\[\dot{x}_3(t) = 2(t) \quad \dot{x}_3(t) \in O(3) \text{ by (a). So now construct a continuous curve from } p \text{ to } q \text{ in } O(3):\]

\[\begin{align*}
y(t) &= (\dot{x}_1(t), t \in [0, t_1]) \\
\dot{x}_1(t) &= \dot{x}_2(t) + t - b - a \\
\dot{x}_2(t) &= \dot{x}_3(t) + b - a \\
\dot{x}_3(t) &= \dot{x}_2(t) + t - b - a
\end{align*}\]

7.2 \[\|\dot{x}(t)\| = \text{constant} \Rightarrow \dot{x}(t), \dot{x}(t) = 2\dot{x}(t), \dot{x}(t) = 0, \text{ i.e. } \dot{x}(t) \perp \dot{x}(t)\]

7.3 Let \( S(t) = \int_0^t \|\dot{x}(t)\|dt \). As \( \dot{x}(t) \rightarrow 0 \), so \( S(t) \) monotone increasing so \( S(t) \) is invertible. Let \( h = S^{-1} \). \( h \) is onto by definition \( h' = \frac{1}{S''(h)} > 0 \)

\[\dot{x} = \dot{x}(h(t)), h'(t) = \frac{\dot{x}(h(t))}{\|\dot{x}(h(t))\|} \text{ so } \beta \text{ is unit speed.}\]

7.4 "For if part is by Example 2 in this chapter only if \( \dot{x}(0) = (r \cos b, r \sin b, 0) \), which has covered all possible points on cylinder \( \dot{x}(0) = (-r \sin b, r \cos b, 0) \).

So \( \dot{x}(0) \) has covered all possible initial velocity in \( \mathcal{S}(0) \).

As geodesic is uniquely determined by initial position and initial velocity these are all possible geodesics on cylinder \( S \).

Another proof is by looking at (6) on page 41. \( N(x, y, z) = (x, y, 0)\)

7.5 "if part" is covered by Example 3 in this chapter only if \( \dot{x}(0) = e^1, \dot{x}(0) = ae^2 \). Since \( e_2 \in \mathcal{S}, e \) allows all norm of velocity \( 0 \), allows all possible initial position, \( \dot{x}(0) \) allows all possible initial velocity due to uniqueness of geodesic by initial position and velocity, these are all possible geodesics on unit n-sphere.

7.7 If part. \[\beta(t) = a \cdot \dot{x}(at+b)\]

(\(e^2 \dot{x}(e^1 h(t)) = \dot{x}(at+b) \cdot a\). \(\dot{x}(at+b)\) is geodesic so \(\dot{x}(t) \in \mathcal{S}(at+b) \text{ Hf. So } L(t) \in \mathcal{S}(at+b) = S^1 \text{ and } \beta \text{ is geodesic only if } \beta(t) = \dot{x}(h(t)) \cdot (h(t))^2 + \dot{x}(h(t)) \cdot h'(t) \text{ if } \beta \text{ is geodesic, } L(t) \in S^1 = S^1 \text{ and } \beta\) \(\dot{x}(h(t)) \text{ are parallel and } \dot{x}(h(t)) \text{ is geodesic. Generally, } \dot{x} \text{ and } \dot{x} \text{ are not parallel, and } h'(t), h'(t) \text{ are scalar} \dot{x}(h(t)) \text{ parallel so we must require } h'(t) = 0 \text{ (E.g. } \dot{x}(t) = -\dot{e}_2 \text{ and } x(t) = -e_2 \text{, } \theta_{e_2} = 0 \text{, } \theta_{e_2} = 0 \text{, } \dot{x} \text{ and } \dot{x} \text{ are never parallel).} \text{ So } h(t) = at + b \text{. We can't see why } a > 0 \text{. Since } x(t) \text{ when } a = 0 \text{, } \beta \text{ is still geodesic.} \)