there must be a subsequence of $(\frac{p_k}{\chi_k}, \frac{p_k}{\chi_k})$ which converges to $V(N'V') = 1$. Without loss of

generality, we assume that subsequence is $(\frac{p_k}{\chi_k})$ itself. Let $k \to \infty$. We have $\nabla V(N'V') = 0$, 

because $\lim_{k \to \infty} \frac{p_k}{\chi_k} = V.(\frac{p_k}{\chi_k} + \lambda \frac{p_k}{\chi_k}) = p = \frac{p_k}{\chi_k}$. $\nabla V(N'V') = 0$ contradicts the 

assumption that the curvature $k(p) \neq 0$. So for $\varepsilon > 0$, sufficiently close to $p$, the 

normal lines to $p$ at $p$ and $q$ intersect at some point $h$ \in $\mathbb{R}^2$.

(b) First derive $h(\varepsilon)$. $p + S \cdot N(p) = p + S_2 \cdot N(q)$. Suppose there is a local parametrization 

of $C$ about $p$: $\alpha(t) = 1 \to C$, $\alpha(0) = p$, $\alpha'(0) = 0$ and suppose $I$ is small enough at $p$. 

$\forall t \in I$, $\alpha(t)$ satisfies (a). So to derive $h(\alpha(t))$, suppose $\alpha(t) + S \cdot N(\alpha(t)) = \alpha(t) + S \cdot N(\alpha(t))$ 

($s, s_2 \in \mathbb{R}$). Multiply both sides by $\alpha'(t)$ and notice $N(\alpha(t)) \cdot \alpha(t) = 0$. So 

$\alpha(t) \cdot \alpha'(t) = \alpha(t) \cdot \alpha'(t) + S \cdot N(\alpha(t)) \cdot \alpha'(t) = \alpha(t) \cdot \alpha'(t)$. By assumption $N(\alpha(t))$ is 

not parallel with $\alpha(t)$, so $N(\alpha(t)) \cdot \alpha'(t) \neq 0$ So $S \cdot N(\alpha(t)) \cdot \alpha'(t) \neq 0$. 

So $k(\alpha(t)) = \frac{\dot{\alpha}(t) \cdot \alpha(t)}{\lVert \dot{\alpha}(t) \rVert^2}$ [$\ddot{\alpha}(t) \cdot \alpha(t) - N(\alpha(t)) \cdot \ddot{\alpha}(t) \cdot \alpha(t)$] 

When $t = 0$, it is equal to $\alpha(t) (-k(t) \lVert \alpha(t) \rVert^2) - k(t) \alpha(t) \cdot \alpha'(t) \cdot \alpha'(t)$ 

$+ k(t) \alpha(t) \cdot \alpha'(t) \cdot \alpha'(t) - N(\alpha(t)) \cdot \lVert \alpha(t) \rVert^2$ 

So $\lim_{t \to 0} h(\alpha(t)) = -\frac{\lVert \alpha(t) \rVert^2}{\lVert \alpha(t) \rVert^2}$ [$\dot{\alpha}(t) \cdot \alpha(t) \cdot \alpha(t) \cdot \alpha(t)$] 

$= \alpha(t) + \frac{1}{\lVert \alpha(t) \rVert^2} (\alpha(t) \cdot \alpha(t) \cdot \alpha(t))$. 

By (b) this is the focal point of $C$ along the normal line through $p$.

16.3 (a) $\dot{\alpha}(t) = \dot{\varphi}(t) + \lambda \dot{\chi}(t)$ [$\dot{(\varphi \dot{\varphi})}(t) \dot{k}(t) - k'(t) (\varphi \dot{\varphi})(t)$] 

as $(\varphi \dot{\varphi})(t) = -\dot{\varphi}(t) = -k(t) \dot{\varphi}(t)$.

So $\dot{\alpha}(t) = \frac{1}{\dot{k}(t)} \dot{k}(t) (\varphi \dot{\varphi})(t)$. So $\dot{\alpha}(t) = 0$ iff $k'(t) = 0$.

(b) As $\dot{\alpha}(t)$ is parallel to $(\varphi \dot{\varphi})(t)$ and by definition $\dot{\alpha}(t)$ is on the normal 

line to image $\varphi$ at $\varphi(t)$ so the latter is tangent at $\alpha(t)$ to the focal locus of $\varphi$ 

and by (3), $\alpha(t)$ is the focal locus of $\varphi$

(c) The sum is $s = \lVert \alpha(t) \rVert \lVert \alpha(t) \rVert + \lVert \alpha(t) \rVert$. Suppose $k'(t) = b k'(t)$ and 

$k(t) = |k(t)|$ be where $a, b \in \{ \pm 1 \}$ as both $k(t)$ and $k'(t)$ keep their sign 

for $t \in (t_1, t_2)$. So $\frac{\dot{s}}{d(t)} = -\lVert \dot{\alpha}(t) \rVert + \frac{\dot{\alpha}(t)}{d^2(t)} \dot{s} = -\lVert \dot{\alpha}(t) \rVert + a \frac{1}{d(t)} k'(t) 

= \frac{1}{d(t)} \dot{k}(t) + a \frac{1}{d^2(t)} k'(t)$. Notice that if $a b = 1$ then the conclusion 
in this exercise doesn't hold. Otherwise if $k(t) k'(t) < 0, \frac{d}{d(t)} = 0$ so $\alpha$ is constant.