## ELECTROMAGNETISM SUMMARY

- Maxwell’s equations
- Transmission lines
- Transmission line transformers
- Skin depth

## Magnetostatics: The static magnetic field

- Gauss's law for the magnetic field:

$$\oint_A \mathbf{B}.\mathbf{dA} = 0$$

- There is no static sink or source of the magnetic field. Also generally true.

- However **current** is the source of a static magnetic field,

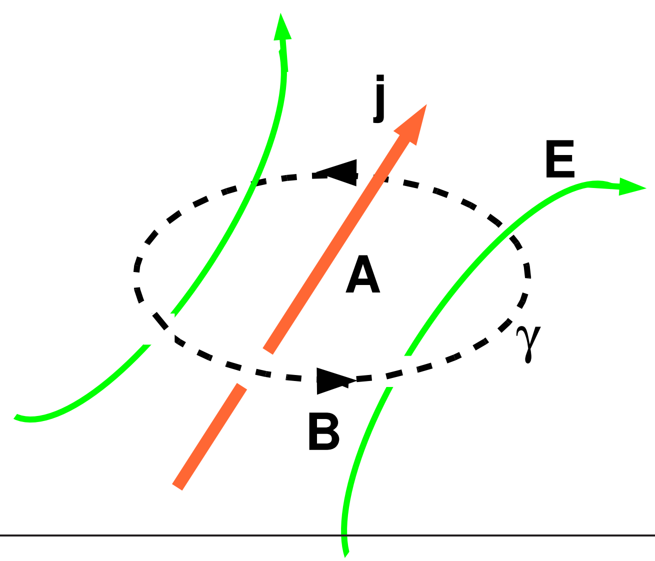
$$\oint_\gamma \mathbf{B}.\mathbf{dl} = \mu_0 I$$

The line integral of **B** around a closed circuit $\gamma$ bounding a surface **A** is equal to the current flowing across **A**. **Ampere's law**

## Important Correction to Ampere’s Law

- **A time varying Electric field** is also a source of the time varying magnetic field,

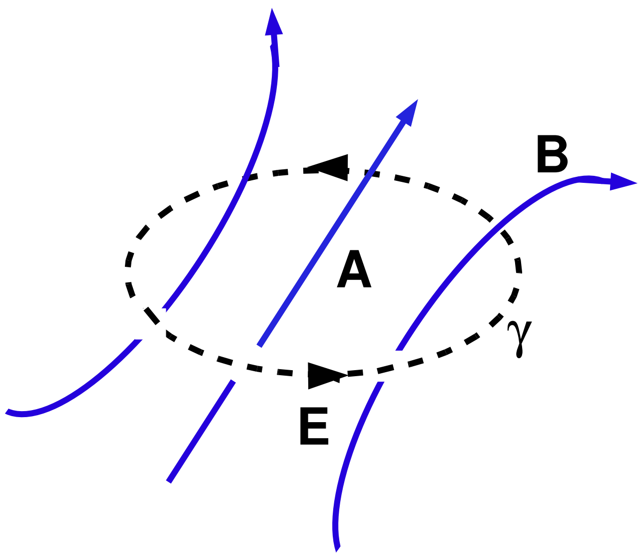
$$\oint_{\gamma} \mathbf{B}.\mathbf{dl} = \int_{A} \mu_0 \left( \mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right) .\mathbf{dA}$$



## Faraday's Law

- The law of electromagnetic induction or Lenz's law
- **A time varying magnetic field** is also the source of the electric field,

$$\oint_{\gamma} \mathbf{E}.\mathbf{dl} = -\int_{A} \frac{\partial \mathbf{B}}{\partial t}.\mathbf{dA}$$



## Maxwell's Equations: Integral Form

- **Gauss's law for the electric field.** Charge is the source of electric field:

$$\oint_A \mathbf{E}.\mathrm{d}\mathbf{A} = \frac{q}{\epsilon_0}$$

- **Faraday's law.** A changing magnetic flux causes an electromotive force:

$$\oint_\gamma \mathbf{E}.\mathrm{d}\mathbf{l} = -\int_A \frac{\partial \mathbf{B}}{\partial t}.\mathrm{d}\mathbf{A}$$

