Four Ports

- Theorems of four port networks
- 180^o and 90^o Hybrids
- Directional couplers
- Transmission line transformers, Four port striplines
- Rat-races, Wilkinsons, Magic-T's
- Circulators, Diplexers and Filter Diplexers

Definition of a Four Port Device

- Until now transformers and transmission lines have been treated as two port devices.. (deliberate)
- Can also define a 4×4 S-parameter scattering matrix where S_{ij} describes the scattered voltage from port i to port j.
- Four port devices with $S_{ij} = S_{ji}$ are termed *reciprocal*.

Passive vs Active Devices

A passive device contains no source that adds power to your signal. Conservation of energy, implies that a passive device can't oscillate. The important properties of a passive network are:

- Whether it is reciprocal or non-reciprocal
- Whether it is lossy or lossless. For a four port device insertion loss is due to internal dissipation or resistance in the device.
- Whether it is impedance matched or unmatched.

Exceptions are *Mixers*. **Why?**

Reciprocal vs Non-Reciprocal Devices

- A reciprocal network is one in which the power losses are the same between any two ports regardless of direction of propagation (scattering parameter $S_{ij} = S_{ji}$).
- A network is known to be reciprocal if it is passive and contains only isotropic materials.
- Examples of reciprocal networks include cables, attenuators, couplers, and antenna systems.

Non-Reciprocal and Non-Unilateral Devices.

Anisotopic materials lead to non-reciprocal devices. Examples of anisotropic media are *magnetised ferrites*, certain *composite and photonic materials* (birefringence) and *magnetised plasmas*.

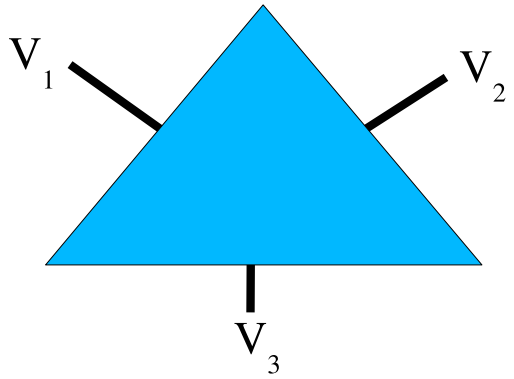
- An *anisotropic material* in one in which the values of the *relative dielectric constant* (and/or *relative permeability*) depends on the orientation (*polarisation*) of the electric (and/or magnetic) field vector.
- This implies that waves of different polarisation propagate at different phase speeds.

Other non-reciprocal devices are non-unilateral devices such as transistors.

Three Port Devices

Theorem: It is impossible for a three-port network to be at the same time **reciprocal, lossless and matched.**

- You can only have two of these properties at the same time!.



Proof

- The network is matched and lossless so, input 1 Watt into port 1, port 2, port 3 in turn, then

$$1 = |S_{21}|^2 + |S_{31}|^2,\ 1 = |S_{12}|^2 + |S_{32}|^2,\ 1 = |S_{23}|^2 + |S_{13}|^2$$

- Suppose that the network is reciprocal. Then S_{21} = S_{12}, etc. We conclude that, $|S_{31}| = |S_{32}|$, $|S_{13}| = |S_{12}|$ and $|S_{21}| = |S_{23}|$.
- If $|S_{31}| = |S_{32}|$ then we can send 1 Watt into ports 1 amd 2 but with a phase such that the power emerging from port 3 is zero.
- Then the power emerging from ports 1 and 2 respectively are S_{21} and S_{12}. Power conservation implies that,

$$2 = |S_{21}|^2 + |S_{12}|^2 = 2|S_{21}|^2 < 2$$

because $|S_{21}|$ is at most unity. It cannot equal unity because from the above $1 = |S_{21}|^2 + |S_{31}|^2$, means $|S_{31}|^2 = 0$. But $|S_{13}| = |S_{12}|$

- Contradiction!

