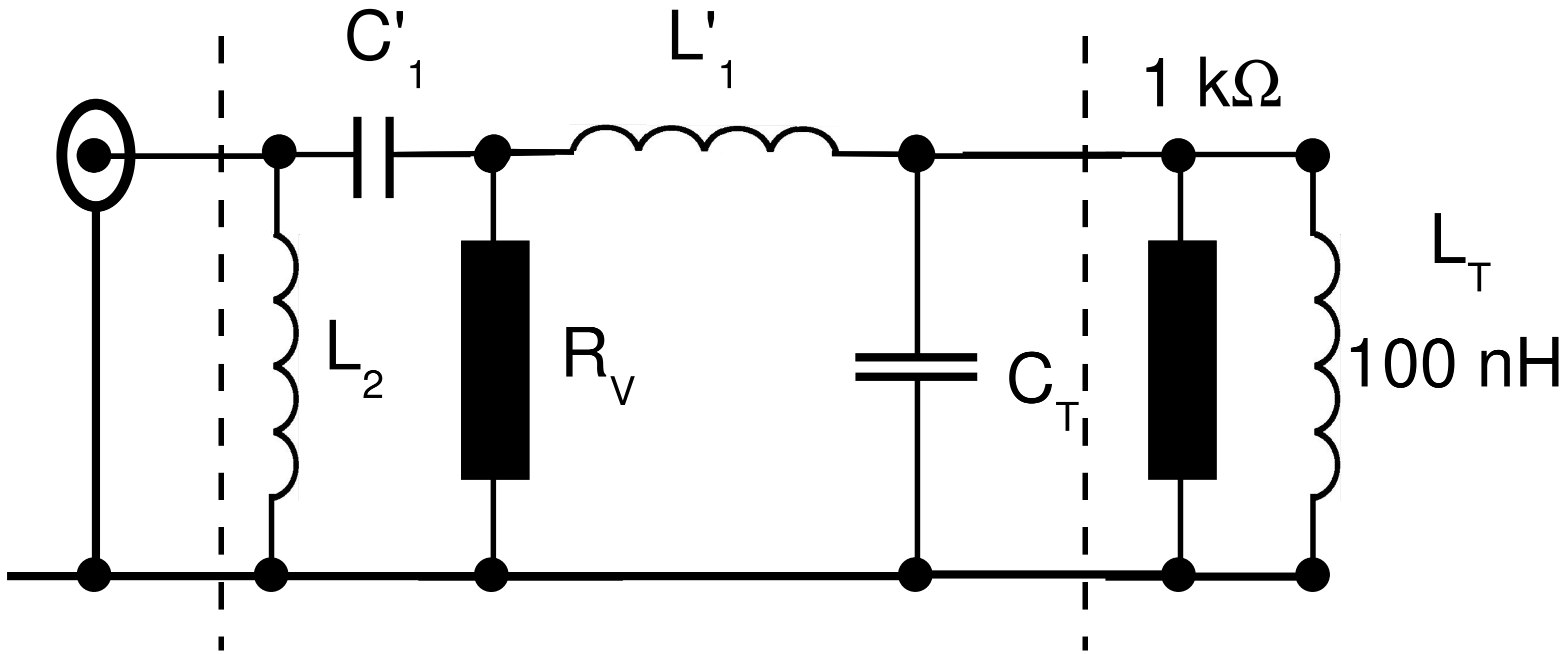
PI Matching Example

- Match 75Ω source to the 1 kΩ / 100 nH load with a Q of 10.



