Transmission Lines

- Transformation of voltage, current and impedance
- Impedance
- Application of transmission lines

The Telegraphist Equations

- We can rewrite the above equations as (**Telegraphist Equations**)

$$\frac{\partial V}{\partial z} = \left(\frac{i\omega Z_o}{v}\right) I$$

$$\frac{\partial I}{\partial z} = \left(\frac{i\omega}{Z_o v}\right) V$$

- See the equivalent web brick derivation in terms of the inductance and capacitance per unit length along the line.
- The Telegraphist Equations become

$$\frac{\partial V}{\partial z} = i\omega L I, \quad L = \frac{Z_o}{v} \quad = \frac{\mu_o d}{w}$$

$$\frac{\partial I}{\partial z} = i\omega C V, \quad C = \frac{1}{Z_o v} \quad = \frac{\epsilon_o \epsilon_r w}{d}$$

The Telegraphist Equations

- The velocity and characteristic impedance of the line can be expressed in terms of L and C.

$$v = \sqrt{\frac{1}{LC}} \qquad Z_o = \sqrt{\frac{L}{C}}$$

- L and C are the inductance and capacitance per unit length along the line.
- For coaxial cable the formula is quite different (a,b inner, outer radii).

$$C = \frac{2\pi\epsilon_o\epsilon_r}{ln(b/a)} \qquad L = \frac{2\pi ln(b/a)}{\mu_o}$$

$$Z_o = \sqrt{\frac{\mu_o}{\epsilon_o\epsilon_r}}\frac{ln(b/a)}{2\pi} \qquad v = \sqrt{\frac{1}{\mu_o\epsilon_o\epsilon_r}}$$

Proof of the Coaxial Cable Relations

- L and C are the inductance and capacitance per unit length along the line.
- For coaxial cable C and L are given by,

$$C = \frac{2\pi\epsilon_o\epsilon_r}{ln(b/a)} \qquad L = \frac{2\pi ln(b/a)}{\mu_o}$$

$$Z_o = \sqrt{\frac{\mu_o}{\epsilon_o\epsilon_r}}\frac{ln(b/a)}{2\pi} \qquad v = \sqrt{\frac{1}{\mu_o\epsilon_o\epsilon_r}}$$

- On board.

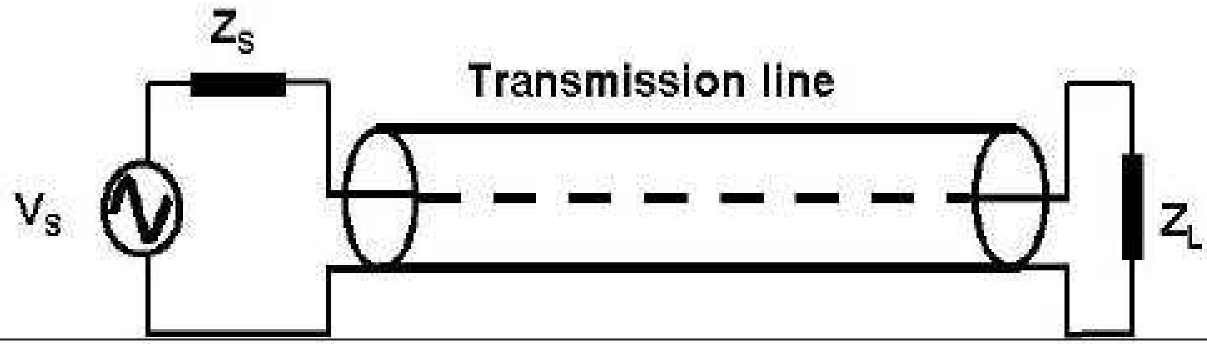
Reflection Coefficient

- Consider a wave propagating toward a load
- In general there is a wave reflected at the load. The total voltage and current at the load are given by,

$$V_{load} = V_f + V_r \qquad I_{load} = I_f + I_r$$

where

$$V_f = Z_o I_f \qquad V_r = -Z_o I_r$$



Reflection Coefficient

- At the load,

$$V_{load} = Z_L I_{load} = Z_L \left(I_f + I_r\right) = V_f + V_r = \frac{Z_L}{Z_o}\left(V_f - V_r\right)$$