Three Port Devices: The Circulator

- An example of a three port device that is both matched and lossless is the *The Circulator*
- In a circulator, power incident on one port always goes to the **right!** (or the left)

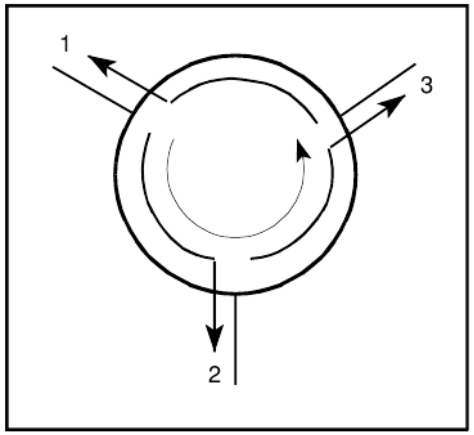


Figure 1. Symbolic Expression for a Y-Junction Circulator

Fundamentals of Four Port Devices

- There are two fundamental types of lossless, matched and reciprocal four port devices known as *180^o and 90^o Hybrids*

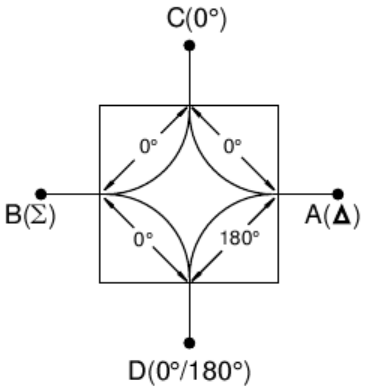


Figure 1a: RF Frequencies
(Up to 2000 MHz)

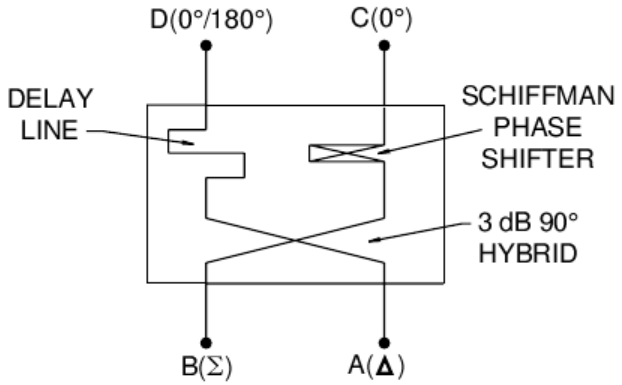


Figure 1b: Microwave Frequencies
(500 MHz and Up)

Fundamental Observations About Four Port Devices

Four Port Devices

- Put 1 Watt into ports A and B with arbitrary phases ϕ_{AD}, ϕ_{AC}, ϕ_{BC} and ϕ_{BD}. Then the output powers can be written,

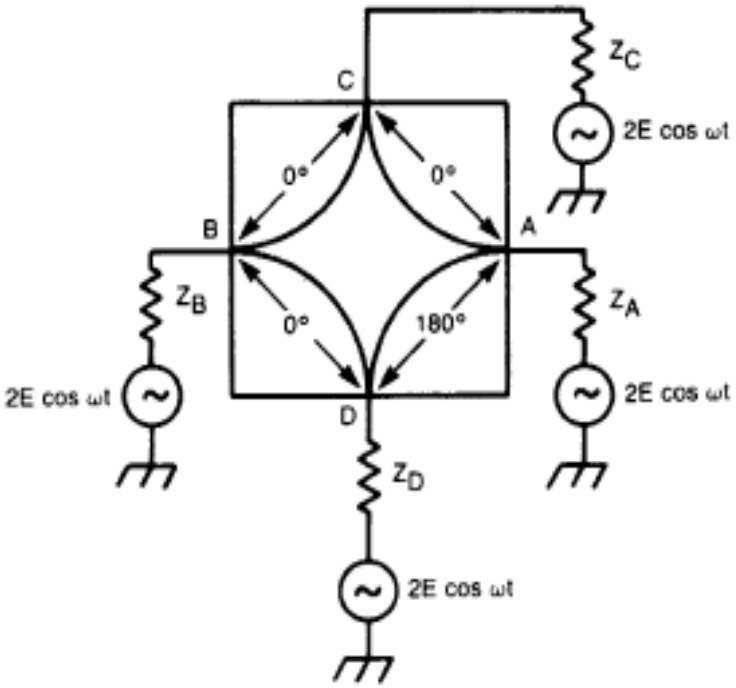
$$C = 1 + \cos(\phi_{AC} - \phi_{BC}), \;\; D = 1 + \cos(\phi_{AD} - \phi_{BD})$$

