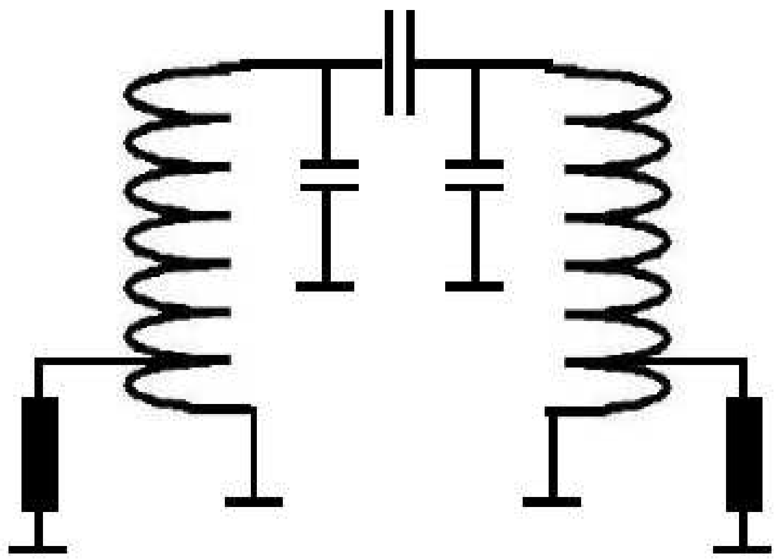
Project ENGN4545

- More on transformers: LC resonator filters
- Transistors
- Transistor S-parameters and Y-parameters
- Transistors and transformers in solve.

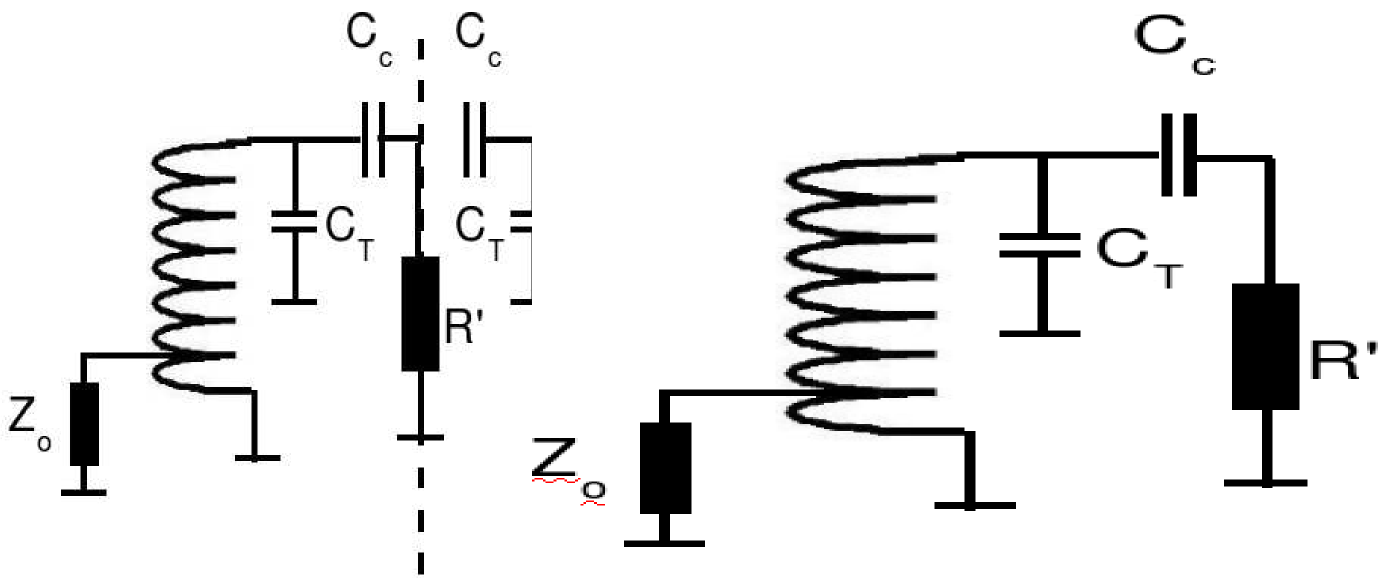
LC (Helical) Resonator Filters

- Back to back electrostatically coupled transformers.
- Make excellent narrowband filtes.