- Solving for $\rho = V_r/V_f$, we obtain,

$$\rho = \frac{Z_L - Z_o}{Z_L + Z_o}$$

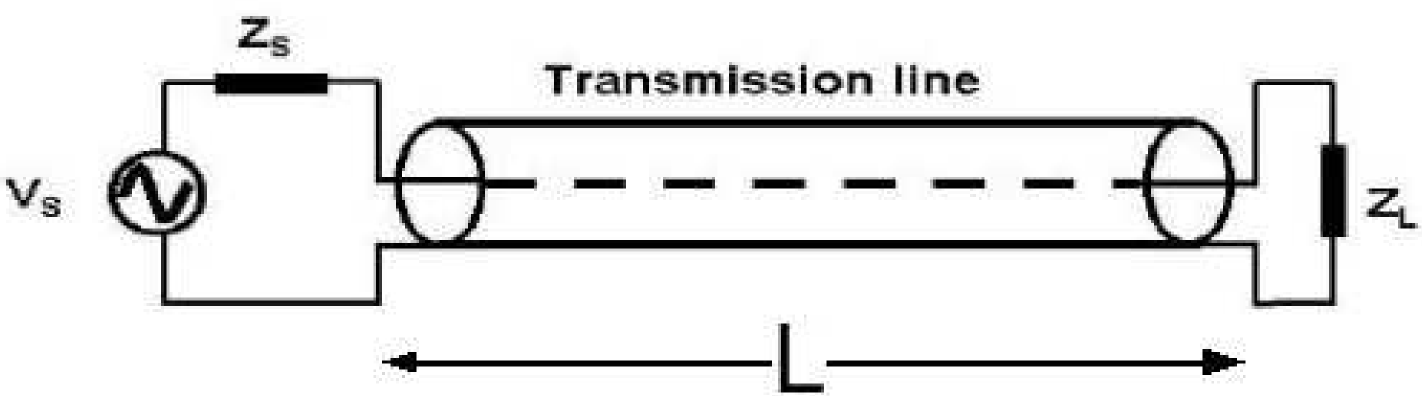
- ρ is the reflection coefficient.
- If $Z_L = Z_o$ there is **no** reflected wave.
- A line terminated in a pure reactance always has $|\rho| = 1$

Impedance Transformation Along a Line

- Consider a transmission line terminated in an arbitrary impedance Z_L.
- The impedance Z_{in} seen at the input to the line is given by

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan kL}{Z_o + jZ_L \tan kL}$$

- If $Z_L = Z_o$, then $Z_{in} = Z_o$.
- If $Z_L = 0$, then $Z_{in} = jZ_o \tan kL$
- If $Z_L = \infty$, then $Z_{in} = Z_o/(j \tan kL)$



Voltage and Current Transformation Along a Line

- Consider a transmission line terminated in an arbitrary impedance Z_L.
- The voltage V_{in} and current I_{in} at the input to the line are given by

$$V_{in} = V_{end} \cos kL + jZ_o I_{end} \sin kL$$

$$I_{in} = I_{end} \cos kL + j\frac{V_{end}}{Z_o} \sin kL$$

- If a line is **unterminated** then the voltage and current vary along line.

Proof of Voltage and Current Transformation

- Consider a transmission line terminated in an arbitrary impedance Z_L.
- The voltage and current waves propagating on the line are,

$$V(z,t) = \exp j(\omega t - kz) + \rho \exp j(\omega t + kz)$$

$$I(z,t) = \frac{1}{Z_o} \exp j(\omega t - kz) - \frac{\rho}{Z_o} \exp j(\omega t + kz)$$

Proof of Voltage and Current Transformation

- At $z = 0$,

$$V(0,t) = \exp j(\omega t) + \rho \exp j(\omega t)$$

$$I(0,t) = \frac{1}{Z_o} \exp j(\omega t) - \frac{\rho}{Z_o} \exp j(\omega t)$$

- At $z = +L$,

$$V(L,t) = \exp j(\omega t - kL) + \rho \exp j(\omega t + kL)$$

$$I(L,t) = \frac{1}{Z_o} \exp j(\omega t - kL) - \frac{\rho}{Z_o} \exp j(\omega t + kL)$$

Proof of Voltage and Current Transformation

- Compute $V(0,t)$ in terms of $V(L,t)$ and $I(L,t)$ ...

