(b) If \( S \) crosses \( S \) for an odd number of times \( t_1, \ldots, t_n \), then by (a)
\[
g(t_1)g(t_n) > 0.\]
With out loss of generality, suppose \( g(t_1) > 0, g(t_n) > 0 \).
Since \( g(t) = g(t_n) = 2 \times \text{constant} \), we have \( g(t) > 0 \) for all \( t \), \( t > t_n \).
However, as \( S \) is compact and \( g \) goes to \( 0 \) in both directions, we can find \( f: R^\infty R \)
there is a \( f \) such that if \( S \) is contained in sphere \( S: ||x||^2 = r^2 \), then pick any point \( P \)
and consider \( S' \) to \( 0 \) in both directions.

- There must be \( t_0, t_0+1 \) with \( t_0 < t_1 < t_0+1 < t_n \), such that \( g(t_0) \) and \( g(t_0+1) \) \( 0 \).

As \( f(x(t)) < 0 < f(x(t_0+1)) \) and \( f \) is continuous on \( S' \), so \( f(x(t_0)) < 0 < f(x(t_0+1)) \).
As \( S' \) is connected (see Ex. 5.1), there is a parametrized curve \( \beta(t) \in S' \).

\[
\beta(t)^1 = x(t), \beta(t)^2 = x(t_0+1), \text{ and } \beta \text{ is continuous on } S'.
\]

there must be a \( t^3 \in (t^1, t_0+1) \) st. \( f(\beta(t^3)) = 0 \).

But \( \beta(t^3) \in S \). This is contradiction!

6.10 (a) \( \beta(0) \in O(S) \), there exists a continuous map \( \alpha: [0, l] \to R^\infty S \)
\[
\text{s.t. } \alpha(0) = \beta(0), \lim_{l \to 0} \alpha(t) = \beta(0).
\]

For \( \forall \beta(t) \), construct curve \( \gamma(t) = \beta(t) - \beta(t)^0 \in O(S) \).

then \( \gamma(t) \) is continuous from \([0, l] \to R^\infty S \).

\( \gamma(0) = \beta(0), \gamma(l) = \beta(l) \) to is arbitrary so \( \beta(t) \in O(S) \) for all \( t \in [a, b] \).

(b) \( \beta(0) \in O(S) \). As \( S \) is a compact n-surface, then can find a n-sphere with a large enough radius \( \Gamma \) which strictly subsumes \( S \).

Then pick one point on the n-sphere \( p \), construct continuous map \( \alpha(t) = p \circ t \in R^\infty S \).

\[
\text{So } \alpha(t) \in R^\infty S, \lim_{l \to 0} \alpha(t) = \beta(0).
\]

So \( p \in O(S) \).

(2) open set \( V \) \( \in O(S) \), \( p \in R^\infty S \), as \( R^\infty S \) is open (due to
\( S = f^{-1}(c) \)) is n-surface and by definition \( f \) is smooth). So there exists an \( \epsilon \)-ball around \( p \), \( \epsilon \in (p, e) \), such that \( V \times e \) \( \in e \) \( R^\infty S \).

We can easily construct a continuous map \( \chi: R^\infty S \to S \).

(3) connected: \( V \) \( p \in O(S) \), Suppose there is a n-sphere \( S \) with radius \( \Gamma \)
such that \( p \in S \) are all contained in it. \( S \) compact, \( \Gamma > (p, e) \).

As \( p \in O(S) \) there is a continuous map \( \alpha: (0, l) \to R^\infty S, \alpha(0) = p, \lim_{l \to 0} \alpha(t) = 0 ).

Suppose \( \beta(t) \in R^\infty S \) (i.e. \( \beta(t) \in O(S) \)). Likewise, we define \( \alpha(t) \) and \( \beta(t) \).

As \( S \) is connected and \( S \subset R^\infty S \), there's a curve \( \alpha(t) \) on \( S \), s.t. \( \alpha(0) = \alpha(1) \).

\[ \therefore \]