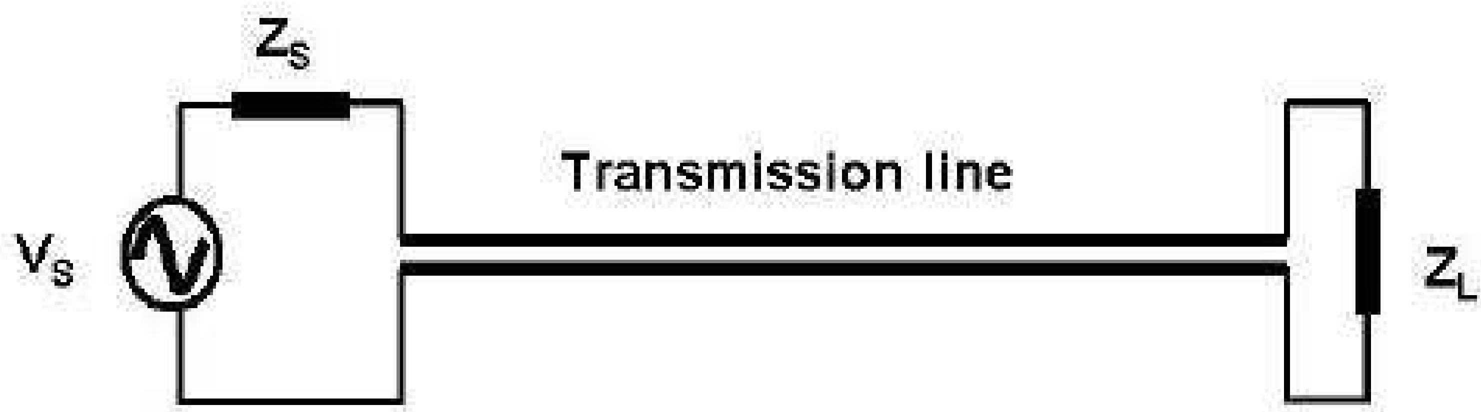
Transmission Lines

- Transmission lines
- Telegraphist Equations
- Reflection Coefficient
- Transformation of voltage, current and impedance
- Application of trasnmission lines

Definition

- If stray inductance and capacitance causes all wires to be dispersive (frequency dependent). Then how can we transport any power at radiofrequency?
- A trasnmission line is a waveguide in which stray capacitance and inductance lead to non dispersive propagation. (discrete picture)
- A trasnmission line is a waveguide that allows plane electromagnetic waves to propagate. (wave picture)