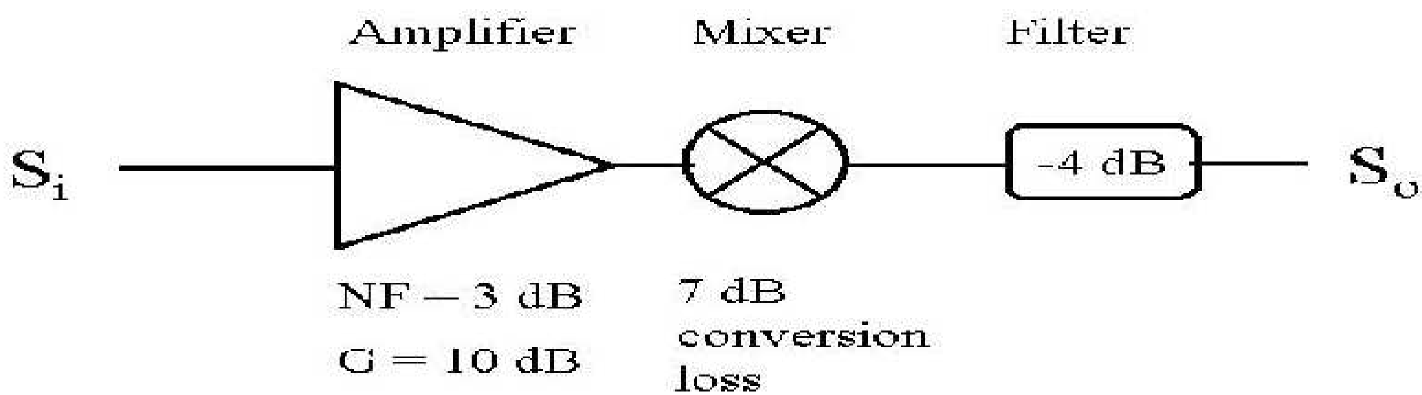
PI Matching Example

- Split into to two halves with R_V in the middle.



Example Noise Power Calculation.

- Consider the following receiver chain which is typical of that in a wireless receiver.
- The noise figure of the mixer and filter (both passive devices with the given insertion losses) is 11dB.
- **Find the overall noise figure of the receiver**



Example Noise Power Calculation. (Contd)

- The noise factor of the amplifier is 2 (=$10log_{10}(3)$).
- The noise figure of the mixer and filter is 11 dB and so the noise factor is 12.6 (=$10log_{10}(11)$). Thus,

$$F_{TOT} = F_1 + \frac{F_2 - 1}{G_1} = 2 + (12.6 - 1)/10 = 3.16.$$

- Finally we obtain

$$F_{TOT} = 10log_{10}(3.16) = 5dB.$$

Receiver Noise Calculations

- The thermal noise added to a signal when passing through a system is given by,

$$N_o = k_B T B$$

- In dBm

$$N_o = 10 log_{10} \frac{k_B T B}{1 \times 10^{-3}}$$

- If N_o and the NF are known, then the required input signal level for a given output SNR can be calculated,

$$S_i = NF + N_o + SNR_o$$

Specifying Phase Noise

- Common to specify phase noise as,

$$S_c(f) = \frac{S_N(\Delta f)/2}{Carrier\ Power}$$

where $S_N(f) = V_o^2 S_{\Delta\theta}(f)$ and the carrier power = V_o^2.

- The factor of 2 dividing the P.S.D. arises because we only consider one sideband in the definition of $S_c(f)$.

- $S_c(f)$ has the units of dBc/Hz.