$$V(0,t) \;=\; \exp j(\omega t) \;+\; \rho \exp j(\omega t)$$

$$V(L,t) \;=\; \exp j(\omega t - kL) \;+\; \rho \exp j(\omega t + kL)$$

$$jZ_o I(L,t) \;=\; j \exp j(\omega t - kL) \;-\; j\rho \exp j(\omega t + kL)$$

Proof of Voltage and Current Transformation

- Multiply the equations for $V(L,t)$ and $I(L,t)$ by $\cos kL$ and $\sin kL$,

$$V(0,t) = \exp j(\omega t) + \rho \exp j(\omega t)$$

$$V(L,t)\cos kL = \exp j(\omega t - kL)\cos kL + \rho \exp j(\omega t + kL)\cos kL$$
$$jZ_oI(L,t)\sin kL = j\exp j(\omega t - kL)\sin kL - j\rho \exp j(\omega t + kL)\sin kL$$

$$V(L,t)\cos kL + jZ_oI(L,t)\sin kL =$$
$$\exp j(\omega t - kL)\,(\cos kL + j\sin kL) + \rho \exp j(\omega t + kL)\,(\cos kL - j\sin kL)$$

$$V(L,t)\cos kL + jZ_oI(L,t)\sin kL = V(0,t)$$

Impedance Transformation Along a Line

- The voltage V_{in} and current I_{in} at the input to the line are given by

$$V(0,t) = V(L,t)\cos kL + jZ_o I(L,t)\sin kL$$
$$I(0,t) = I(L,t)\cos kL + j\frac{V(L,t)}{Z_o}\sin kL$$

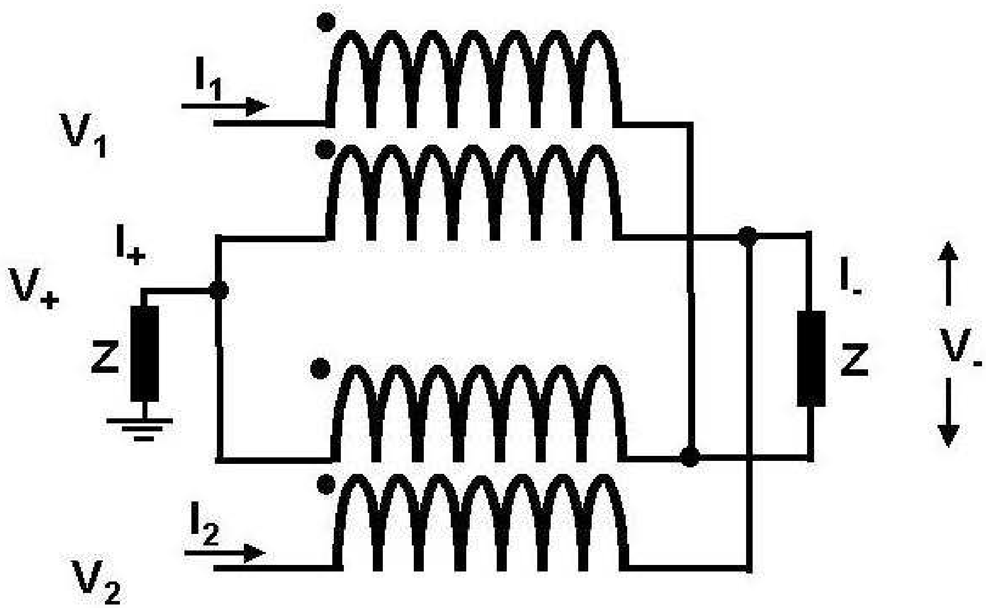
- Divide these

$$\frac{V(0,t)}{I(0,t)} = \frac{V(L,t)\cos kL + jZ_o I(L,t)\sin kL}{I(L,t)\cos kL + j\frac{V(L,t)}{Z_o}\sin kL}$$

$$Z_{in} = \frac{Z_L\cos kL + jZ_o\sin kL}{\cos kL + j\frac{Z_L}{Z_o}\sin kL} = Z_o\frac{Z_L + jZ_o\tan kL}{Z_o + jZ_L\tan kL}$$