LC resonator filter design technique

- We design in the same way as Pi and T matching networks.
- A semi intuitive design procedure.
- Divide the circuit into two halves through the coupling capacitor C_c.
- Consider matching the source (or load) Z_o to a *virtual resistance*, R'.

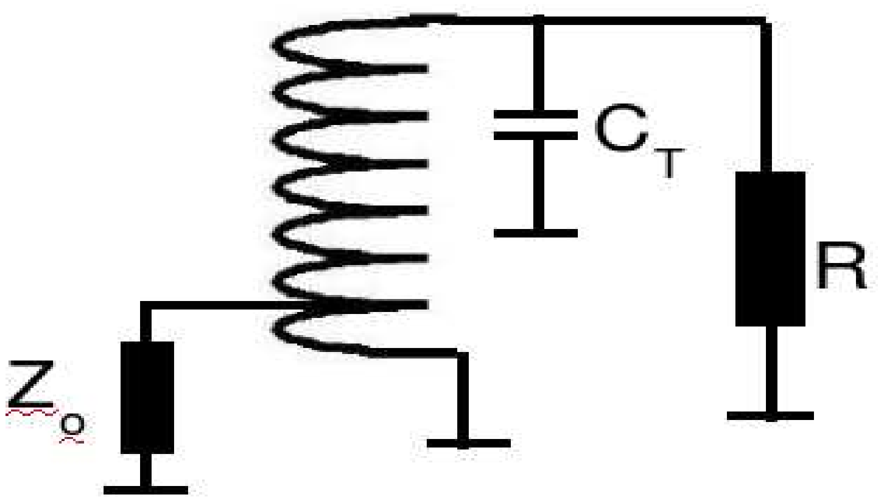


LC resonator filter design technique

- Absorb the coupling capacitor C_c into the tuning capacitor C_T.

$$R = R' + \frac{1}{\omega^2 C_c^2 R'} \quad C_c -> \frac{C_c}{1+(\omega C_c R')^2}$$

- We can now simplify to the following,

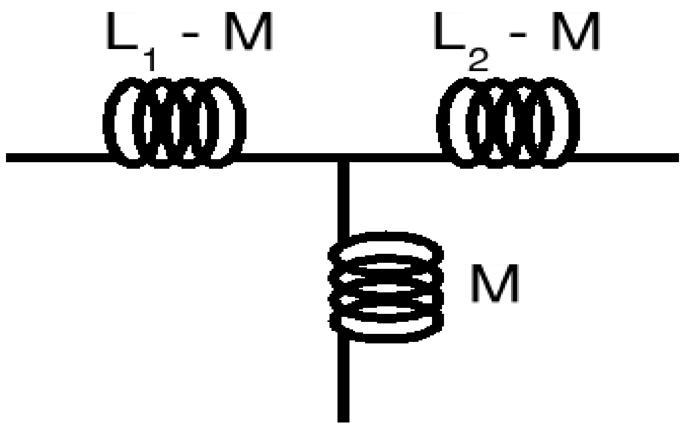


LC resonator filters: Things to notice...

- We now have a matching network which matches Z_o to R.
- The coupling capacitor C_c has been eliminated from the design flow. This means that we will not be able to choose it mathematically. Instead, We choose it *physically* using MATLAB.
- C_c nonetheless plays an important role in the circuit, for without it, no signal can pass from one side of the circuit to the other.
- It should be clear that for a **narrowband** filter we are going to design a **high Q** L-matching network.

LC resonator filter design technique using autotransformer theory

- We have a matching network to match Z_o to R.
- We do not know R. Instead we choose Q.
- Specifying Q is the **only** step in the process!
- First we must model the autotransformer.
- Represent the autotransformer as the T-circuit equivalent of inductors...



The Autotransformer

- An autotransformer is a special transformer in which there is only **one turn**. The “primary” is formed by a tap on the “secondary”.
- In an autotransformer $V_1 = \nu V_2$, where V_1 is the primary tap voltage and V_2 is the voltage across the whole transformer. (the secondary!).
- ν is the ratio of the tap turn number to the total number of turns on the transformer.
- The mutual inductance, $M = \nu L$, where L is the total inductance of the transformer and $L_1 = \nu^2 L$.