$$S_c(f) = S_{\Delta\theta}(f)/2 : \quad S_c(f)(dB) = 10 log_{10}(S_c(f))$$

Specifying Phase Noise

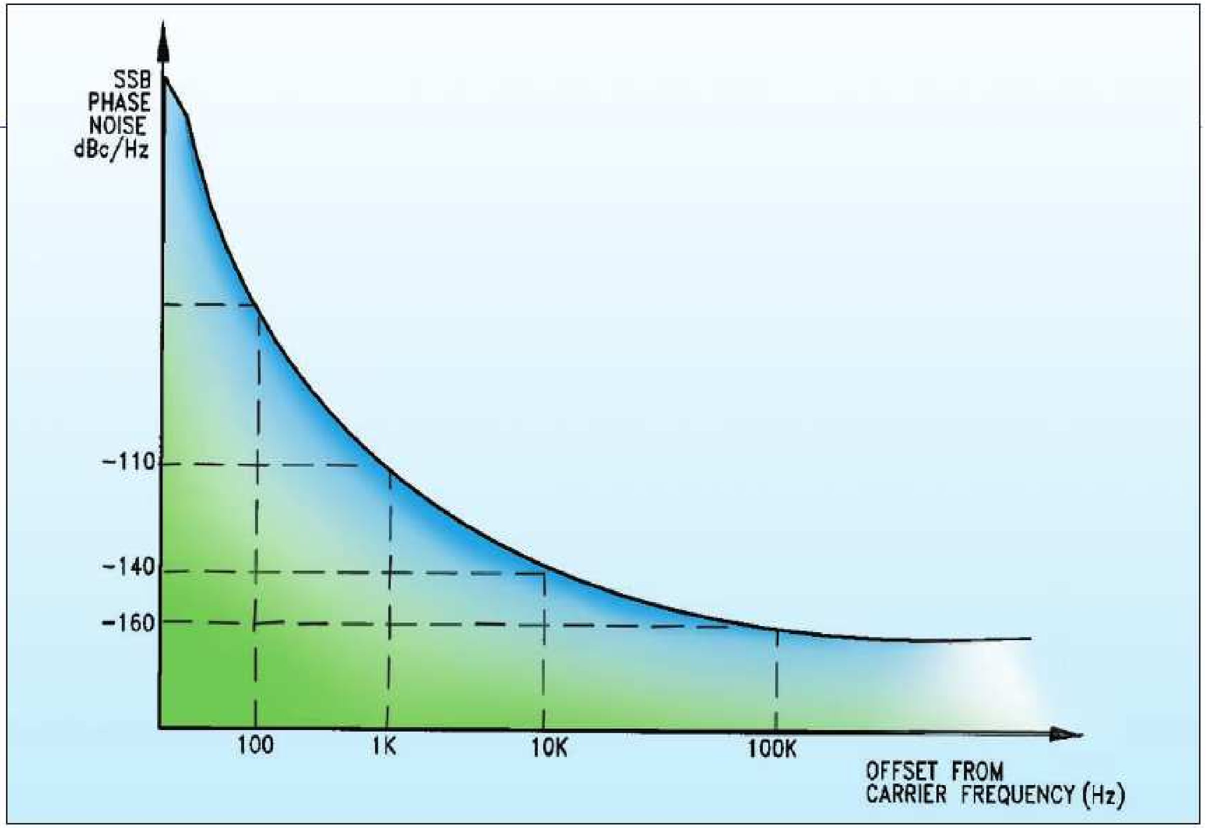
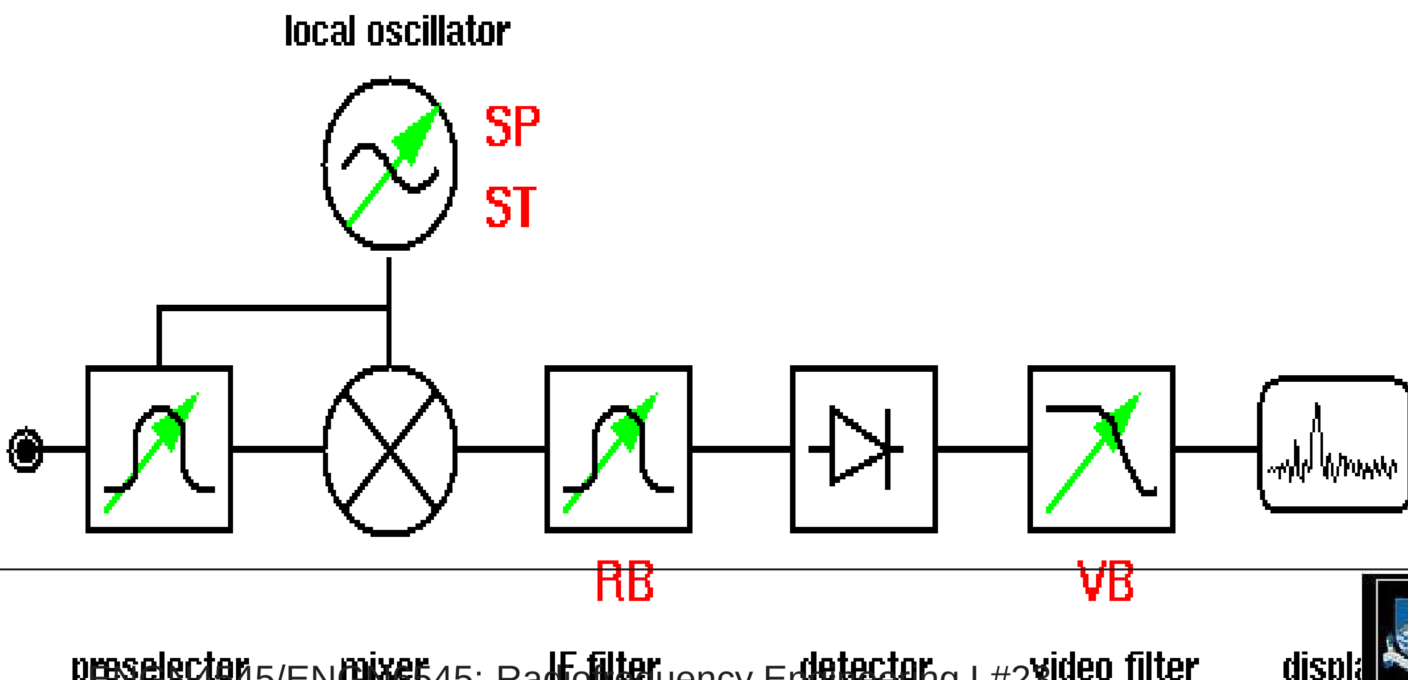