- **Gauss's law for the magnetic field.** Magnetic fields are source free:

$$\oint_A \mathbf{B}.\mathrm{d}\mathbf{A} = 0$$

- **Ampere's law:**

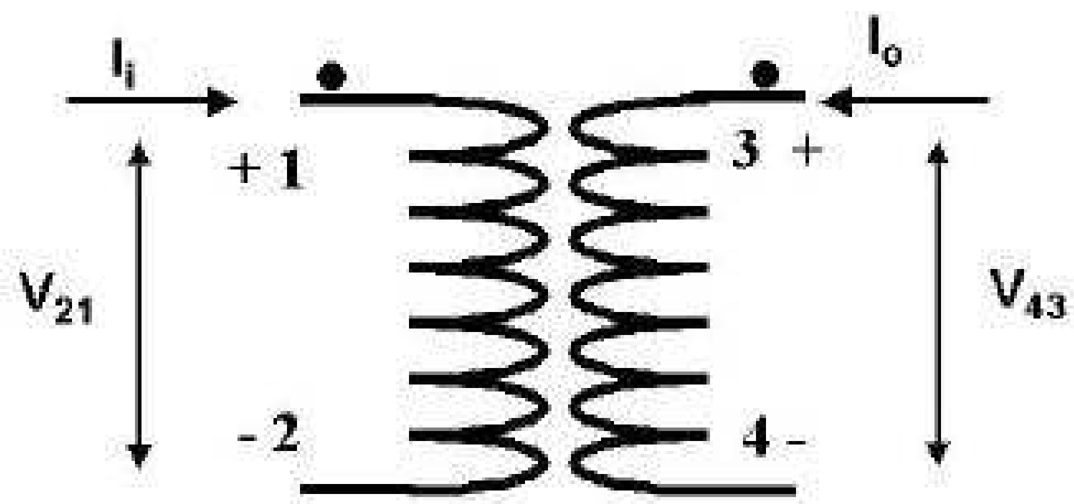
$$\oint_\gamma \mathbf{B}.\mathrm{d}\mathbf{l} = \int_A \mu_0 \left( \mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right).\mathrm{d}\mathbf{A}$$

## Transformers

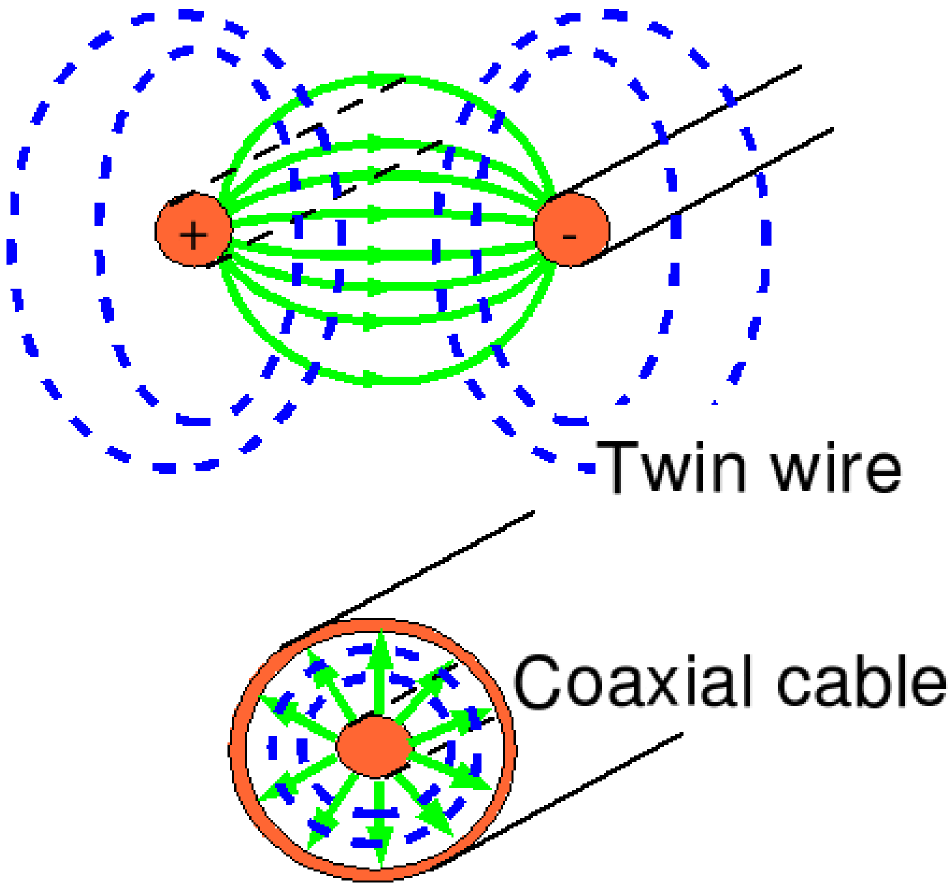
- Combining the effects of self and mutual inductance, the equations of a transformer are given by,