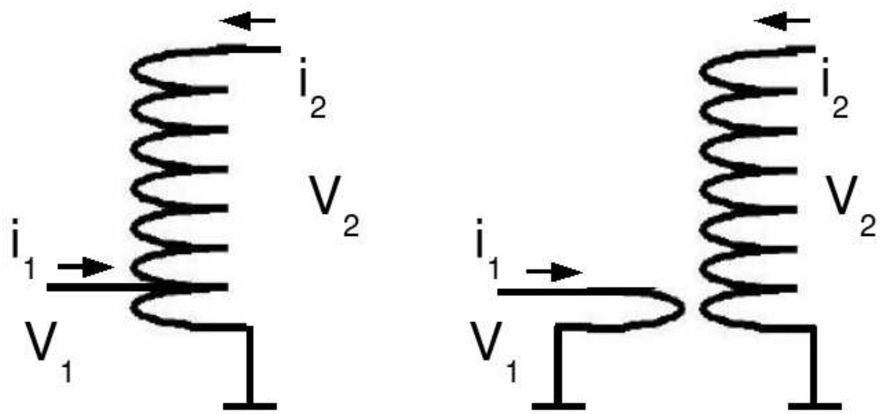
Proof of these relations for the Autotransformer

- The equations of a general transformer are given by,

$$V_1 = j\omega L_1 i_1 + j\omega M i_2$$

$$V_2 = j\omega M i_1 + j\omega L_2 i_2$$

- The autotransformer and its transformer equivalent:



Proof of these relations for the Autotransformer

- Multiply the second equation by ν.

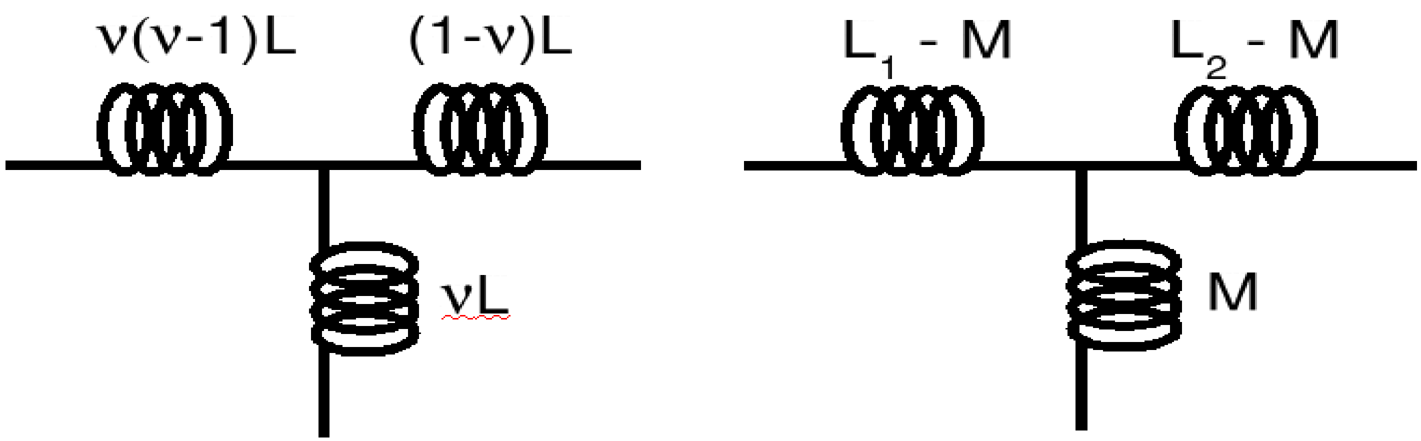
$$V_1 = j\omega L_1 i_1 + j\omega M i_2$$

$$\nu V_2 = V_1 = j\omega\nu M i_1 + j\omega\nu L_2 i_2$$

- If we use $V_1 = \nu V_2$ to replace V_2, then we have two identical equations in V_1.
- Equate the coefficients of i_1 and i_2 and we obtain, $L_1 = \nu M$ and $M = \nu L_2$.
- Finally substitute L for L_2 (the total inductance) and then $M = \nu L$ and $L_1 = \nu M = \nu^2 L$. -Q.E.D.