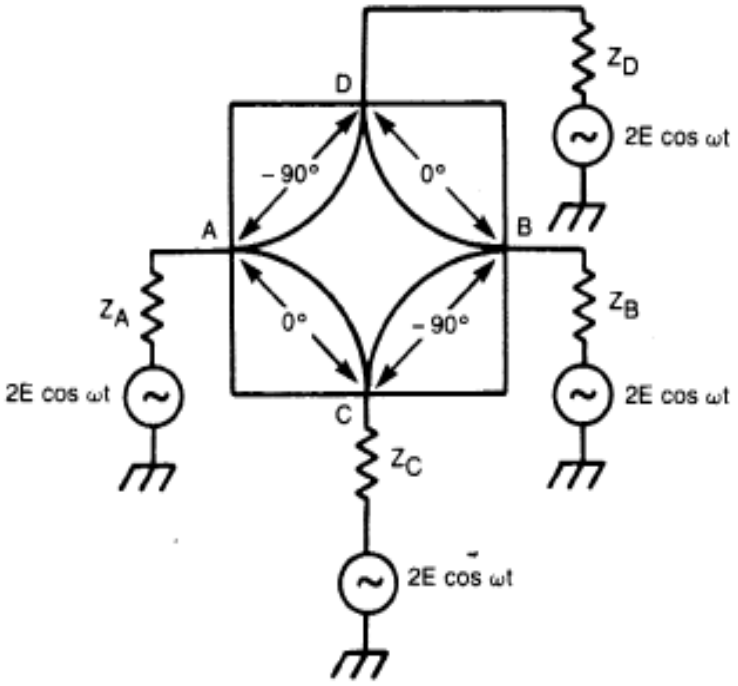
From which energy conservation implies,

$$0 = \cos(\phi_{AC} - \phi_{BC}) + \cos(\phi_{AD} - \phi_{BD})$$

- We distinguish two cases, ($\phi_{AC} = 0^o$, $\phi_{AD} = 0^o$) and ($\phi_{AC} = 0^o$, $\phi_{AD} = 90^o$).

The 180^o and 90^o Hybrids





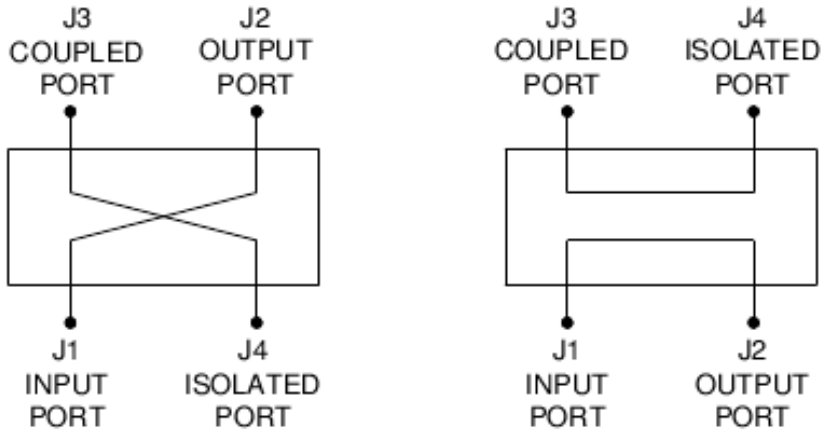
The 180^{o} Hybrid

Table 1 - Power Divider Relationships for 180° Hybrids

		Output Signals			
Input Signal	**Input Port**	**Port A**	**Port B**	**Port C**	**Port D**
E cos(ωt)	A	–	0	0.707 E cos(ωt)	
	B	0	–	0.707 E cos(ωt)	0.707 E cos(ωt)
	C	0.707 E cos(ωt)	0.707 E cos(ωt)	–	0
	D	0.707 E cos(ωt-180°)	0.707 E cos(ωt)	0	–