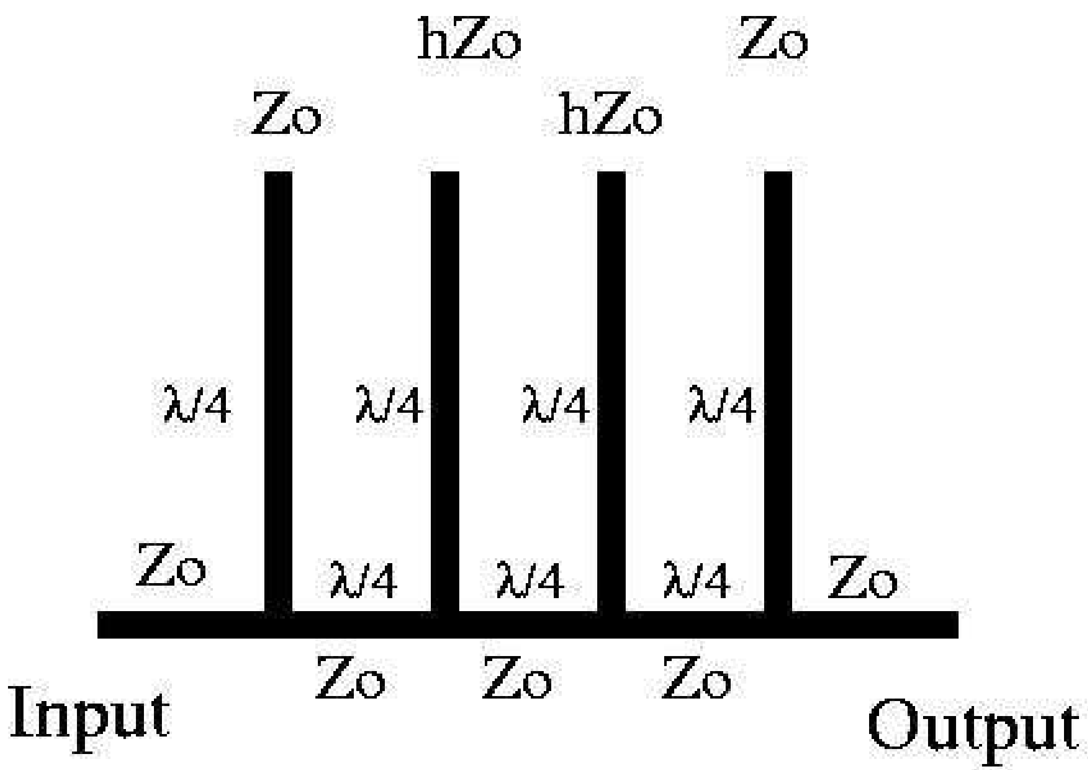
Applications of Transmission Lines

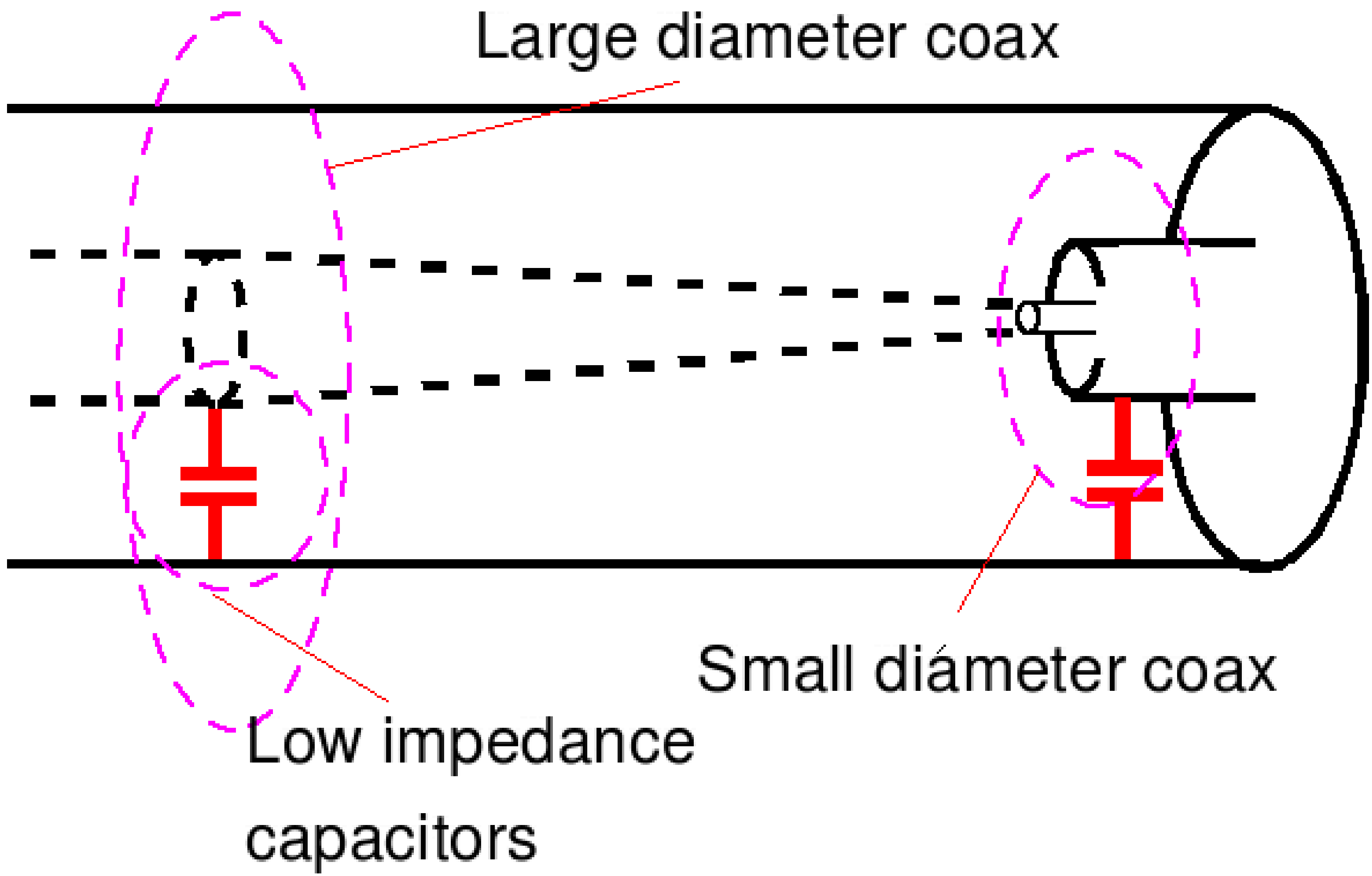
- Hybrids and baluns

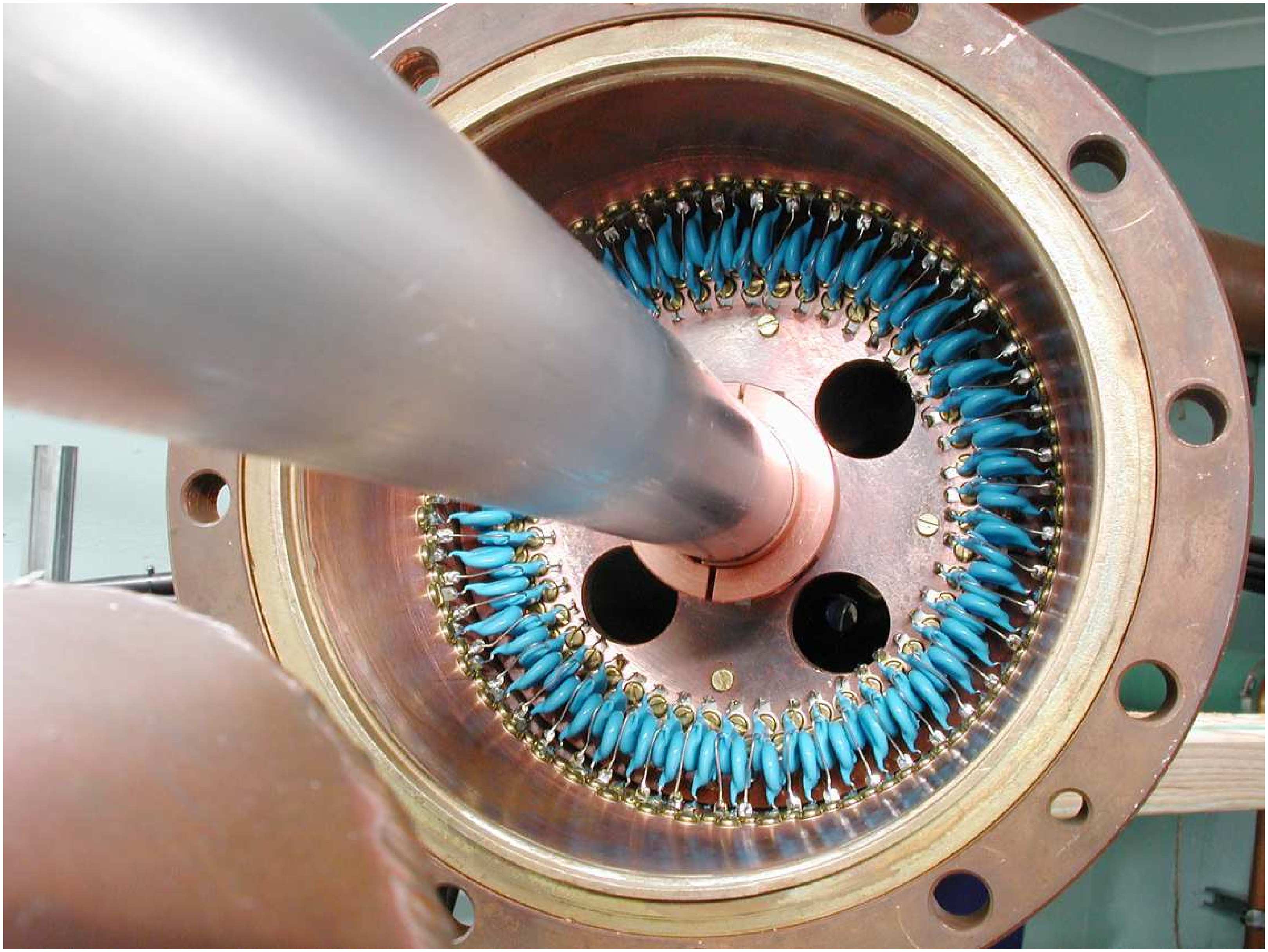


Applications of Transmission Lines

- Filters




Large diameter coax
Small diameter coax
Low impedance
capacitors

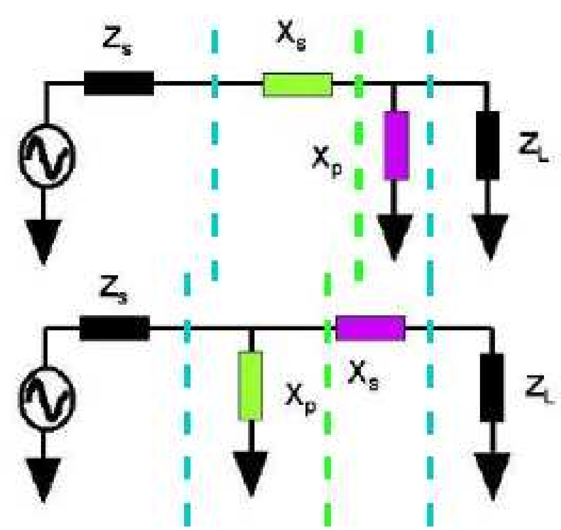


Matching Networks

- Use a matching network to match a source to a load for maximum transferred power.
- $Z_L = Z_S^*$
- Two different types we consider: L-networks and Pi/T networks