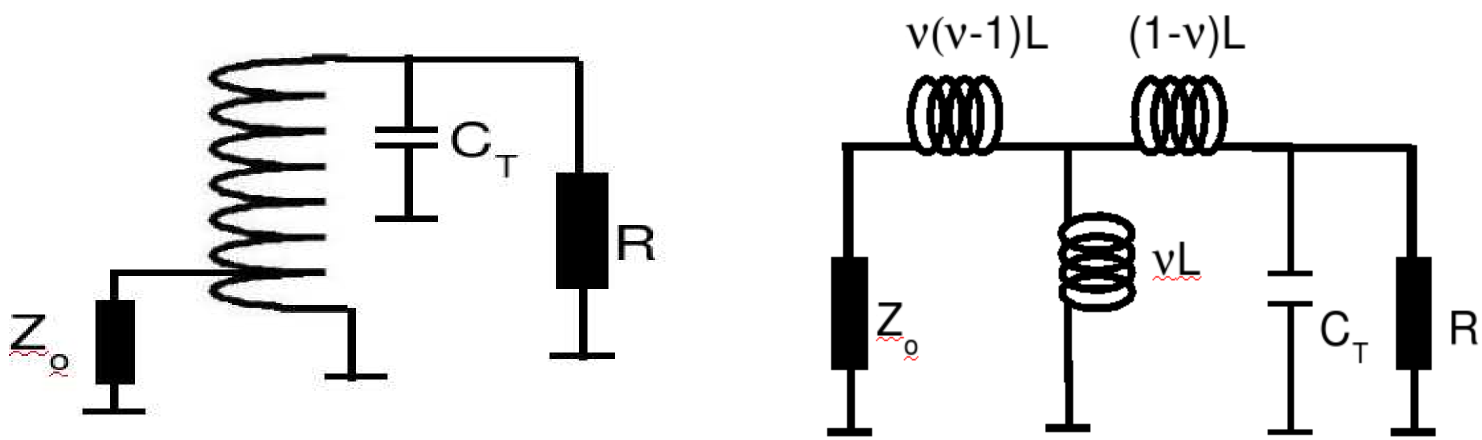
Proof of these relations for the Autotransformer

- We can replace the transformer T-circuit as follows.



Back to the LC resonator

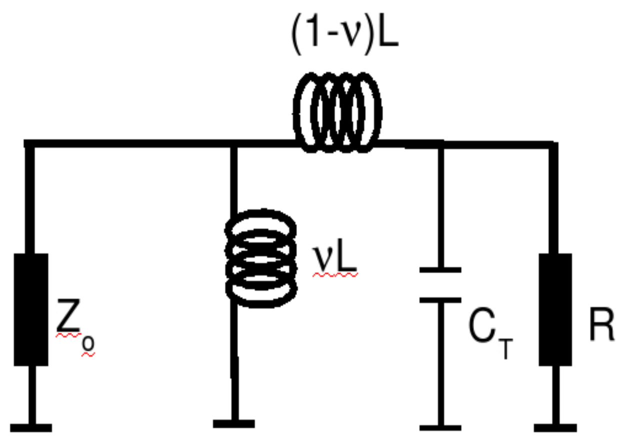
- The LC resonator now has the following simple form,