$$V_{21} = j\omega L_1 I_i + j\omega M I_o$$

$$V_{43} = j\omega M I_i + j\omega L_2 I_o$$



## Transmission Lines

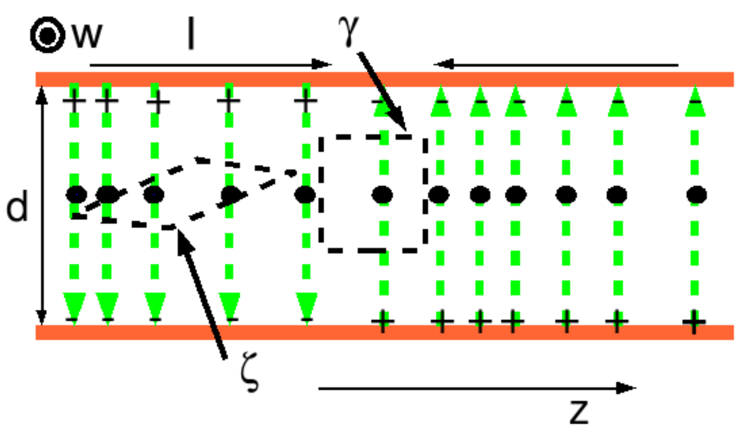


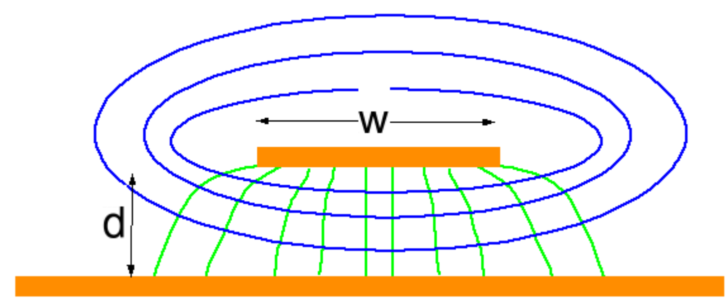
## The Planar Transmission Line: Identical to the plane EM wave

➤ Same derivation as for plane waves (but include a dielectric)...

$$\frac{\partial E}{\partial z} = i\omega B \qquad - Integral\, over\, \gamma$$

$$\frac{\partial B}{\partial z} = \frac{i\omega}{v^2} E \qquad - Integral\, over\, \zeta$$




Stripline



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## The Planar Transmission Line

- From the definition of potential difference and Ampere's law we obtain

$$V = Ed \quad and \quad B = \frac{\mu_o I}{w}$$

- Substitute these in the differential equations for E and B.

$$\frac{\partial E}{\partial z} = i\omega B => \frac{\partial V}{\partial z} = \left(\frac{i\omega\mu_o d}{w}\right) I$$

$$\frac{\partial B}{\partial z} = \frac{i\omega}{v^2} E => \frac{\partial I}{\partial z} = \left(\frac{i\omega w}{\mu_o d v^2}\right) V$$

- For sinusoidal dependence $V, I = \exp i\left(\pm kz - \omega t\right)$

$$\frac{\omega}{k} = \pm v = \pm\sqrt{\frac{1}{\mu_o \epsilon_o \epsilon_r}}$$

$$\frac{V}{I} = \pm\frac{\mu_o v d}{w} = \pm\sqrt{\frac{\mu_o}{\epsilon_o \epsilon_r}}\frac{d}{w}$$

## The Planar Transmission Line

- **For a wave travelling in one direction only, the ratio of V to I is a constant**
- For a given wave, the ratio $|V/I|$ is referred to as the **Characteristic Impedance**.
- The planar transmission line is also called a **stripline**.
- For the stripline:

$$Z_o = \sqrt{\frac{\mu_o}{\epsilon_o \epsilon_r}}\frac{d}{w}$$

- Note that the characteristic impedance can be complex if the dielectric material separating the conductors is lossy (has finite loss tangent).
- In the latter case the propagation speed is also complex.

## The Telegraphist Equations

- We can rewrite the above equations as (**Telegraphist Equations**)

$$\frac{\partial V}{\partial z} = \left(\frac{i\omega Z_o}{v}\right) I$$

$$\frac{\partial I}{\partial z} = \left(\frac{i\omega}{Z_o v}\right) V$$

- See the equivalent web brick derivation in terms of the inductance and capacitance per unit length along the line.
- The Telegraphist Equations become

$$\frac{\partial V}{\partial z} = i\omega L I, \quad L = \frac{Z_o}{v} \quad = \frac{\mu_o d}{w}$$

$$\frac{\partial I}{\partial z} = i\omega C V, \quad C = \frac{1}{Z_o v} \quad = \frac{\epsilon_o \epsilon_r w}{d}$$

## The Telegraphist Equations

- The velocity and characteristic impedance of the line can be expressed in terms of L and C.

$$v = \sqrt{\frac{1}{LC}} \qquad Z_o = \sqrt{\frac{L}{C}}$$

- L and C are the inductance and capacitance per unit length along the line.
- For coaxial cable the formula is quite different (a,b inner, outer radii).

