ELECTROMAGNETISM SUMMARY

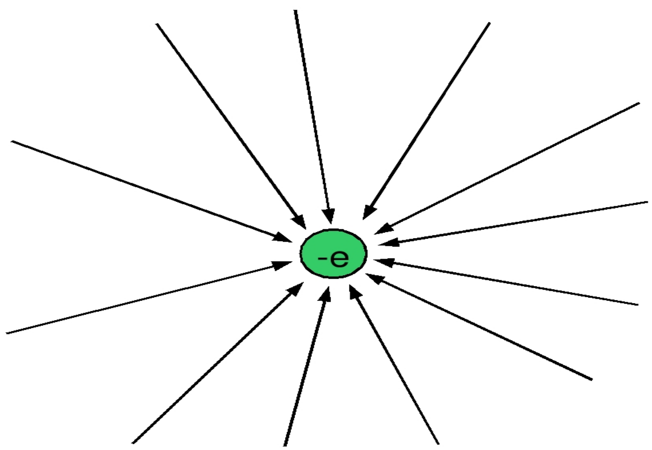
- Electrostatics
- Charge conservation
- Dielectrics
- Electromagnetic Shielding

Electrostatics: The Case of Stationary Charge

- The source of all electromagnetic fields is ultimately the charge.
- When there are no time variations, charge is the source of electric field.
- For a point charge we have Coulomb's law:

$$E_r = \frac{q}{4\pi\epsilon_0 r^2}$$

where the free space permittivity, $\epsilon_0 = 8.85 \times 10^{-12}$ Farads/m and q is in *Coulombs*.

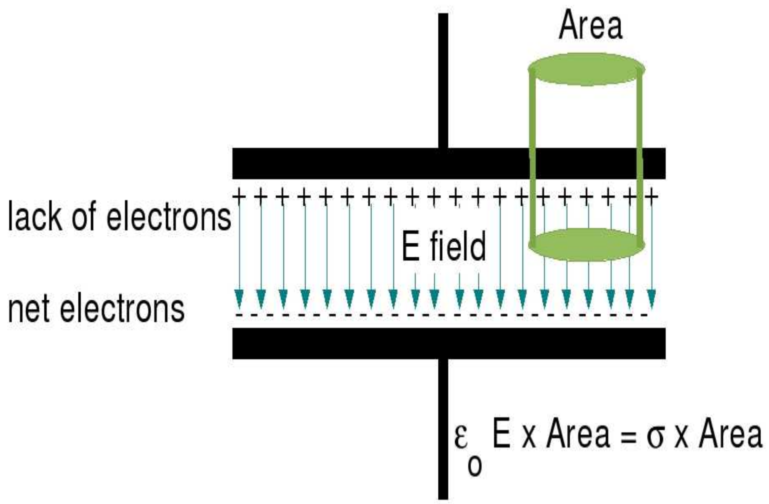


Electrostatics: The static electric field

- In general we have the following closed integral form (Gauss's law):

$$\oint_A \mathbf{E}.\mathbf{dA} = \frac{q}{\epsilon_0}$$

- Example: a charged capacitor. $E = \sigma/\epsilon_o$...



Electrostatics: The Electrostatic Potential

- Definition: The potential difference between two points $\mathbf{x_1}$ and $\mathbf{x_2}$ is given by,

$$\Phi = -\int_{\mathbf{x_1}}^{\mathbf{x_2}} \mathbf{E.dl} = -\int_{\gamma} \mathbf{E.dl}$$

- Since the path γ can be any which connects the points x_1 and x_2 we may conclude that $\mathbf{E} = -\nabla\Phi$.
- Kirchhoffs voltage law.

Charge Conservation

- When current flows out of a region in space, it depletes the charge in that region. The current per unit area j is given by,

$$I = \oint_A \mathbf{j}.\mathbf{dA} = -\frac{\partial q}{\partial t}$$

where the current I is, $I = \int_A \mathbf{j}.\mathbf{dA}$ for any surface (not necessarily closed).

