5.7 (a)(b) just write out (c) take $u = (1,0,0), (0,1,0), (0,0,1)$ then get it

5.8 (a) consistent $\Leftrightarrow \det\begin{pmatrix} v \\ w \\ N(p)\end{pmatrix} > 0 \Leftrightarrow v\cdot(w \times N(p)) > 0 \Leftrightarrow N(p)\cdot(v\times w) > 0$

(b) Denote $\hat{x} \triangleq x/\|x\|$, consistent $\Leftrightarrow \hat{w}\cdot(N(p)\times\hat{v}) > 0$

~~[As $\{v,w\}$ is a basis of S_p~~ ~~As $\|\hat{w}\|=1$~~ ~~so there must exist θ. $\hat{w} = \cos\theta\,\hat{v} + \sin\theta\, N(p)\times\hat{v}$~~

(Proof) As $N(p)\cdot(N(p)\times\hat{v}) = \det\begin{pmatrix} N(p) \\ N(p) \\ \hat{v}\end{pmatrix} = 0$ So $N(p)\times\hat{v} \in S_p$.

$\hat{v}\cdot(N(p)\times\hat{v}) = \det\begin{pmatrix}\hat{v} \\ N(p) \\ \hat{v}\end{pmatrix} = 0$. So $\{N(p)\times\hat{v}, \hat{v}\}$ is an orthornormal basis of S_p

As $\|\hat{w}\| = 1$ So there exists θ s.t. $\hat{w} = \cos\theta\cdot\hat{v} + \sin\theta\cdot N(p)\times\hat{v}$

So $\hat{w}\cdot(N(p)\times\hat{v}) = \sin\theta$

So $\theta \in (0,\pi) \Leftrightarrow \hat{w}\cdot(N(p)\times\hat{v}) > 0 \Leftrightarrow \{v,w\}$ is consistent with N

5.9 (a) take $u = (1,0,0,0), (0,1,0,0) \cdots (0001)$ (b) just check

5.10 (a) $\det\begin{pmatrix} v_1 \\ v_n \\ N\end{pmatrix} < 0 \Leftrightarrow \det\begin{pmatrix} v_1 \\ v_n \\ -N\end{pmatrix} > 0$

(b) let $v = \begin{pmatrix} v_1 \\ v_n\end{pmatrix}$, $w = \begin{pmatrix} w_1 \\ w_n\end{pmatrix}$ $\begin{pmatrix} W \\ N\end{pmatrix} = \begin{pmatrix} AV \\ N\end{pmatrix} = \begin{pmatrix} A & 0 \\ 0 & 1\end{pmatrix}\begin{pmatrix} V \\ N\end{pmatrix}$ where $W = \begin{pmatrix} w_1 \\ w_n\end{pmatrix}$, $V = \begin{pmatrix} v_1 \\ v_n\end{pmatrix}$

So $\det\begin{pmatrix} W \\ N\end{pmatrix} = \det A\cdot\det\begin{pmatrix} V \\ N\end{pmatrix}$, thus consistency of W with N is identical to the consistency of V with N iff $\det A > 0$

6.1 ~~$N(S) = \{v \mid \|v\|$~~ $n=1$ $N(S) = \{(0,1),(0,-1)\}$; $n=2$ $N(S) = \{(0, x_2, x_3) \mid x_2^2 + x_3^2 = 1\} \in \mathbb{R}^3$

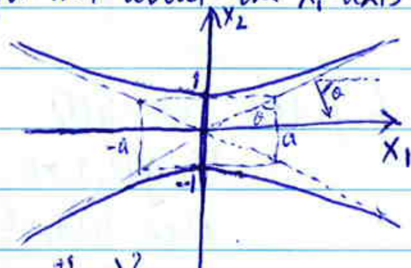
6.2 $n=1$ $N(S) = \{(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}), (-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})\}$; $n=2$ $N(S) = \{(-\frac{\sqrt{2}}{2}, u, v) \mid u^2 + v^2 = \frac{1}{2}\}$

6.3 $n=1$ $N(S) = \{(x_1, x_2) \mid x_1^2 + x_2^2 = 1\}$; $n=2$. $N(S) = \{(x_1\ x_2\ x_3) \mid x_1^2 + x_2^2 + x_3^2 = 1\}$

6.4 $n=1$ $N(S) = \{(x_1, x_2) \mid x_1^2 + x_2^2 = 1, x_1 < 0\}$; $n=2$ $N(S) = \{(x_1, x_2, x_3) \mid \sum_{i=1}^{3} x_i^2 = 1, x_1 < 0\}$

6.5 We only ~~heed~~ need to analyze $n=1$, the cases for $n \geq 2$ can be ~~derived by~~ viewed as the surface of revolution obtained by rotating the curve for $n=1$ about the x_1-axis then about (x_1, x_2) plane, then about (x_1, x_2, x_3) ...

For $n=1$ $-\frac{x_1^2}{a^2} + x_2^2 = 1$ like the right figure.



The spherical image is  $\theta = tg^{-1} a$. or formally $\{(x_1, x_2) \in S^1 \mid x_1 \in (\frac{-1}{\sqrt{a^2+1}}, \frac{1}{\sqrt{a^2+1}})\}$

For $n \geq 2$. The spherical image is $\{(x_1, \cdots x_{n+1}) \in S^n \mid x_1 \in (\frac{-1}{\sqrt{a^2+1}}, \frac{+1}{\sqrt{a^2+1}})\}$

When $a \to \infty$, it shrinks to a narrow band

when $a \to 0$, it extends to the whole S^n