15.13 Let \( \tilde{S} = \{ \tilde{g} \in S \mid \tilde{g} \) can be joined to \( p \) by a continuous curve in \( S \} \). Let \( S = f^{-1}(c) \). 
First \( \tilde{S} \) is obviously connected. If \( \tilde{g}_1, \tilde{g}_2 \in \tilde{S} \), just concatenate their curve joining \( p \) will yield a continuous curve between \( \tilde{g}_1 \) and \( \tilde{g}_2 \). Since \( \tilde{S} \subseteq \tilde{S} \), so \( \tilde{g}_1, \tilde{g}_2 \). 
\( \tilde{S} = f^{-1}(c) \). Now we only need to prove that \( \tilde{S} \) is an open set \( U, \) s.t. 
\( \tilde{S} = \{ x \in U \mid f(x) = c \} \). We mimic the proof of Thm3. 
For each \( \tilde{g} \in \tilde{S} \), let \( \tilde{V}_\tilde{g}: U_\tilde{g} \rightarrow S \) be a local parametrization of \( S \) whose image contains \( \tilde{g} \) and let \( \tilde{V}_\tilde{g}: U_\tilde{g} \times \mathbb{R} \rightarrow \mathbb{R}^n \) be defined by \( \tilde{V}_\tilde{g}(r, \tilde{s}) = \tilde{V}_\tilde{g}(r) + s \tilde{N}(\tilde{V}_\tilde{g}(r)), \) where \( N \) is the orientation of \( S \). Then as in the proof of Thm2, we can find an open set \( U_\tilde{g} \) about \( (\tilde{V}_\tilde{g}(0), 0) \) in \( U_\tilde{g} \times \mathbb{R} \) s.t. \( \tilde{V}_\tilde{g} \) maps \( U_\tilde{g} \) one to one onto and open set \( U_\tilde{g}^* \) in \( \mathbb{R}^n \) and \( \phi_\tilde{g}: U_\tilde{g} \rightarrow U_\tilde{g}^* \) is smooth. Furthermore by shrinking \( U_\tilde{g} \) if necessary, we may assume that \( U_\tilde{g}^* \) is open. 
Since \( U_\tilde{g} \) is an open set, then for any \( \tilde{g} \), there must be a \( \tilde{g} \) in an open set \( U_\tilde{g} \) containing \( \tilde{g} \). 
there is a smooth curve \( \alpha(t) \): \([a, b] \rightarrow U_\tilde{g} \), s.t. \( \alpha(t) = \tilde{g} \) \( \alpha(a) = \tilde{g}(t), \alpha(b) = \tilde{g}(t) \). 
By shrinking \( U_\tilde{g} \) further, we may assume that \( U_\tilde{g} = \tilde{V}_\tilde{g}(U_\mathbb{R}) \). 
And \( \tilde{g} \in I \), we have \( \beta(t) = \tilde{V}_\tilde{g}(\tilde{g}(t), t) \in U_\tilde{g} \times \mathbb{R} \). 
We define a continuous map \( \tilde{g} \) through a curve \( \alpha(t) \) on \( S \), so \( \tilde{g} \in S \). In other words, for \( \tilde{g} \in \tilde{S} \), there is an open set \( W_\tilde{g} \) about \( \tilde{g} \) s.t. \( \tilde{g} \in W_\tilde{g} \times \mathbb{R} \). 
Now we define \( U = U_\tilde{g} \cap W_\tilde{g} \) which is open, then \( \tilde{g} \in U \) by definition. 
\( \Sigma \times \mathbb{S} \), we have \( \Sigma \in U \), \( \mathbb{F}(x) = c \). So \( \Sigma \times \{ x \in U \mid f(x) = c \} \). 
\( \Sigma \times \{ x \in U \mid f(x) = c \} \), there must be a \( \Sigma \in \mathbb{S} \), s.t. \( \Sigma \times \mathbb{F} \). As \( \Sigma \in \mathbb{S} \), so \( \Sigma \times \mathbb{F} \times \mathbb{S} \subseteq \mathbb{S} \). Thus, \( \{ x \in U \mid f(x) = c \} \subseteq \mathbb{S} \). 
Hence \( \mathbb{S} = \{ x \in U \mid f(x) = c \} \), i.e. \( \mathbb{S} \) is a surface.

15.14 Suppose \( \alpha(t) = \alpha(t_1) \) for some \( t_1 + t_2 \in I \). Suppose the maximal integral curve of \( X \) through \( \alpha(t) \) is unique, then denoted as \( \beta(t) \) and \( \beta(0) = \alpha(t_1) \), then \( \alpha(t) = \beta(t-t_1) \) and \( \alpha(t) = \beta(t-t_2) \) for all \( t \). Setting \( T = t_1-t \), we have \( \alpha(t) = \beta(t-T-t_1) = \alpha(t_1) \) for all \( t \) such that \( t \) and \( t+T+T \in I \). 
Thus if \( \alpha \) is not one to one then it is periodic. 
To prove that the maximal integral curve \( X \) through \( \alpha(t) \) is unique, we notice