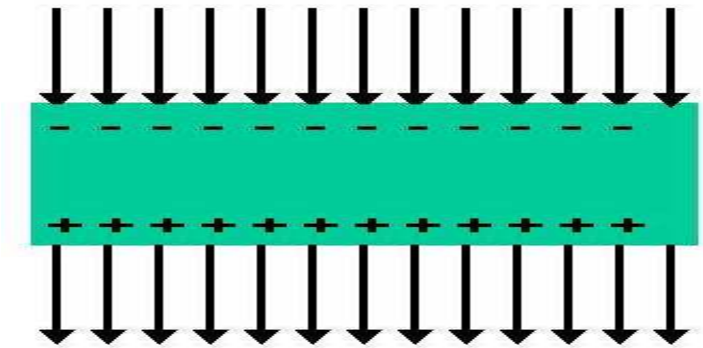
- If q is in **Coulombs** and the time in seconds then I is in **Amperes**.
- A general law... not just electrostatics.
- If current flows round in a closed loop, then there need be no change in the charge: Kirchhoffs current law.
- Quite generally: $\mathbf{j} = nq\mathbf{v}$ where n is the charge carrier density, q their charge and $\mathbf{v}$ their velocity.

Dielectrics and Conductors

- Dielectrics are insulating materials that do not allow D.C. current to flow through them. Usually we just call them insulators.
- In conductors, charge carriers (electrons) are free to move. E.G. metals.
- We study briefly the phenomenology of dielectrics and conductors.

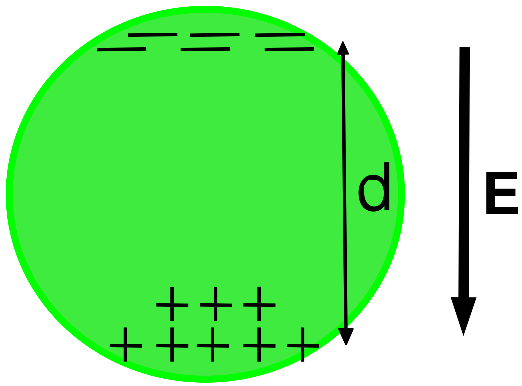
Dielectrics 1

- Dielectrics are insulating materials that do not allow D.C. current to flow through them.
- Electrons and nuclei in the atoms and molecules of dielectrics experience opposing forces in the presence of an imposed electric field.
- Electrons move opposite to the field and nuclei move in the direction of the field. This separation of charge produces a **polarisation**.
- The charge separation induced by the field, acts to **reduce** the electric field within the dielectric.



Dielectrics 2: Polarisation

- Polarisation $\mathbf{P}$ is the dipole moment per unit volume induced by an **imposed, external electric field**
- $\mathbf{P} = \mathbf{nqd}$, where n is the number density of dipoles, q is the charge at each end of the dipole and d is the displacement of the $\pm q$ charges at each end of the dipole.
- Polarisation is a vector quantity.
- Pictorial representation of a dipole moment:



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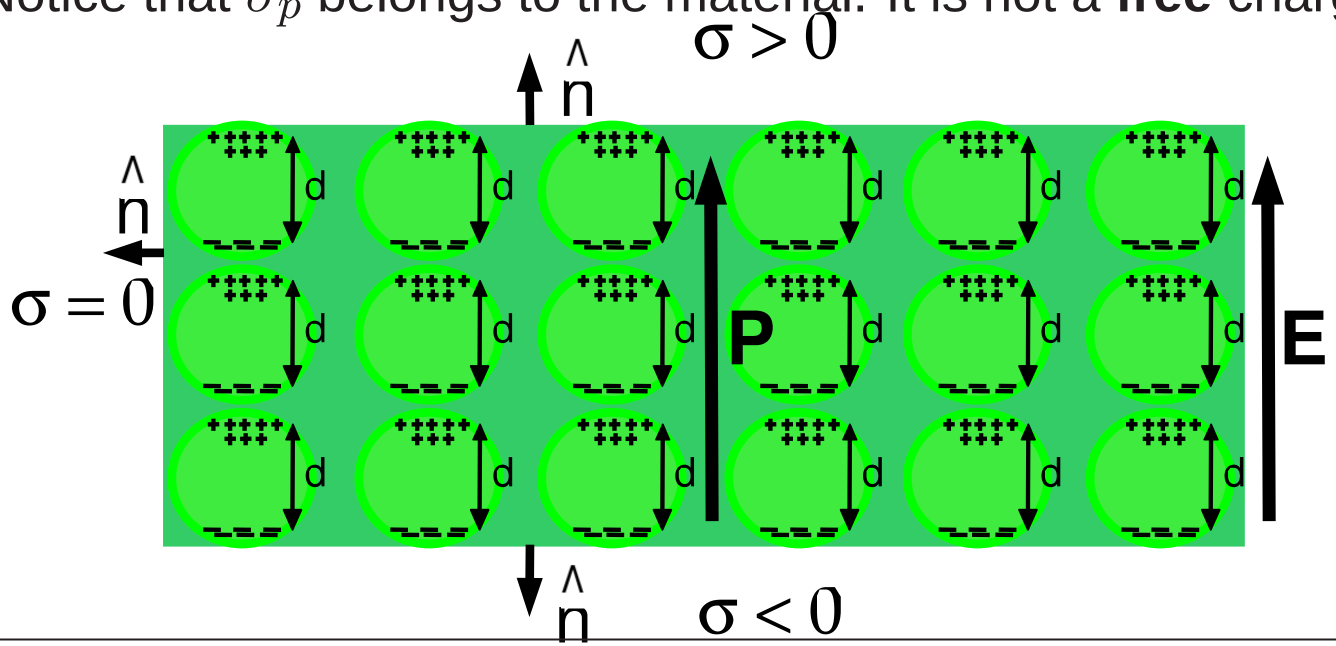
Dielectrics 3: Polarisation