Back to the LC resonator

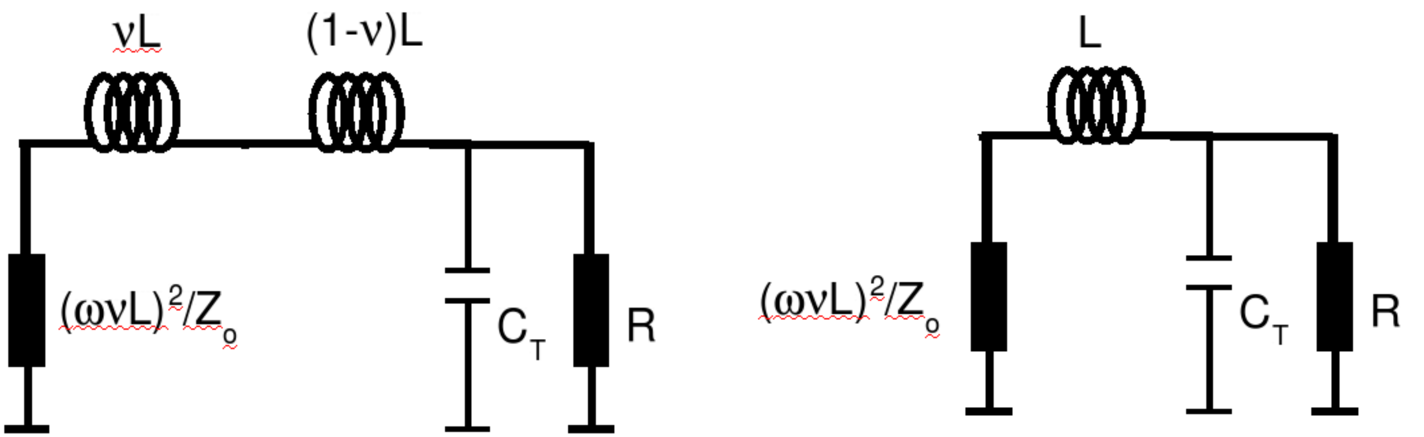
- Can still simplify this.
- In order to get a high Q circuit, the input impedance of the input to the transformer must be much lower than Z_o, hence

$$Z_o \gg \omega\nu(1-\nu)L$$



The LC resonator

- Finally express the left leg parallel combination of Z_o and νL as a series combination,



The LC resonator

- Now apply the matching technique for L-type matching networks.
- The only difference is that because we do not know R we must choose Q instead. **A very convenient situation!**

$$R = \frac{(\omega\nu L)^2}{Z_o}\left(1 + Q^2\right) \approx \frac{(\omega\nu L Q)^2}{Z_o}$$

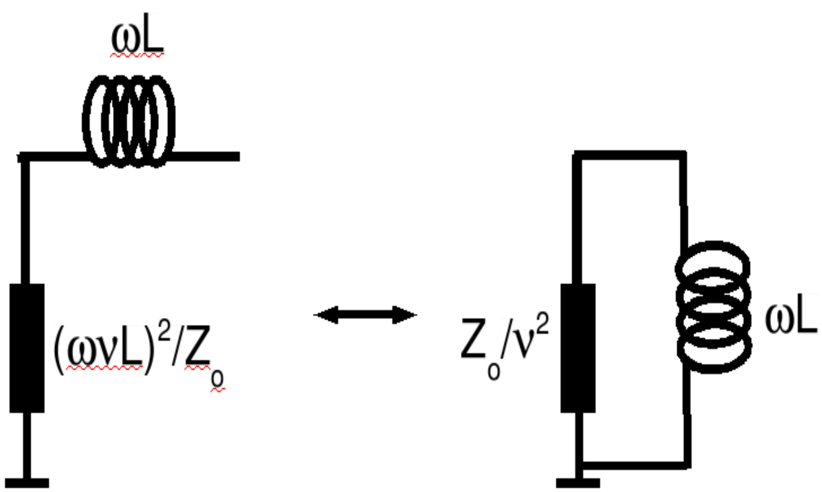
provided $Q \gg 1$. And

$$Q = \omega C_T R = \frac{Z_o \omega L}{(\omega\nu L)^2} = \frac{Z_o}{\omega L \nu^2}$$

The LC resonator

- If we express the left leg as a parallel rather than a series combination we obtain,

$$R = \frac{Z_o}{\nu^2} \quad and \quad \omega^2 L C_T = 1$$



The LC resonator design procedure

- Guess L. Say $L = 1\mu H$.
- Compute $C_T = 1/\omega^2/L$
- Choose Q.
- Compute ν