1) This chart assumes only one generator at a time is operating.
2) Infinite isolation is assumed in the hybrid junction.
3) This is only applicable to RF and Magic T configurations. (Figures 1a and 2) for Microwave frequency Schiffman - type 180° hybrids, use Table 1A.

The 90^{o} Hybrids

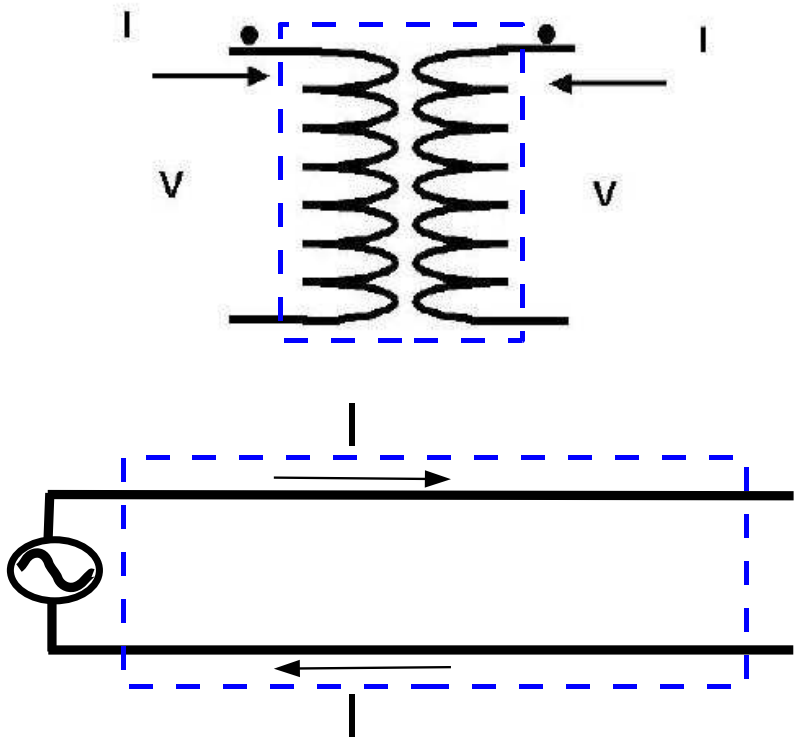


	J1	J2	J3	J4
J1	X	-90°	0°	ISO
J2	-90°	X	ISO	0°
J3	0°	ISO	X	-90°