- At the edge of a polarised dielectric there is a charge density left over by the displacement of the dipoles.
- The surface charge density on the dielectric $\sigma_p = \mathbf{P}.\hat{\mathbf{n}}$ where $\hat{\mathbf{n}}$ is unit vector normal to the surface.
- Notice that σ_p belongs to the material. It is not a **free** charge.



Dielectrics 4: Relative dielectric constant

- The relative dielectric constant ϵ_r is defined by,

$$\mathbf{P} = \epsilon_0(\epsilon_r - 1)\mathbf{E}$$

- The Electric Displacement $\mathbf{D}$ is defined by,

$$\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P} = \epsilon_r\epsilon_0\mathbf{E}$$

- Main advantage of the definition of D is that its source is the free charge only and not the induced polarisation charge,

$$\oint_A \mathbf{D}.\mathbf{dA} = q_{free}$$

- c.f.

$$\oint_A \mathbf{E}.\mathbf{dA} = \frac{q_{total}}{\epsilon_0}$$

Dielectrics 5: Key Points

- The relative dielectric constant ϵ_r describes the behaviour of a dielectric when exposed to an oscillating electric field.
- ϵ_r at D.C. is a positive dimensionless number and $\epsilon_r > 1$ for dielectrics.
- When the electric field oscillates, ϵ_r is a complex function of frequency.
- The ratio of the imaginary to real components of ϵ_r is termed the **loss tangent** of the dielectric:

$$tan\delta = \frac{Im(\epsilon_r)}{Re(\epsilon_r)}$$

Conductors and Ohm's law

- Ohm's law: The current density in a conductor is proportional to the electric field within the conductor.

$$\mathbf{j} = \sigma \mathbf{E}$$

where σ is the conductivity.

- **Conductors are completely specified by** σ.
- For copper, $\sigma = 5.80 \times 10^7 mhos/meter.$.
- Ohm's law is assumed to be an accurate result for metals at all radiofrequencies :).
- I.E. σ is always a real number and independent of frequency.

Conductors vs Dielectrics?

- For metals: $\mathbf{j} = \sigma\mathbf{E}$
- For dielectrics: $\mathbf{P} = \epsilon_0(\epsilon_r - 1)\mathbf{E}$
- If $\mathbf{P}$ oscillates as a function of time then,