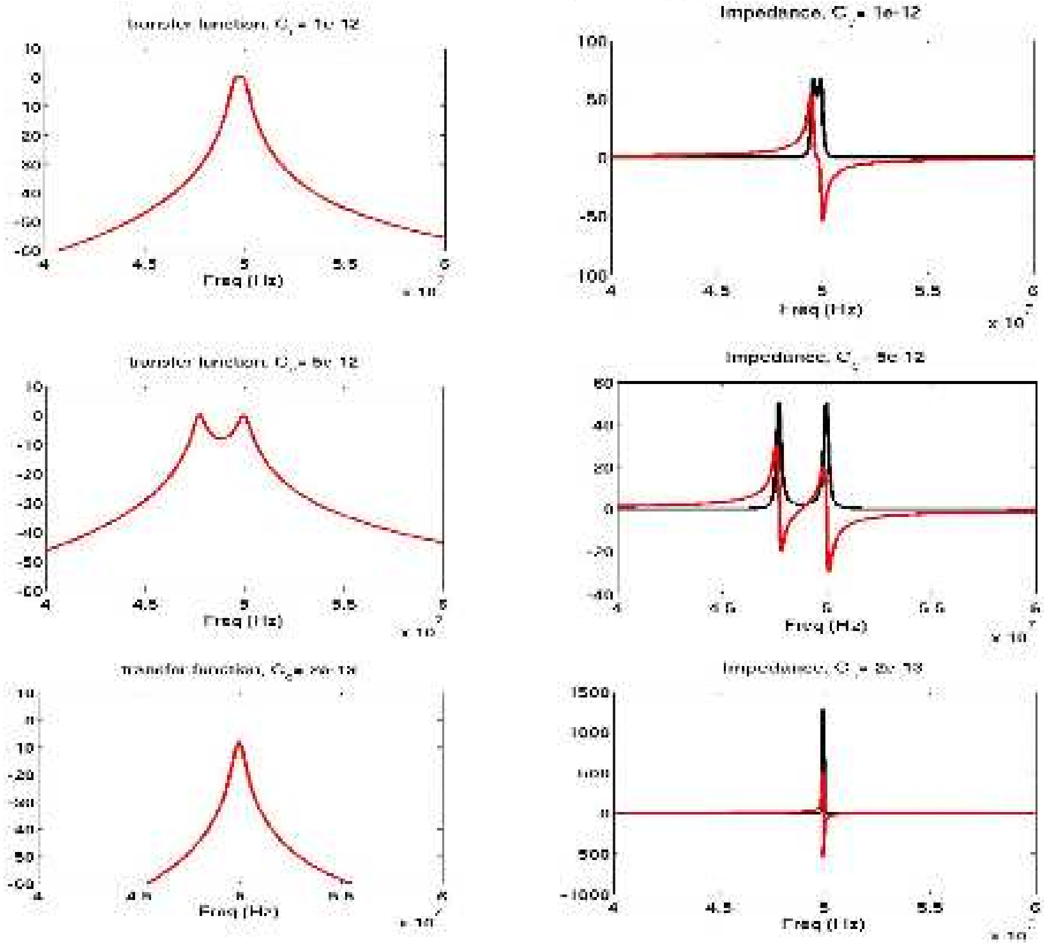
$$\nu = \sqrt{\frac{Z_o}{\omega Q L}}$$

- ν gives us the tap location, $L_1 = \nu^2 L$, the mutual inductance, $M = \nu L$.
- Also $L_2 = L$.

Example

- Design a narrowband filter for 50 MHz.
- Let Q = 100 and L=100 nH.
- Choose a value of C_c