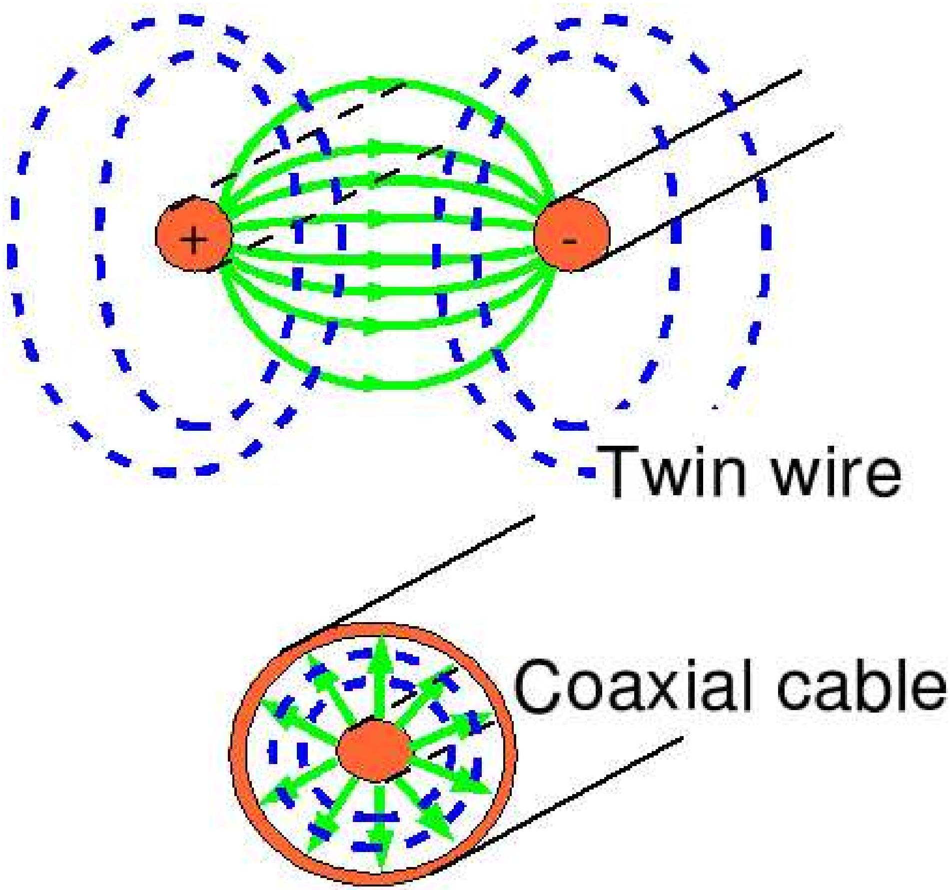
Properties of Transmission Lines

- Many different varieties.. **balanced lines** such as twin wire and **unbalanced lines** such as coaxial cable.
- Transmisssion lines must always have two parallel conductors (a single wire is **always an antenna** at radiofrequency.
- At low powers coax is often flexible and the conductors are separated by a dielectric. Example dielectrics are **polythene** (RG-58 Coax), **foam PFE** high quality sem-rigid (Aluminium jacket) coax and **PTFE** (teflon).
- On PCBs you need to manufacture your own transmission lines... **PCB tracks are designed as striplines**. In this case the dielectric is **FR4** (fibreglass).

Properties of Transmission Lines

- No net current flows along the line.
- In coax, current flows along the outside of the centre conductor and in the opposite direction on the inside of the outer conductor. Why? **Coaxial cables are shielded**.
- Electric fields point radially between the inner and outer conductors. The magnetic fieldlines circle the inner conductor.
- Transmission lines are versatile. Use them in impedance transforming elements (baluns), filters, antennas, DC blocks.

Properties of Transmission Lines

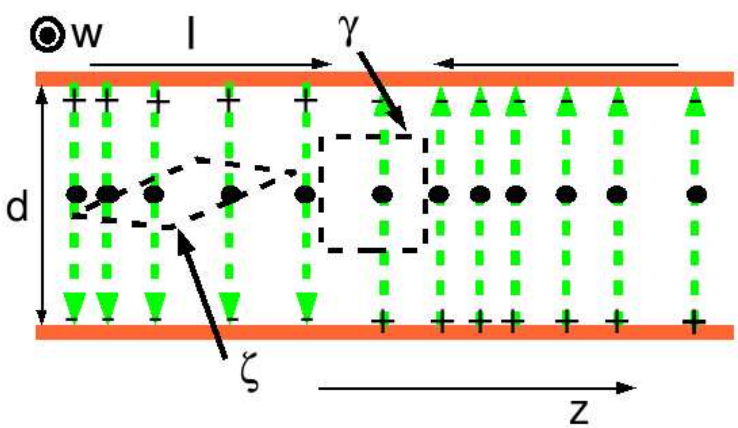


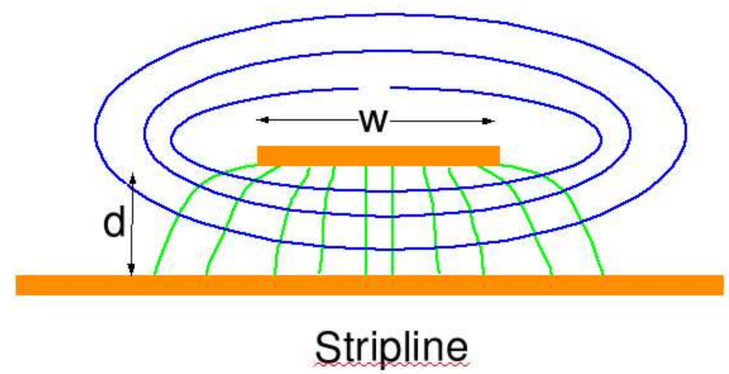
The Planar Transmission Line: Identical to the plane EM wave

- Same derivation as for plane waves (but include a dielectric)...