$$j\omega\mathbf{P} = j\omega\epsilon_0(\epsilon_r - 1)\mathbf{E}$$

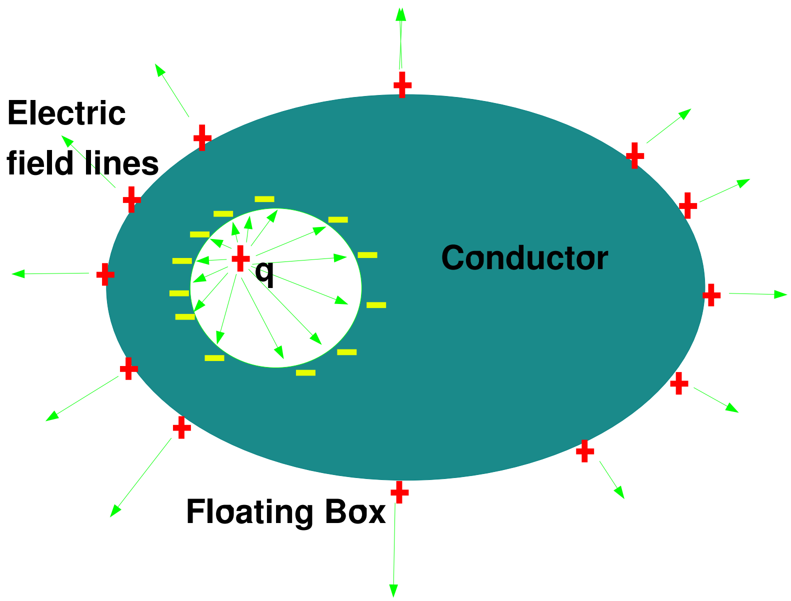
- Since the dipoles are reversing sign at rate ω while traversing a distance d we may write, $\mathbf{j_P} = \mathbf{j}\omega\mathbf{P}$, where $\mathbf{j_P}$ is the polarisation current.
- Thus dielectrics obey a sort of Ohm's law with $\mathbf{j_P} = \sigma_\mathbf{P}\mathbf{E}$ and $\sigma_P = j\omega\epsilon_0(\epsilon_r - 1)$
- The main difference between conductors and insulators is simply that σ is **resistive** for a conductor, but is mainly **reactive** for insulators.
- In fact, generally speaking, ϵ_r is a complex function of frequency.

Electromagnetic Shielding 1

- Ohm’s law for metals and Gauss’s law for the electric field give rise to the concept of **electromagnetic shielding**

Electromagnetic Shielding 2: The Floating Conducting Box

Suppose that there is an isolated positive charge, q, in a cavity and that the conducting box is **floating** (not connected to earth) and **neutral** (has no net charge).



Electromagnetic Shielding 3: The Floating Conducting Box

We can deduce the following...

- Electric current is finite (actually zero here) and Ohm's law implies that the electric field inside the conductor (Turquoise region) is **zero**.
- Gauss's law implies that the net negative charge on the inside of the inner wall of the cavity is equal and opposite in sign to q.
- Charge conservation implies that the amount of positive charge on the outer surface equals the negative charge on the inner surface.

Electromagnetic Shielding 4: The Floating Conducting Box

Observations:

(a) The negative charge arranges itself to give perfect cancellation of the electric field inside the conductor.

(b) The negative surface charge density (σ) is highest on the wall closest to q.

(c) The positive charges on the outer surface see no electric field inside the conductor and therefore do not respond to movement in either q or the negative charge on the cavity wall.

(d) The positive charges on the outside bunch toward surfaces of high curvature and spread out along surfaces of low curvature.

Electromagnetic Shielding 5: The Floating Conducting Box

(e) The charge on the surface of the metal-air interface lies right on the surface and not inside the conductor. (RF burns?)

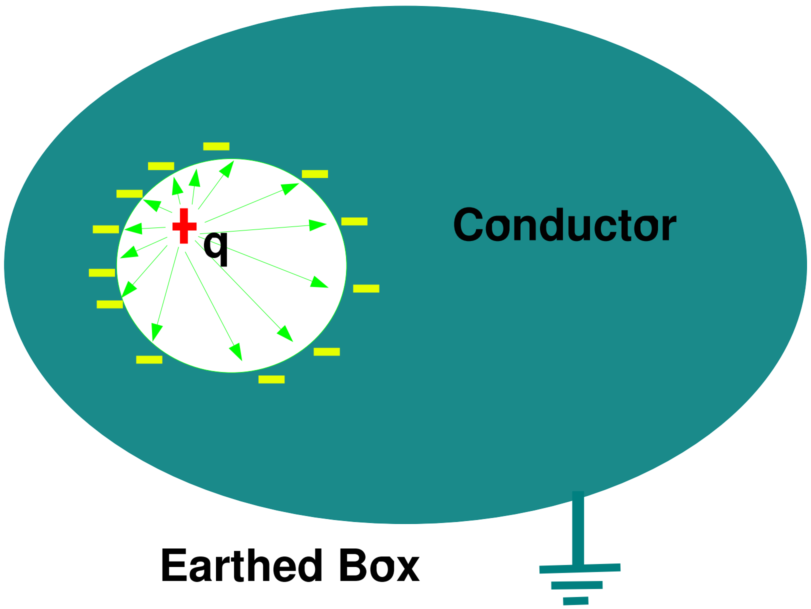
(f) By Gauus's law, here is still an electric field outside the conductor.

(g) If there were no net charge inside the cavity ($q = 0$) then there would be **no electric field outside the conducting box either**. For example there could even be a $+q$ and a $-q$ arbitrarily located. It makes no difference.