- Consist entirely of Ls and Cs.
- How to deal with reactive source and load impedances? Either treat by **absorption** or **resonance**
- Dont forget that if there is a transmission line in between the source and the load network then there are two matching networks: one to match the source impedance to Z_o and one to match Z_o to the load impedance.

L-Networks

- Consist of two matching elements.
- Choose shunt arrangement at Z_L (resp. Z_S) if $R_S < R_L$ (resp. $R_L < R_S$). Use series arrangement on the other side.
- Try to absorb source and load reactances into the matching impedance reactances.
- Since the impedances seen in either direction through the green line must be complex conjugates of each other, t[...] the circuits on either side of the green line.



L-Networks

- The relationships between the r_S, x_S and r_L, x_L are given by

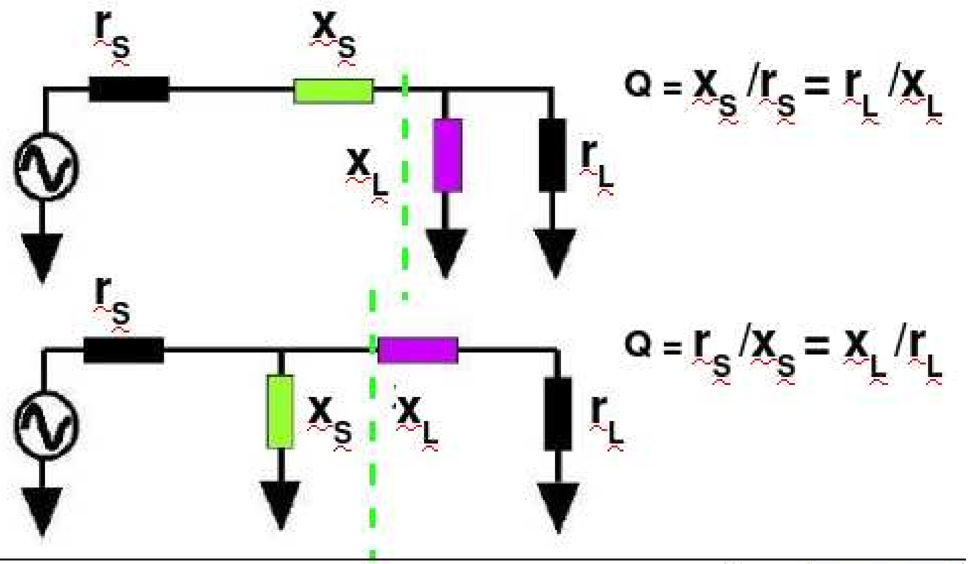
$$\frac{r_L}{r_S} = 1 + Q^2, \quad \frac{x_L}{x_S} = \frac{1 + Q^2}{Q^2}. \quad \mathbf{R_L \text{ shunt. } R_S \text{ series.}}$$