Choice of coupling capacitor

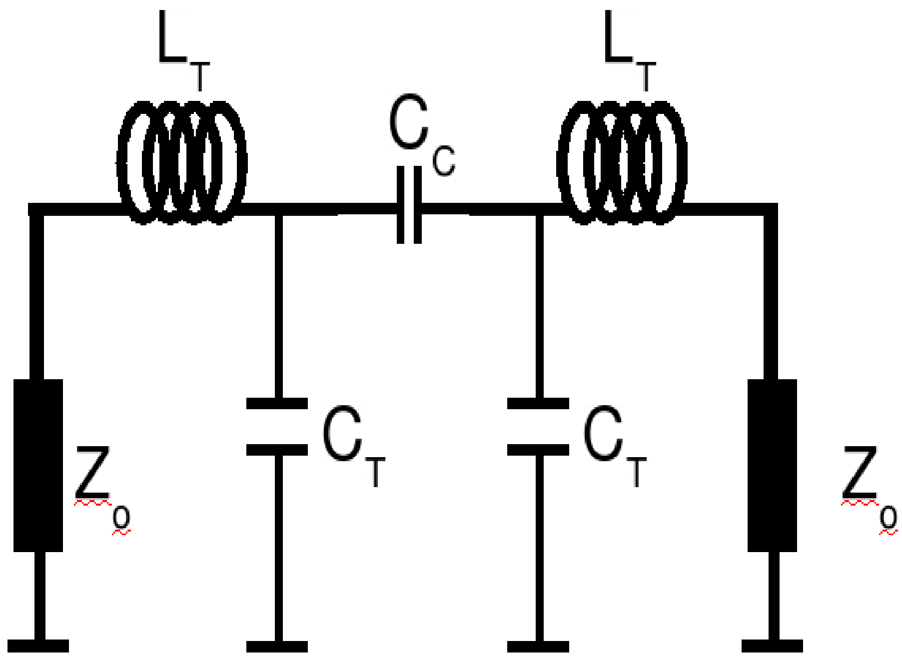


Loose coupling

Tight coupling

A Simpler Narrowband Circuit for low Q

- Consider the following



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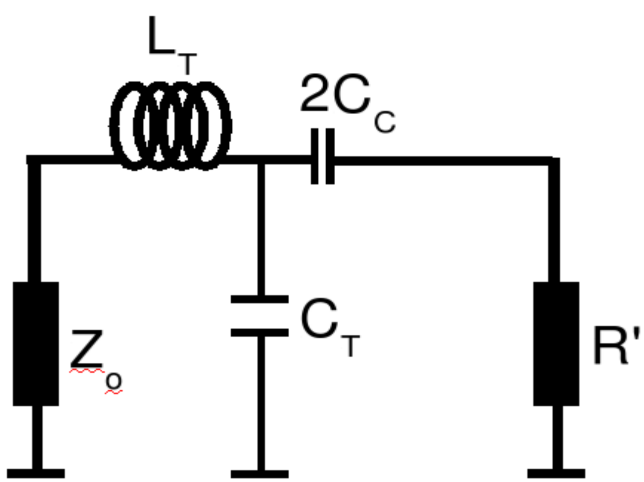
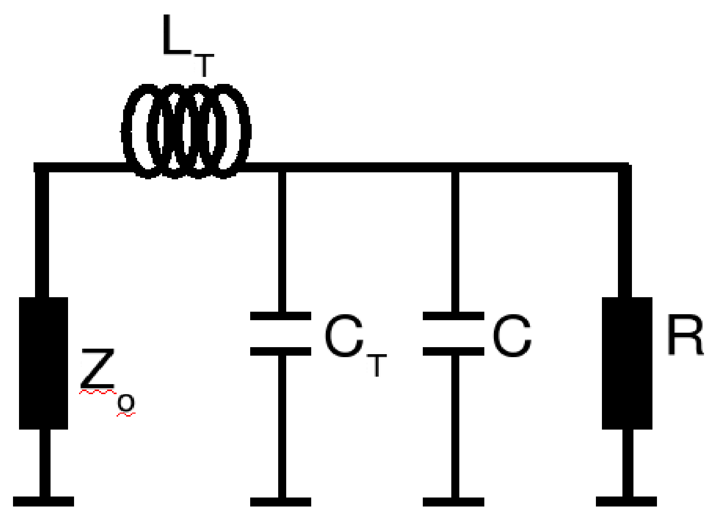
A Simpler Narrowband Circuit for low Q

- Divide in half as usual





A Simpler Narrowband Circuit for low Q

- We can show that

$$C = \frac{2C_c}{1 + (2\omega C_c R)^2}$$

and

$$R = Z_o\left(1 + Q^2\right)$$

$$C + C_T = \frac{Q}{\omega R}, \quad and \quad L_T = \frac{QZ_o}{\omega}$$

A Simpler Narrowband Circuit for low Q

- Once again C_c is not in the design flow so we have to guess it while using MATLAB to improve the performance.
- Note that if we change the centre frequency ω to ω/r, then we can scale the circuit components as follows,

$$L_T = rL_T, \; C_T = rC_T \;\; and \;\; C_c = rC_c.$$

- Thus to do a design for a different frequency just change r.