by definition $Q(v) > 0$ for all $v \neq 0$. As $Q$ is pos. def., both eigenvalues are positive. Thus $\det L = \lambda_1 \lambda_2 > 0$.

(8) $L$ is non-singular $\iff$ $\det L = \prod \lambda_i \neq 0 \iff \lambda_i \neq 0$ (all eigenvalues) i.e. $p$ is non-degenerate.

(9) If $p$ is non-degenerate $\Rightarrow$ $L: v \mapsto L(v \langle \nabla h \rangle)$ is non-singular $\iff$ $\lambda_i \neq 0 \Rightarrow \forall v \ n(v \langle \nabla h \rangle) \neq 0$.

3.4 If $h$ is height function or any function which has constant $|\nabla h|$, then

\[ \langle h(\beta(t)), \dot{\beta}(t) \rangle = \langle \nabla h(\beta(t)), \dot{\beta}(t) \rangle = \| \nabla h(\beta(t)) \| \| \dot{\beta}(t) \| = \| \nabla h(\beta(t)) \| \| \dot{\beta}(t) \| \]

So $\dot{h}(\alpha(b)) = h(\alpha(a)) + \int_a^b \langle h(\beta(t)), \dot{\beta}(t) \rangle \ dt = h(\beta(b))$.

Equality holds $\iff \nabla h(\beta(t)) = \lambda \dot{\beta}(t)$ for all $t$. But $\nabla h(\alpha(t)) = \nabla h(\beta(t))$. So $\lambda = 1$.

So $\dot{h}(\beta(t)) = \dot{\beta}(t)$, i.e. $\beta$ is also a gradient line passing thru $\omega(\alpha(x))$, but such a line is unique, so $\beta = \alpha$.

If $\| \nabla h \| = \text{const}$ is not guaranteed, WE FEEL that this proposition may not hold.

The following is an counter-example. Let $h(x_1, x_2) = x_1 \ f(x_1, x_2) = x_2$ then $\nabla h = (h(x_1), 0)$ $\nabla f = (0, 1)$ so $S = \nabla f$ is a surface. $\nabla \nabla \nabla f \Rightarrow \nabla h = \alpha(t), \beta(t) \in S$. So we can write in brief $\omega(t) = (\alpha(t), 0), \beta(t) = (\beta(t), 0)$

So now $\dot{\alpha}(t) = h'(\alpha(t)) \mid \dot{\beta}(t) = |\dot{\beta}(t)|$.

If $\beta(t)$ appears in the conclusion only inside $h(\beta(t))$, the only constraint on $\beta$ is actually $\langle h(\beta(t)), \dot{\beta}(t) \rangle \ dt = \int_{a(\alpha)}^{a(\beta)} \langle h(t), \dot{t} \rangle \ dt = \ell(\alpha)$. Now we define $\dot{\alpha}(a) = a(0)$ $\dot{\beta}(a) = 0$ defined on $(R^+, R)$ which is open. Let $\alpha(t) = a(t)$

the first peak to the left of $t_0$ is $x$, where $t_0 = \frac{\pi}{2}$ $x = \frac{\pi}{2} + \pi$. $h(t-x) = \frac{t-x}{t_0} + \pi, h(t+x) = \frac{t+x}{t_0} + \pi$.

Besides, the first peak to the right of $t_0$ is $t+s$, i.e. $t+s = \frac{\pi}{2} - \pi, s = \frac{\pi}{2} - t_0 > X$ in $(t_0 + s) \ \omega 0$. Now suppose $\omega(t) = t + \pi$ where $\omega$ is sufficiently small, $\omega$. For $t > a$, $\omega(t)$ monotonically increases, and $\beta(t)$ is forced to decrease monotonically.

As $\omega$ can be arbitrarily small, by above discussion $\beta(t)$ first reaches $t_0$, then $\omega(t)$ monotonically increases, and $\beta(t)$ is forced to decrease monotonically.

Suppose $b$ is chosen such a moment, then we have $h(a(b)) < h(\beta(b))$.

which contradicts the exercise assertion.