$$\frac{\partial E}{\partial z} = i\omega B \qquad - Integral\, over\, \gamma$$

$$\frac{\partial B}{\partial z} = \frac{i\omega}{v^2} E \qquad - Integral\, over\, \zeta$$





The Planar Transmission Line

- From the definition of potential difference and Ampere's law we obtain

$$V = Ed \quad and \quad B = \frac{\mu_o I}{w}$$

- Substitute these in the differential equations for E and B.

$$\frac{\partial E}{\partial z} = i\omega B => \frac{\partial V}{\partial z} = \left(\frac{i\omega\mu_o d}{w}\right) I$$

$$\frac{\partial B}{\partial z} = \frac{i\omega}{v^2} E => \frac{\partial I}{\partial z} = \left(\frac{i\omega w}{\mu_o d v^2}\right) V$$

- For sinusoidal dependence $V, I = \exp i\,(\pm kz - \omega t)$

$$\frac{\omega}{k} = \pm v = \pm\sqrt{\frac{1}{\mu_o \epsilon_o \epsilon_r}}$$

$$\frac{V}{I} = \pm\frac{\mu_o v d}{w} = \pm\sqrt{\frac{\mu_o}{\epsilon_o \epsilon_r}}\frac{d}{w}$$

The Planar Transmission Line

- **For a wave travelling in one direction only, the ratio of V to I is a constant**
- For a given wave, the ratio $|V/I|$ is referred to as the **Characteristic Impedance**.
- The planar transmission line is also called a **stripline**.
- For the stripline:

$$Z_o = \sqrt{\frac{\mu_o}{\epsilon_o \epsilon_r}} \frac{d}{w}$$

- Note that the characteristic impedance can be complex if the dielectric material separating the conductors is lossy (has finite loss tangent).
- In the latter case the propagation speed is also complex.

The Telegraphist Equations

- We can rewrite the above equations as (**Telegraphist Equations**)

$$\frac{\partial V}{\partial z} = \left(\frac{i\omega Z_o}{v}\right) I$$

$$\frac{\partial I}{\partial z} = \left(\frac{i\omega}{Z_o v}\right) V$$

- See the equivalent web brick derivation in terms of the inductance and capacitance per unit length along the line.
- The Telegraphist Equations become

$$\frac{\partial V}{\partial z} = i\omega L I, \quad L = \frac{Z_o}{v} \quad = \frac{\mu_o d}{w}$$

$$\frac{\partial I}{\partial z} = i\omega C V, \quad C = \frac{1}{Z_o v} \quad = \frac{\epsilon_o \epsilon_r w}{d}$$

The Telegraphist Equations

- The velocity and characteristic impedance of the line can be expressed in terms of L and C.

$$v = \sqrt{\frac{1}{LC}} \qquad Z_o = \sqrt{\frac{L}{C}}$$

- L and C are the inductance and capacitance per unit length along the line.
- For coaxial cable the formula is quite different (a,b inner, outer radii).

$$C = \frac{2\pi\epsilon_o\epsilon_r}{ln(b/a)} \qquad L = \frac{\mu_o ln(b/a)}{2\pi}$$

$$Z_o = \sqrt{\frac{\mu_o}{\epsilon_o\epsilon_r}}\frac{ln(b/a)}{2\pi} \qquad v = \sqrt{\frac{1}{\mu_o\epsilon_o\epsilon_r}}$$