(h) At the surface of the metal, the electric field vector is always normal. There is **never** any tangential component of electric field on the surface of a conductor.

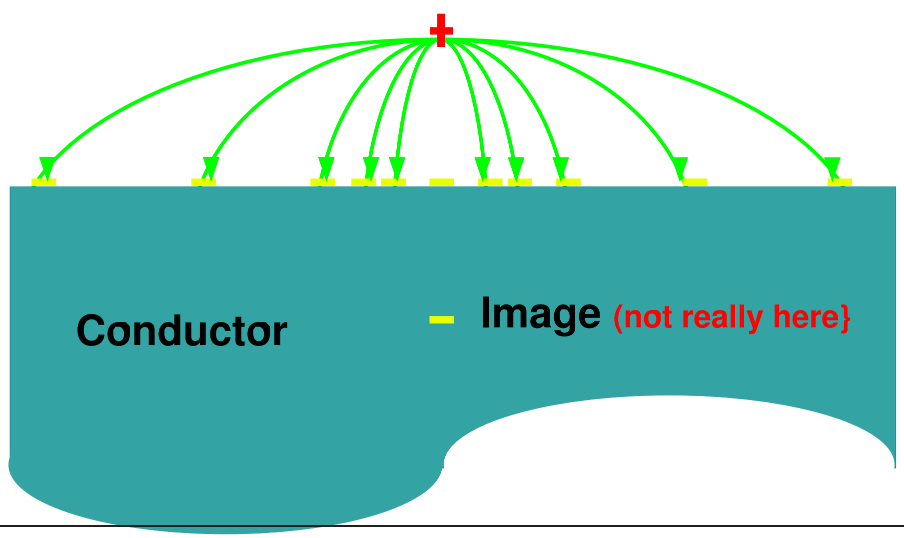
Electromagnetic Shielding 6: The Earthed Conducting Box

- Largely the same observations as for the floating case, except that now there is **no field outside the conductor** under any condition.
- Main practical conclusion: **Closed conducting boxes isolate the outside world from electromagnetic influences within the box and vice versa.**



Electromagnetic Shielding 7: Proximity to a Conductor

- A charge near a conducting surface attracts charge of the opposite sign to the nearest point on the surface.
- These surface charges arrange themselves so that the **tangntial component** of electric field is zero on the surface.
- One may compute the electric field outside the conductor by assuming (mathematically) that there is an **image** charge within the conductor.



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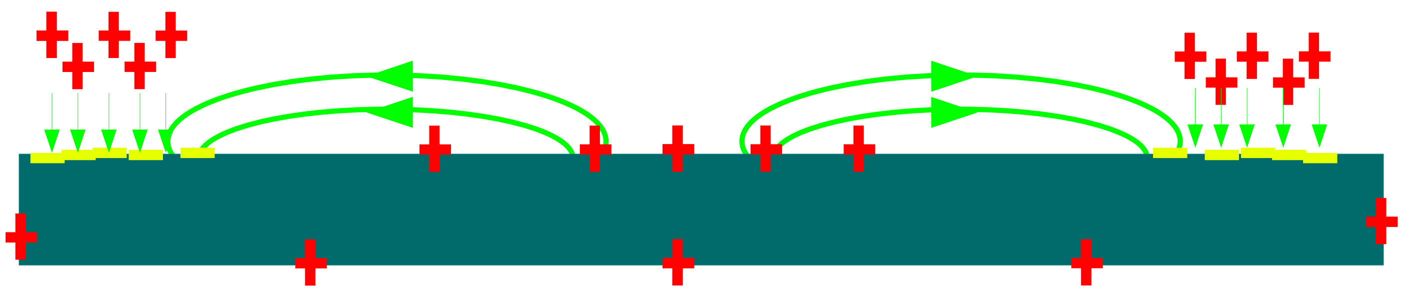
Electromagnetic Shielding 8: Proximity to Floating Conductors

- Two charges (electronic components) near a floating conductor must **share** the charge within the conductor.
- Electromagnetic fields at opposite ends of the plane cause charge separations that **increase** the influence of each component on the other.
- If the electronic components are themselves neutral then there is not such a large influence of the plane.



Floating Conductor

Electromagnetic Shielding 9: Proximity to Earthed Conductors

- Earth is an inexhaustible source of charge.
- Two charges (electronic components) near an earthed conductor attract charge from earth.
- Electromagnetic fields at opposite ends of the plane cause charge separations that **decrease** the influence of each component on the other.