$$C = \frac{2\pi\epsilon_o\epsilon_r}{ln(b/a)} \qquad L = \frac{2\pi ln(b/a)}{\mu_o}$$

$$Z_o = \sqrt{\frac{\mu_o}{\epsilon_o\epsilon_r}\frac{ln(b/a)}{2\pi}} \qquad v = \sqrt{\frac{1}{\mu_o\epsilon_o\epsilon_r}}$$

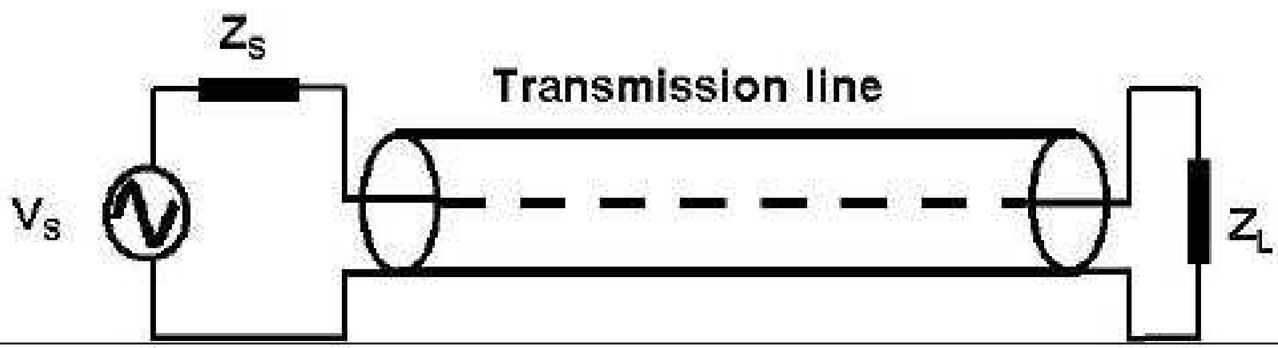
## Reflection Coefficient

- Consider a wave propagating toward a load
- In general there is a wave reflected at the load. The total voltage and current at the load are given by,

$$V_{load} = V_f + V_r \qquad I_{load} = I_f + I_r$$

where

$$V_f = Z_o I_f \qquad V_r = -Z_o I_r$$



## Reflection Coefficient

- At the load,

$$V_{load} = Z_L I_{load} = Z_L \left( I_f + I_r \right) = V_f + V_r = \frac{Z_L}{Z_o} \left( V_f - V_r \right)$$

- Solving for $\rho = V_r / V_f$, we obtain,

$$\rho = \frac{Z_L - Z_o}{Z_L + Z_o}$$

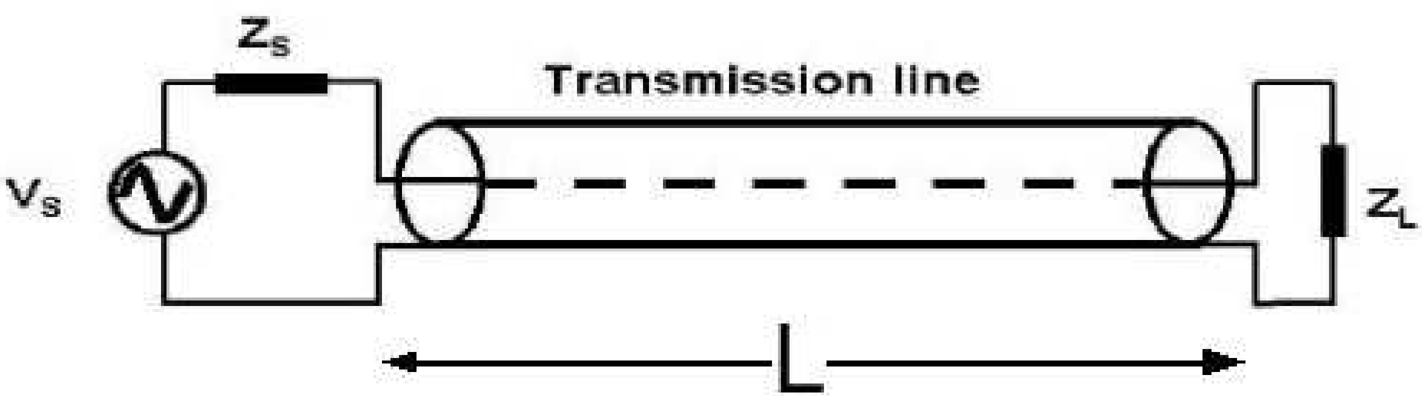
- $\rho$ is the reflection coefficient.
- If $Z_L = Z_o$ there is **no** reflected wave.
- A line terminated in a pure reactance always has $|\rho| = 1$

## Impedance Transformation Along a Line

- Consider a transmission line terminated in an arbitrary impedance $Z_L$.
- The impedance $Z_{in}$ seen at the input to the line is given by