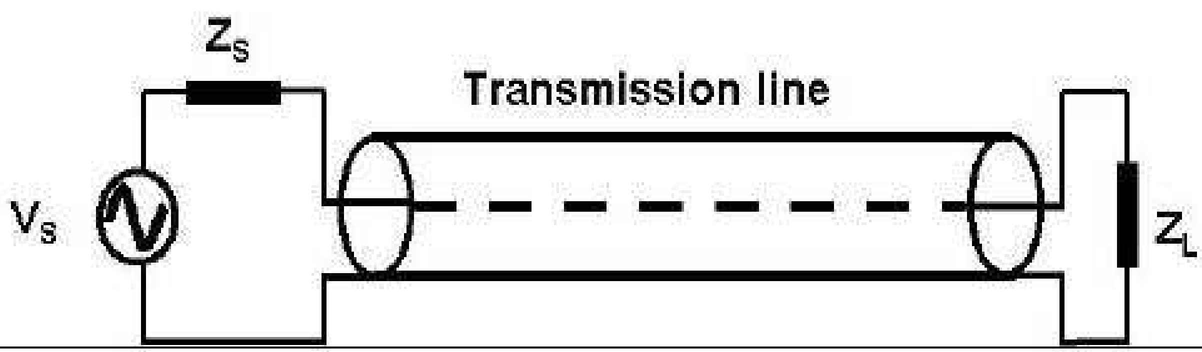
Reflection Coefficient

- Consider a wave propagating toward a load
- In general there is a wave reflected at the load. The total voltage and current at the load are given by,

$$V_{load} = V_f + V_r \qquad I_{load} = I_f + I_r$$

where

$$V_f = Z_o I_f \qquad V_r = -Z_o I_r$$



Reflection Coefficient

- At the load,

$$V_{load} = Z_L I_{load} = Z_L \left(I_f + I_r\right) = V_f + V_r = \frac{Z_L}{Z_o}\left(V_f - V_r\right)$$

- Solving for $\rho = V_r/V_f$, we obtain,

$$\rho = \frac{Z_L - Z_o}{Z_L + Z_o}$$

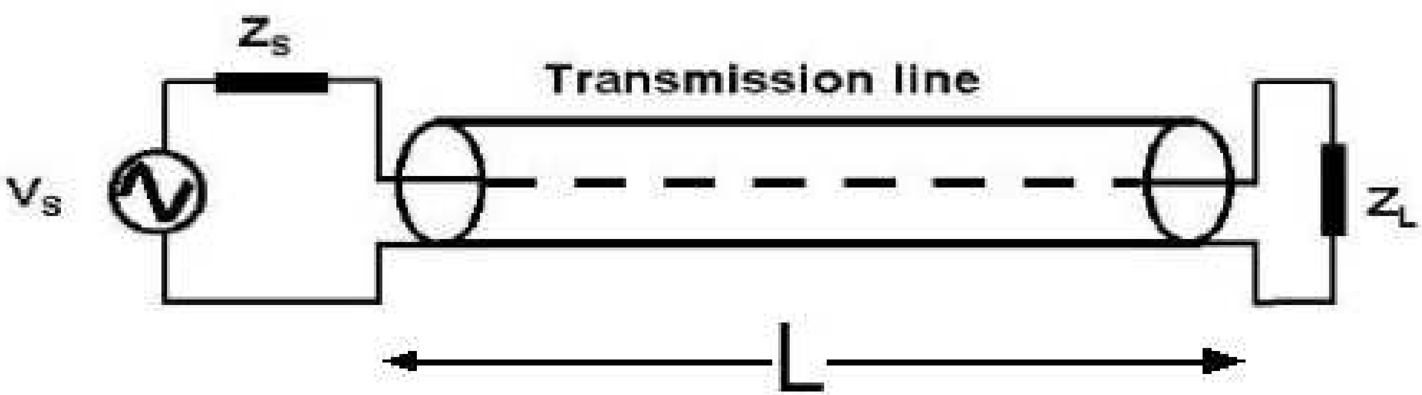
- ρ is the reflection coefficient.
- If $Z_L = Z_o$ there is **no** reflected wave.
- A line terminated in a pure reactance always has $|\rho| = 1$

Impedance Transformation Along a Line

- Consider a transmission line terminated in an arbitrary impedance Z_L.
- The impedance Z_{in} seen at the input to the line is given by

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan kL}{Z_o + jZ_L \tan kL}$$

- If $Z_L = Z_o$, then $Z_{in} = Z_o$.
- If $Z_L = 0$, then $Z_{in} = jZ_o \tan kL$
- If $Z_L = \infty$, then $Z_{in} = Z_o/(j \tan kL)$



Voltage and Current Transformation Along a Line

- Consider a transmission line terminated in an arbitrary impedance Z_L.
- The voltage V_{in} and current I_{in} at the input to the line are given by