$$\frac{r_S}{r_L} = 1 + Q^2, \quad \frac{x_S}{x_L} = \frac{1 + Q^2}{Q^2}. \quad \mathbf{R_S \text{ shunt. } R_L \text{ series.}}$$

- Q obtained from,

$$Q = \sqrt{\frac{r_L}{r_S} - 1}.$$

$$Q = \sqrt{\frac{r_S}{r_L} - 1}.$$



L-Networks: Summary

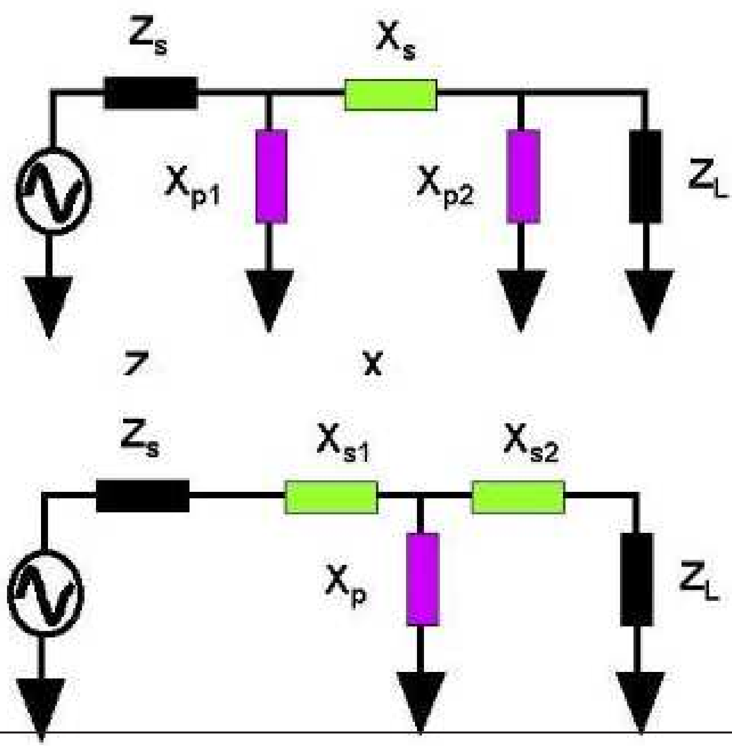
- Place the shunt of the L-network across the highest resistance and the series of the L-network in series with the lower resistance.
- Compute the Q required to match the source and load resistances.
- Use the Q to find x_S and x_L from r_S and r_L.
- Remember to place inductors in series with capacitors and vice versa in order to allow for complex conjugates.
- **Absorb** or **resonate** the source and load stray reactances X_S and X_L of the matching network with x_S and x_L.
- Whether we absorb or resonate depends on how large the strays are.

L-Networks: Limitations

- The value of Q arises from the calculation.
- But what if we need to specify Q?
- **Solution:** T and Pi networks.