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan kL}{Z_o + jZ_L \tan kL}$$

- If $Z_L = Z_o$, then $Z_{in} = Z_o$.
- If $Z_L = 0$, then $Z_{in} = jZ_o \tan kL$
- If $Z_L = \infty$, then $Z_{in} = Z_o/(j \tan kL)$



## Voltage and Current Transformation Along a Line

- Consider a transmission line terminated in an arbitrary impedance $Z_L$.
- The voltage $V_{in}$ and current $I_{in}$ at the input to the line are given by

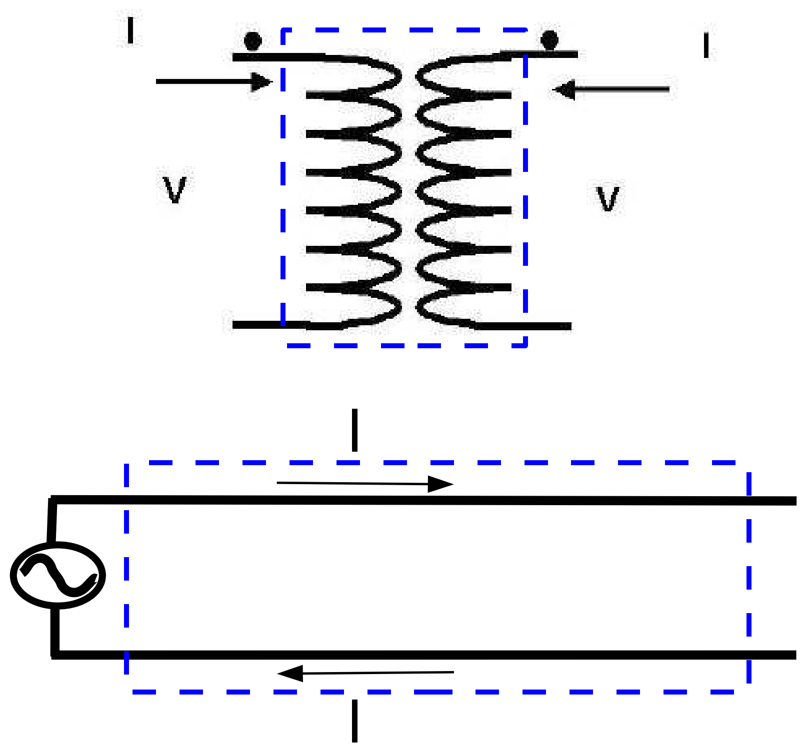
$$V_{in} = V_{end} \cos kL + jZ_o I_{end} \sin kL$$

$$I_{in} = I_{end} \cos kL + j\frac{V_{end}}{Z_o} \sin kL$$

- If a line is **unterminated** then the voltage and current vary along line.

# Transmission Line Transformers

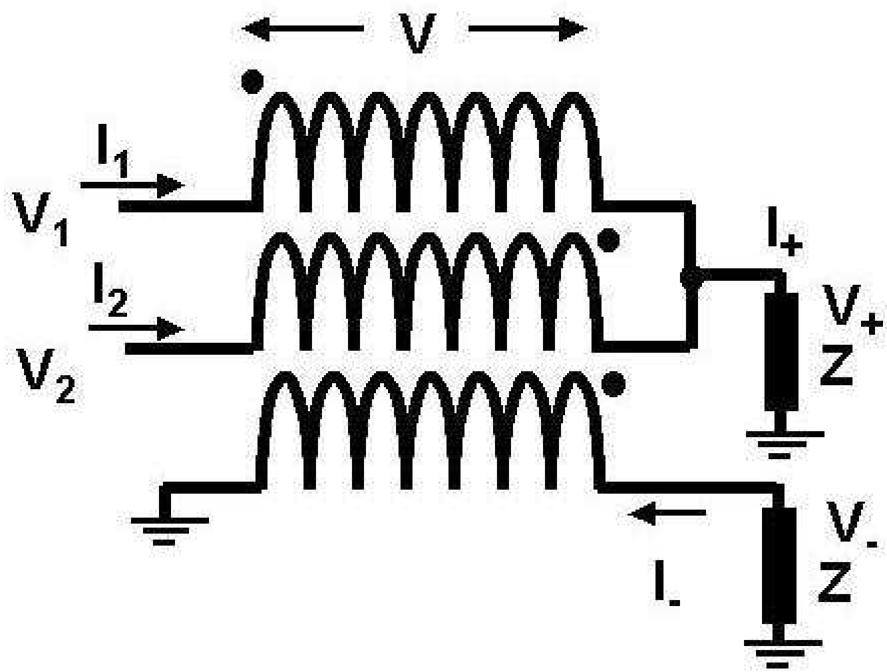
- One way to make $180^o$ Hybrids.
- Transformers and transmission lines are equivalent at Radiofrequency!