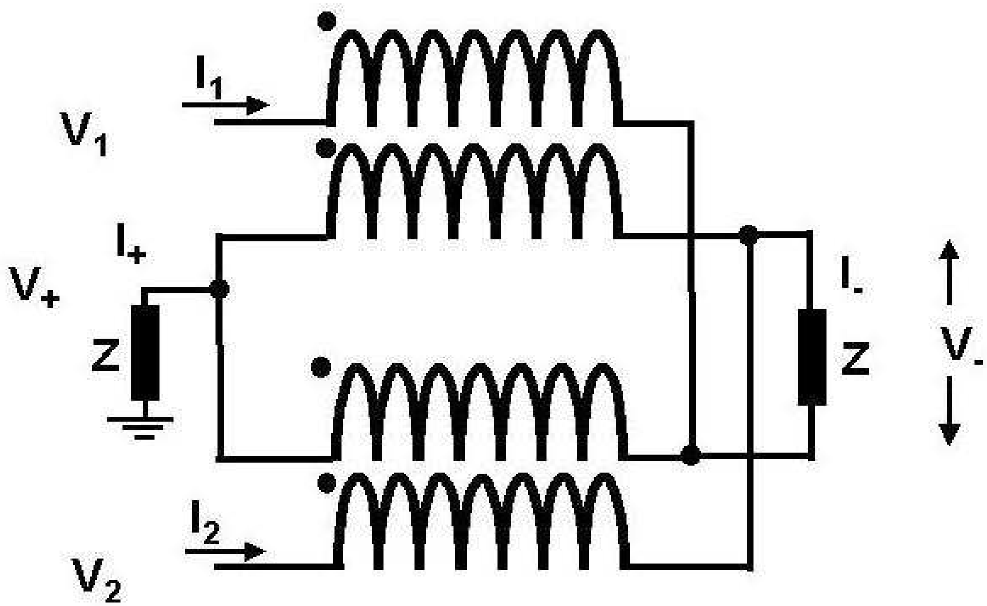
$$V_{in} = V_{end} \cos kL + jZ_o I_{end} \sin kL$$

$$I_{in} = I_{end} \cos kL + j\frac{V_{end}}{Z_o} \sin kL$$

- If a line is **unterminated** then the voltage and current vary along line.