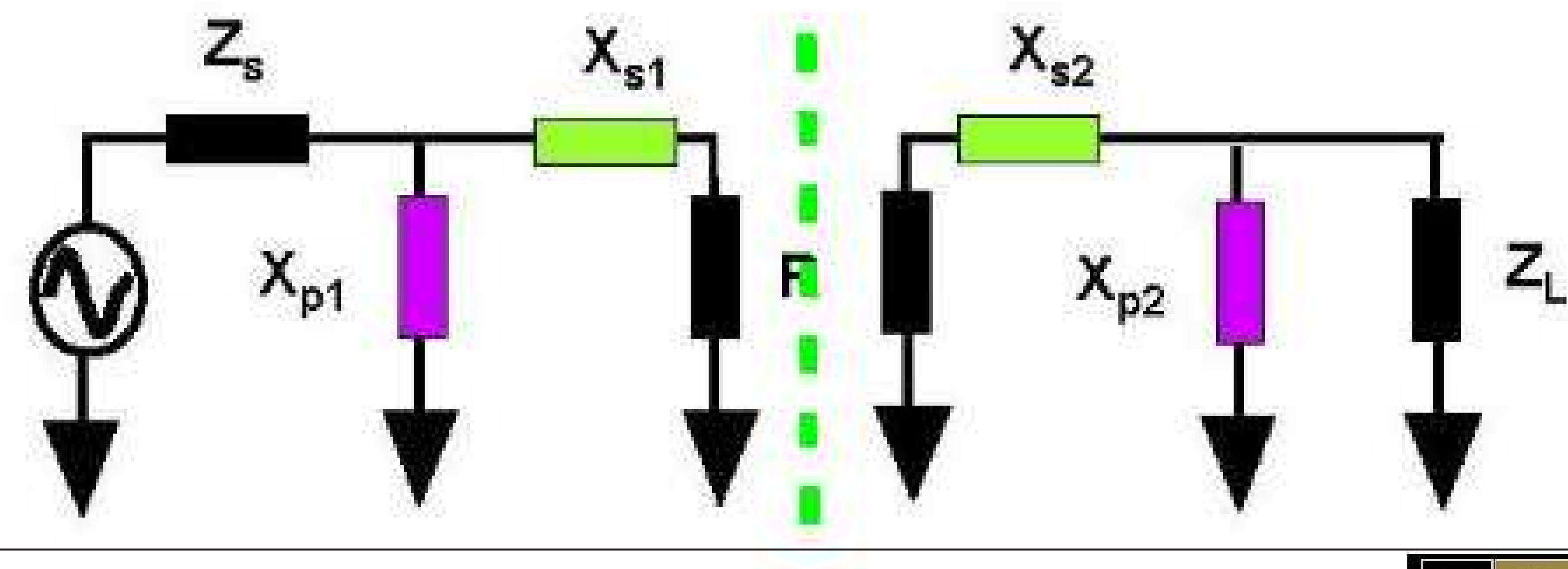
T and Pi: High Q Networks

- Can allow us to choose Q.
- Q however is always **higher** than for an L-network. Why?



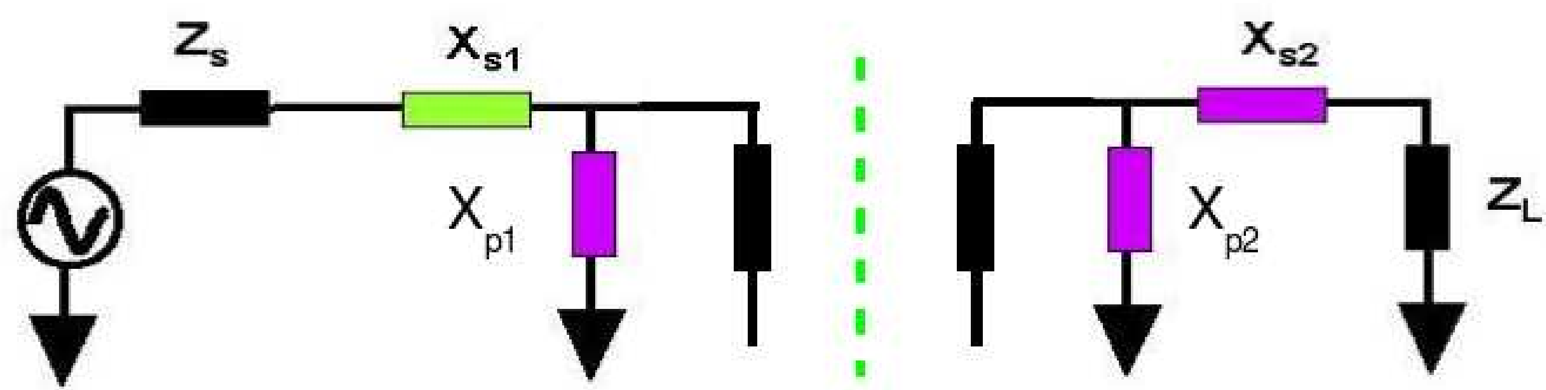
Analysis of T and Pi Networks

- Choose Q.
- Consider the T or Pi network to be a pair of back to back L networks.
- The virtual resistance in a Pi network must be smaller that those on the source and load.



T Networks

- The virtual resistance in a T network must be larger that those on the source and load.



Low Q Networks

- Q however is always **lower** than for an L-network.
- OK for broadband match.