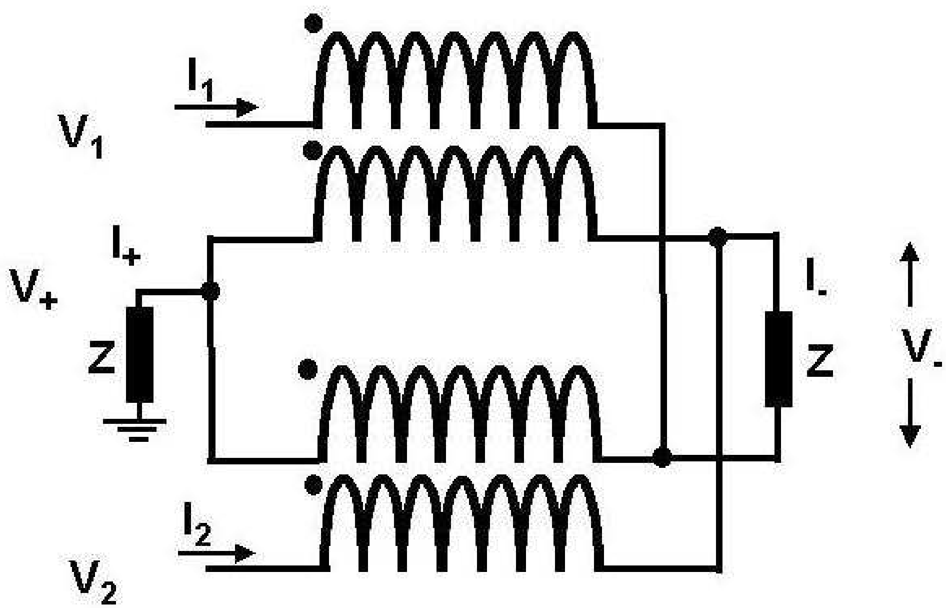
# Rules for Transmission Line Transformers

- Always wind the windings in multifilar fashion.
- Can use either toroidal or linear or whatever shaped ferrites. Toroidal ferrites are usually the best.
- The dot on the transformer diagram points to one end of the wires at one end of the transformer.
- The voltage drop across all windings must be same. **WHY?**
- The currents in the same direction in the windings must sum to zero. **WHY?**
- Respects phase delays along the transmission line when doing its sums?