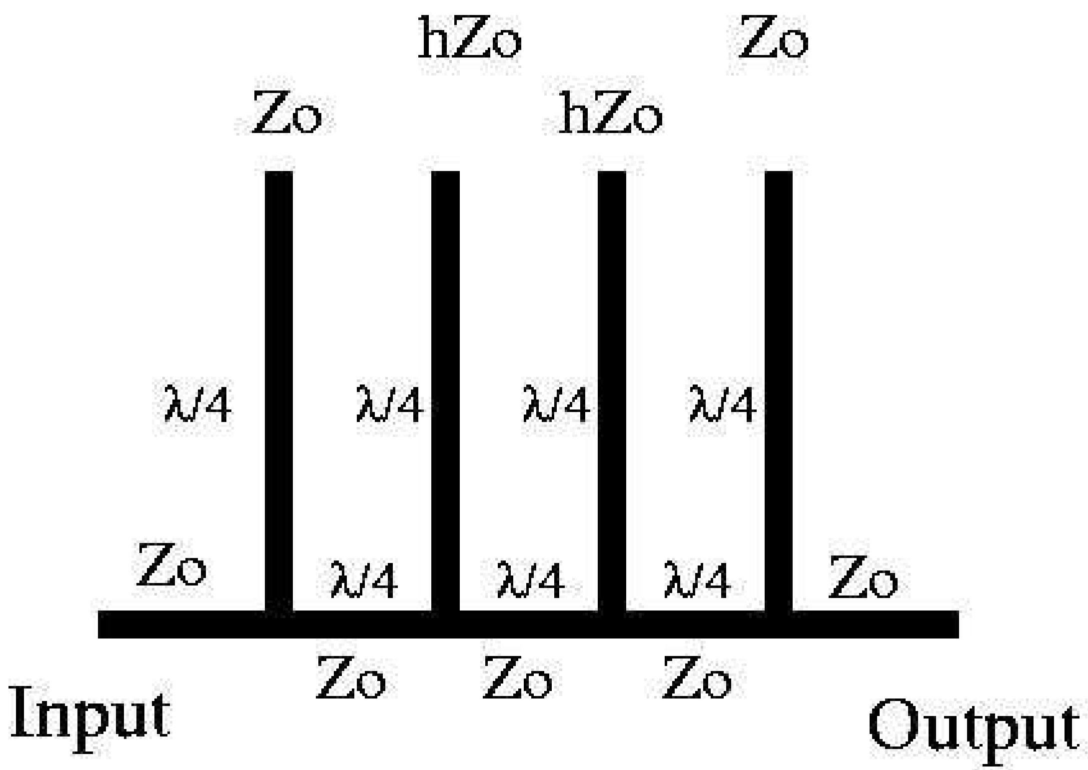
Applications of Transmission Lines

- Hybrids and baluns



Applications of Transmission Lines

- Filters



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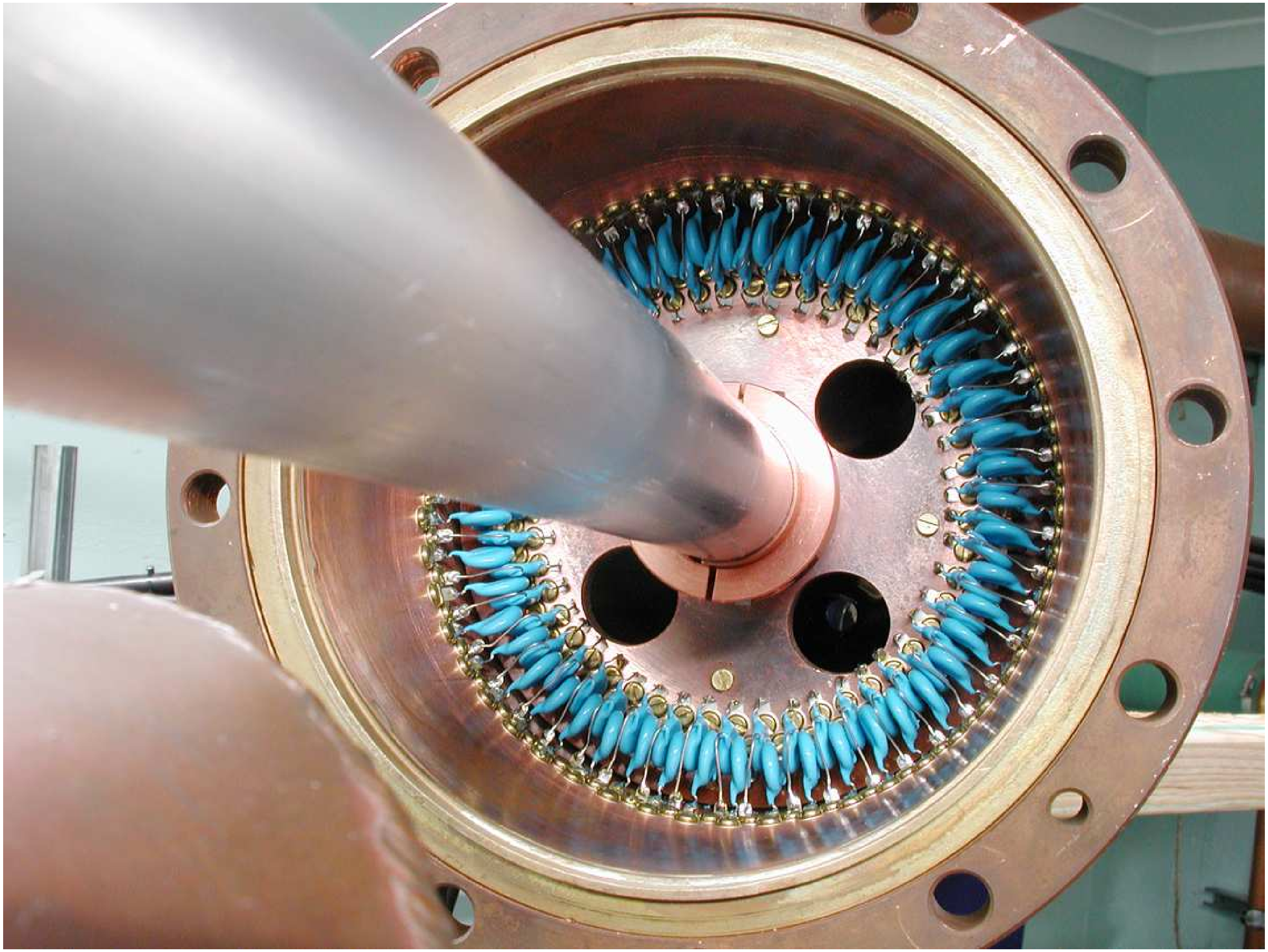