Figure 4. Single-sideband phase noise representation

Spectrum Analyser Revision

- LO Sweep generator is mixed with incoming signal
- IF signal is passed through two filters.
- *IF filter* : Resolution Bandwidth.
- *DC filter* : Video Bandwidth.
- **Thus be wary when measuring the phase noise with a spectrum analyser.**

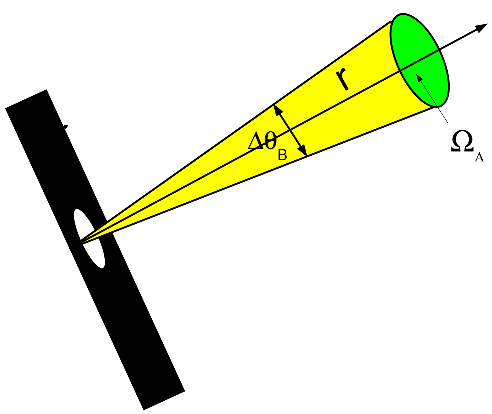


Effective Aperture of a Dish Antenna

- Imagine a planar light beam illuminating a round hole on a black screen at normal incidence.
- The Rayleigh condition for a diffraction limited aperture describes the angle of expansion of the beam on exit from the hole.

$$\Delta\theta_B = \frac{4\lambda}{\pi d}$$

where λ is the wavelength, $\Delta\theta_B$ is the opening angle of the beam and d is the diameter of the aperture.



Antenna Aperture: Useful to compute received power.

- The effective aperture of **any antenna** is given by:

$$A_e = \frac{G\lambda^2}{4\pi}$$

where λ is the wavelength, G is the antenna gain.

- Effective aperture only depends on antenna gain and the wavelength of operation.

- E.G. A low gain monopole tuned to 3 MHz has an aperture $A_e = G\lambda^2/4/\pi \approx 100^2/4/\pi = 800m^2$!!!

Antenna Aperture

- *If an antenna is oriented for maximum signal and correctly tuned* $Z_{load} = Z^*_{ANT}$, *it will intercept a maximum signal power equal to:*

$$P = S_i A_e$$

- where S_i is the incident power flux density (Watts per m^2) and A_e is the *antenna effective aperture*.

- An antenna absorbs half this power into a matched load and reradiates (scatters) the other half **WHY?**.

The Friis Transmission Formula

- We know how to calculate the power radiated by an antenna, the maximum flux density of an antenna from its gain and the power intercepted by an antenna from A_e

- If we assume that the antennas are aligned for maximum transmission and reception, then in free space,

$$P_r = \frac{G_t A_r P_t}{4\pi r^2}$$

  where A_r is the receiving aperture of the receiving antenna.

- Since $A_r = G_r \lambda^2/(4\pi)$

$$P_r = G_t G_r P_t \left[\frac{\lambda}{4\pi r}\right]^2$$

Antennas (Cont.): Antenna Noise

- Random noise comes from the sky: E.G. The *cosmic radiation background at $3^o K$*.

- Black body radiation $=>$ *it must be there at finite temperature even in a vacuum!*

- This noise can be picked up by antennas. In a receiver it adds to the noise of the receiver electronics.

- PSD = N_o = KT where K = $1.38 \times 10^{-23} J/^o K$ and T is the absolute temperature. Thus the noise power is

$$P_n = kTB$$

- Such noise picked up by the antenna leads to the definition of antenna *temperature*.

Link Budget: Friis transmission

- The Friis transmission formula describes e.m. propagation between line of sight antennas:

$$P_r = P_t \frac{G_1 G_2 \lambda^2}{(4\pi r)^2}$$

where P_t and P_r are the transmit and received powers, $G(= G_1, G_2)$ is the gains of the antennas at each end of the link, r is the distance between the antennas and λ the wavelength.

- Note in particular the dB with respect to 1 mW.. dBm

$$P(dBm) = 10 \log_{10} \frac{P(Watts)}{.001}$$

Link Budgets

- Consider communications between a dish and an arbitrary antenna each matched to a pair of signal generators.