# Transmission Line Transformer 180$^O$ Hybrid

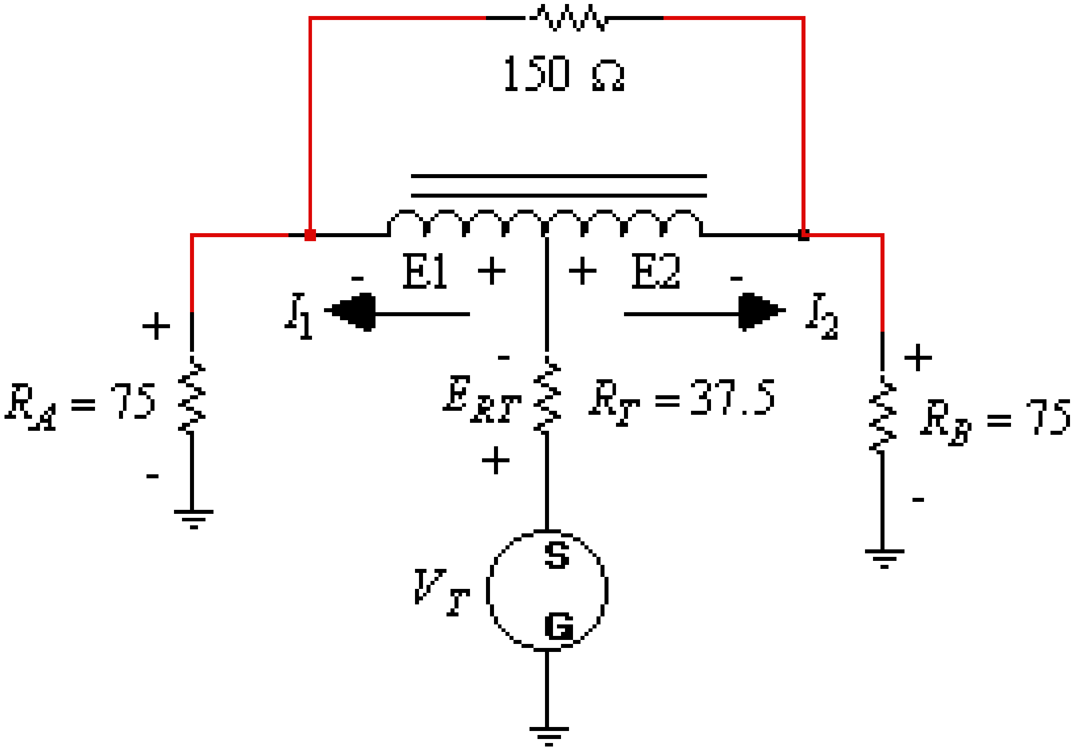


# The Linear Phase Shift Combiner.

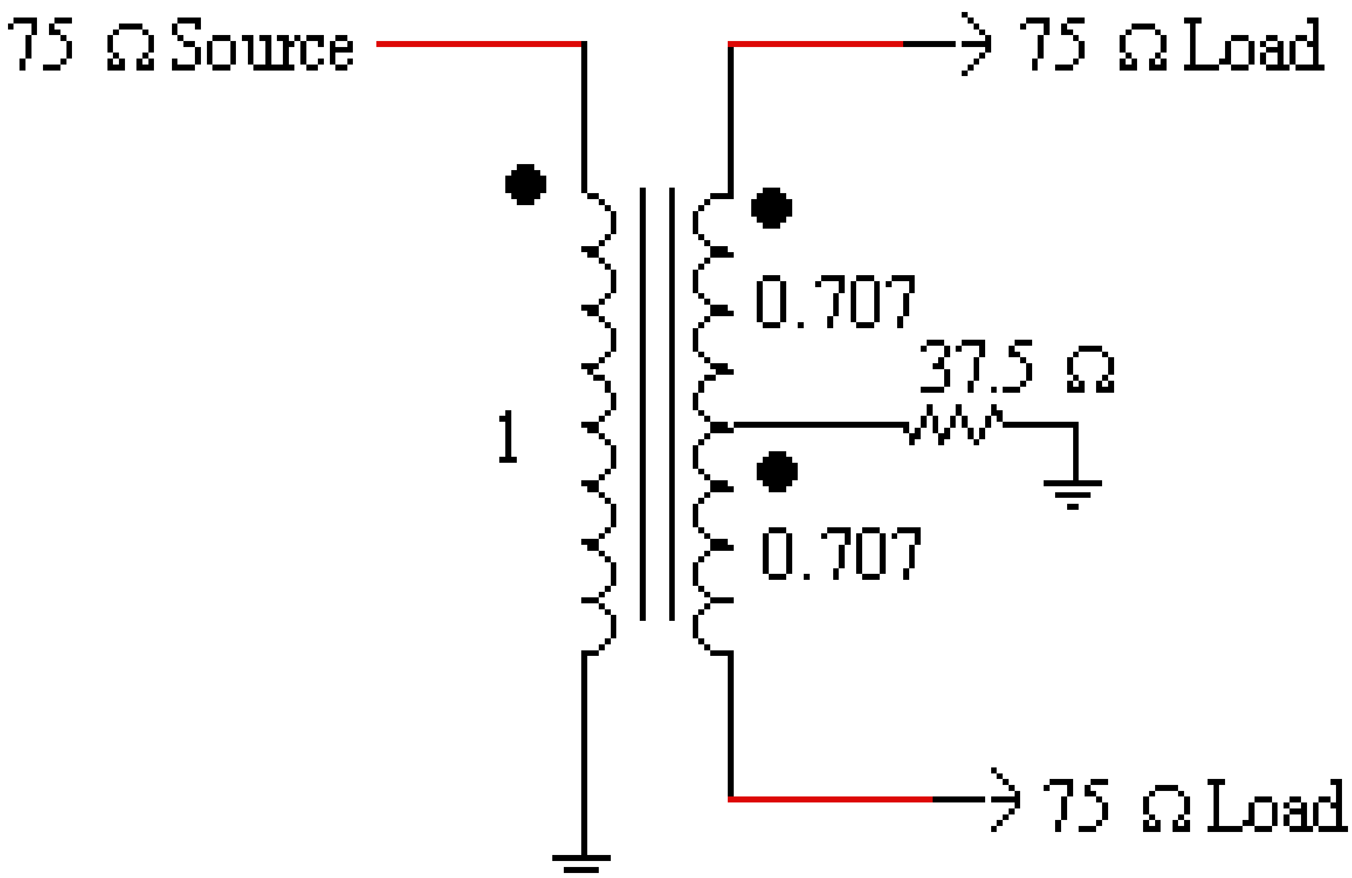


# The Magic-T (Wilkinson)

- The magic-T is a three port device that uses lossiness in order to achieve reciprocity and matching.