J4	ISO	0°	-90°	X

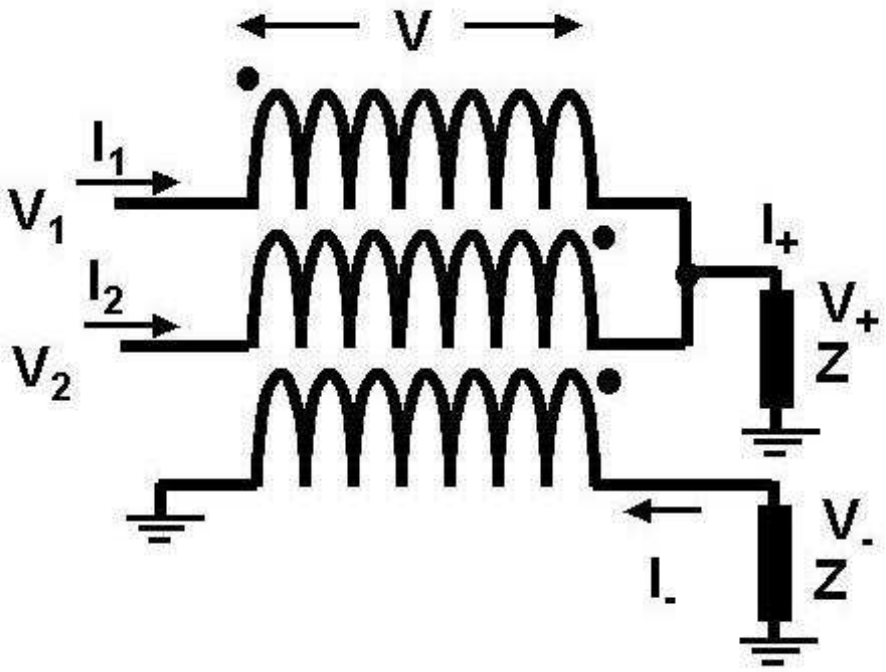
Figure 6. Microwave Frequency 90° Hybrid Schematics and Phase Truth Table

Transmission Line Transformers

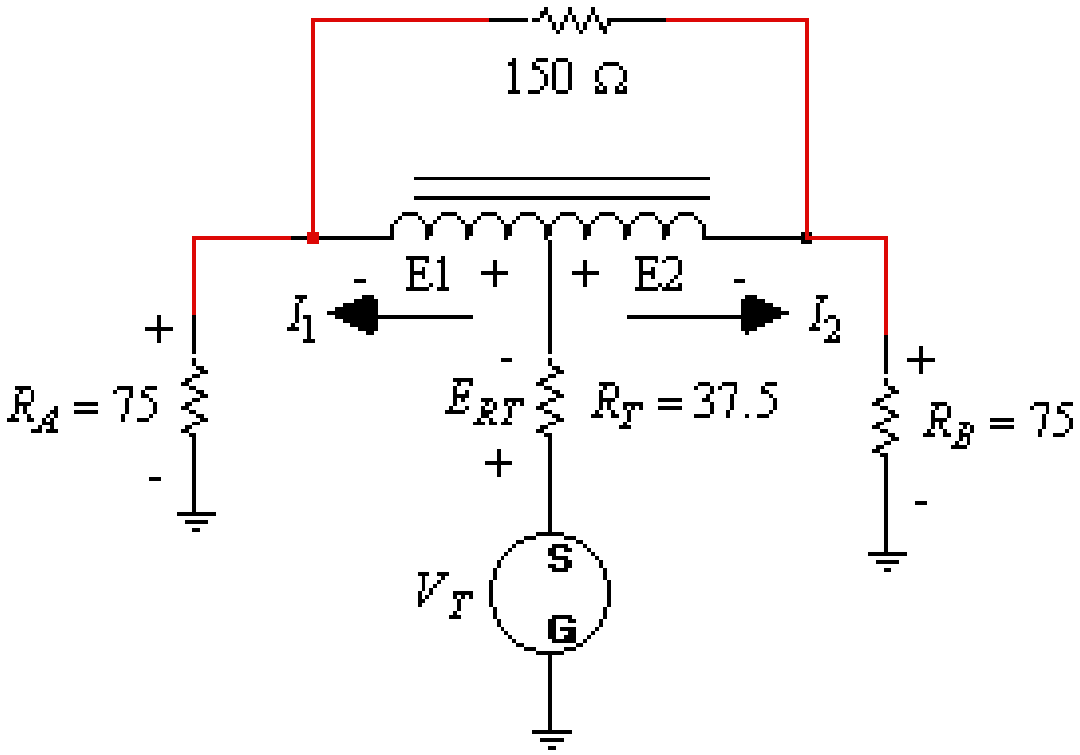
- One way to make 180^o Hybrids.
- Transformers and transmission lines are equivalent at Radiofrequency!



Transmission Line Transformer 180^{o} Hybrid



The Magic-T



The Magic-T