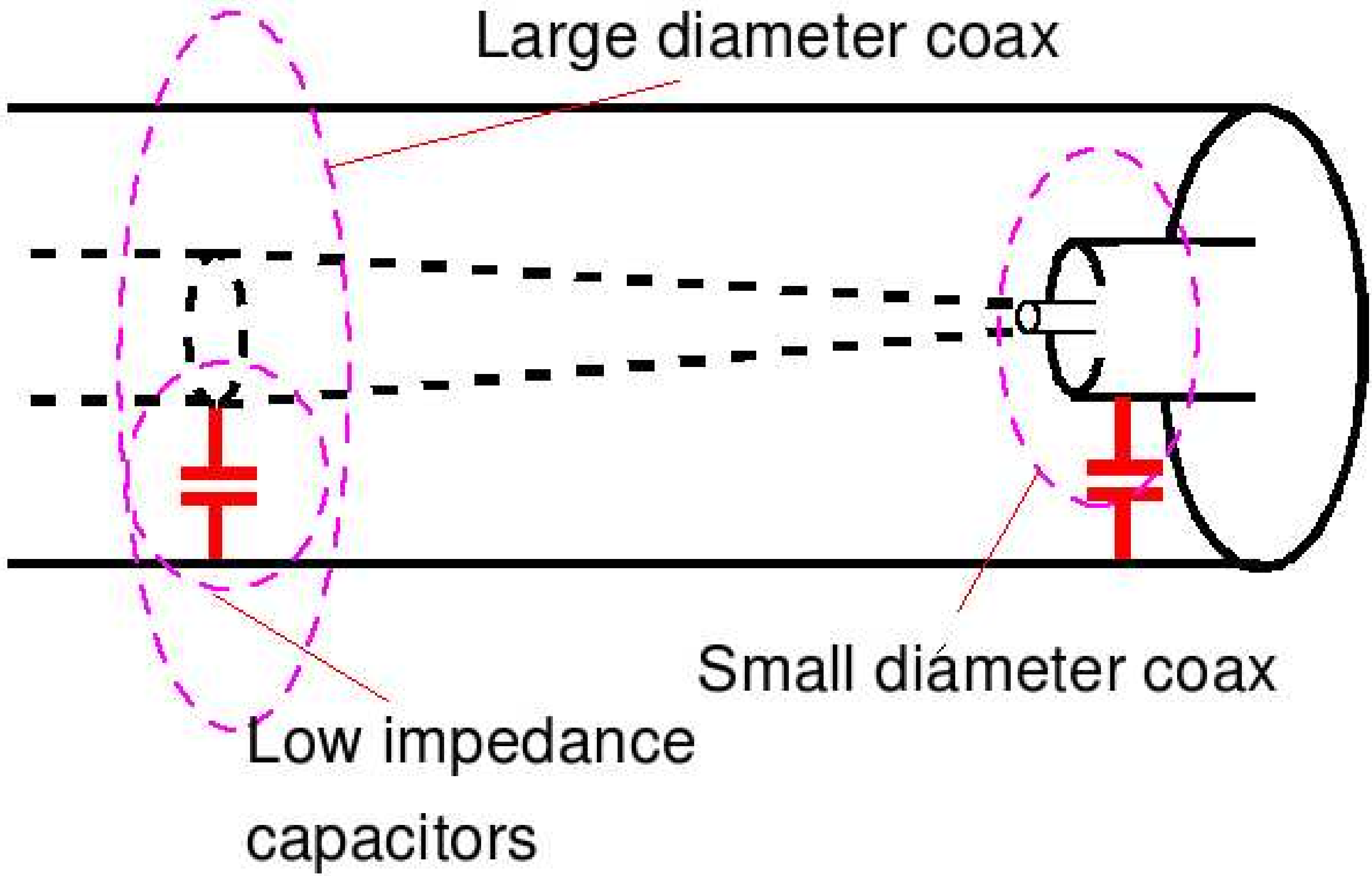
Large diameter coax
Small diameter coax
Low impedance
capacitors