# The Magic-T (Wilkinson)



## Skin Depth

- **Electromagnetic waves, j, E, B, ... only penetrate a distance $\delta$ into a metal. Check the magnitude of $\delta$ in lab and web exercises.**
- The wave equation for metals simplifies to...

$$\frac{\partial^2 E_y(z)}{\partial z^2} = j\omega\sigma\mu_0 \, E_y(z)$$

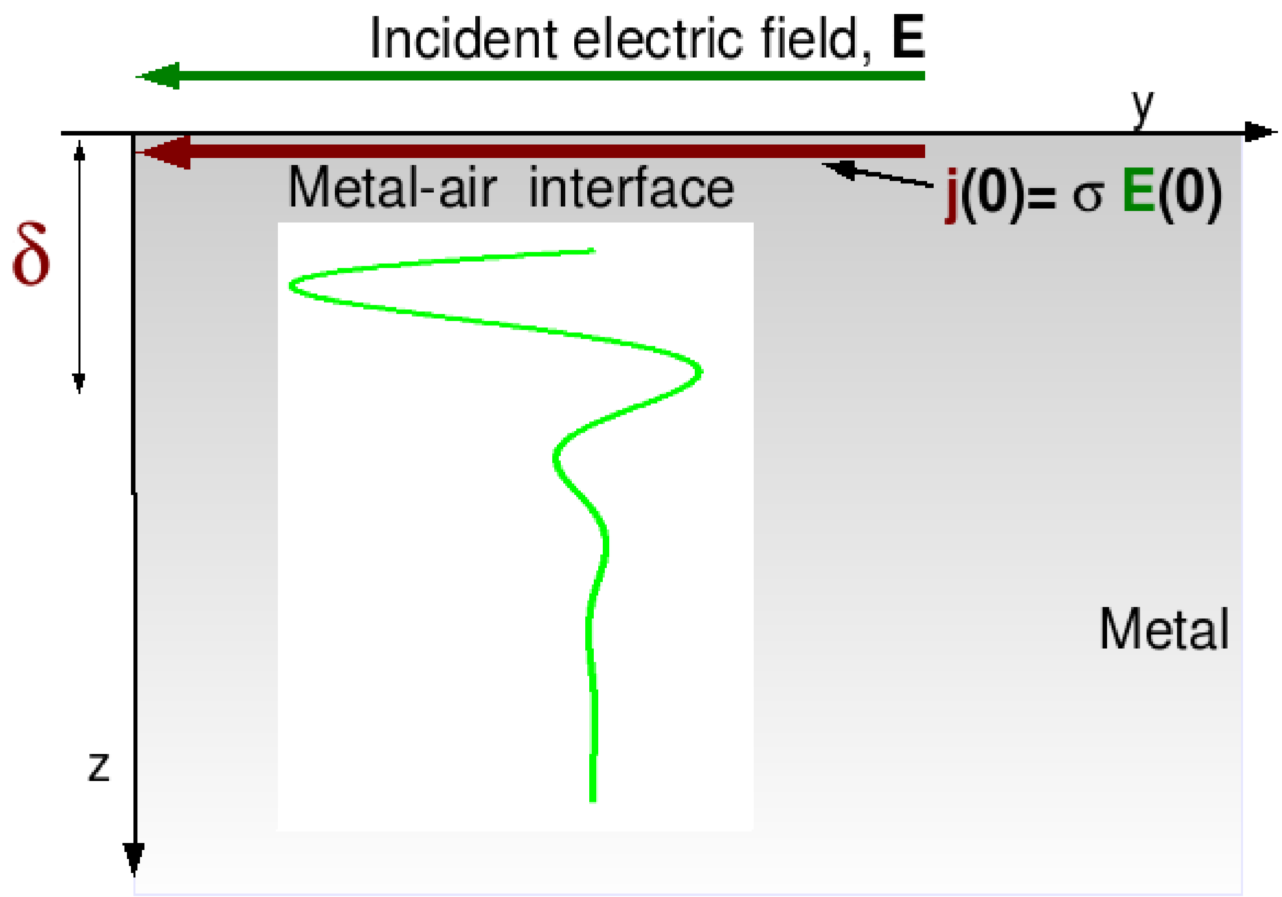
- The solution...

$$E_y(z) = \exp\left(-\frac{1+j}{\delta}\, z\right)$$

- where $\delta$ the **skin depth** is given by...

$$\delta = \sqrt{\frac{2}{\omega\sigma\mu_0}}$$

## Skin Depth



Incident electric field, E
y
Metal-air interface
j(0)= σ E(0)
δ
Metal
z

## Impedance per Square

- By integrating the formula for the electric field inside a metal,

$$E_y(z) = \exp\left(-\frac{1+j}{\delta}z\right)$$

to find the current per unit width $I_s$ we defined the impedance per square as

$$Z_s = E_y(0)/I_s = \frac{1+j}{\sigma\delta} = \sqrt{\frac{\pi\mu_0 f}{\sigma}}(1+j)$$

- For a wire of radius, $a$, length $L$ and circumference $2\pi a$, we obtain

$$Z = \frac{L}{2\pi a}Z_s$$