by plugging into \( (1.1) \)

\[
\frac{d^N}{dt^N} j_p = \frac{1}{a} \frac{d}{dt} j_p
\]

So \( \overline{K}(v) = \frac{k(v)}{a} \), which is true at all stationary points.

But mean curvature \( H \) is the average of \( K \) at stationary points, thus \( H(\alpha p) = \frac{1}{a} H(p) \).

(a) \( K \) (Gauss-Kronecker curvature) is the product of \( k(v) \) at stationary points

So \( K(\alpha p) = a^n k(p) \)

Remark: Above argument based on stationary points is not strict enough, especially considering the multiplicity of \( g_p \)'s eigenvalues. A better proof is: \( H = V - 2\phi p \). As \( \nabla g_p \) is \( \frac{1}{a} \) \( \nabla f(p) \)

\[
\text{So } Sp = g_p, \forall \nu, \nu \in Sp \quad \forall p(v) W = \frac{1}{2} [k(v + W) - k(v) - k(W)] = \frac{1}{a} \overline{K}(\nu) \quad \forall \nu \in Sp
\]

Let \( \overline{q} \) be mapping on \( S \) at \( \overline{q} : \overline{q}(v) W = \frac{1}{2} [k(v + W) - k(v) - k(W)] = \frac{1}{a} \overline{K}(\nu) \).

Since \( W \) is arbitrary in \( Sp \), so \( \overline{q}(v) W = \frac{1}{a} (k(v))^n \). So each eigenvalue \( \lambda_i \) of \( k(v) \) corresponds to the eigenvalue \( \lambda_i/a \) of \( k(v) \). As \( H \) and \( K \) are average / product of eigenvalues, we have

\[
H(\alpha p) = \frac{1}{a} H(p) \quad K(\alpha p) = a^n k(p)
\]

So \( \overline{K}(\nu) = \frac{1}{a} \overline{K}(\nu) \).

Then one has \( \overline{q}(v) - \frac{1}{a} \overline{q}(v) = 0 \) So \( \overline{q}(v) = 0 \).

13.1 If \( S \) is convex at \( p \), then \( H(\nu = N(p) \text{ Gauss map}) \) attains local max/min at \( p \).

So \( g_p \) is semi-definite, so \( Sp = g_p \) is also semi-definite. But the Gauss-Kronecker curvature, which is equal to the product of all eigenvalues of \( g_p \), is negative.

As \( Sp \) is even dimensional, there must be one positive and one negative eigenvalue, which contradicts the fact that \( Sp \) is semi-definite. So \( S \) is not convex at \( p \).

13.2 \( \forall \nu, \nu \in Sp \quad \overline{q}(\text{grad}\, h \nu) W = \overline{q}(\nu - \nu N) W = \overline{q}(\nu W) - (\nu \cdot N) \overline{q}(W) \overline{q}(W)

\[
\overline{q}(\text{grad}\, h \nu) V = \overline{q}(\nu - \nu N) V = \overline{q}(\nu V) - (\nu \cdot N) \overline{q}(W)
\]

We know \( \overline{q}\) is self-adjoint, i.e., \( \overline{q} N \nu = \overline{q} \nu \cdot N \nu \). Besides,

\[
\overline{q}(\nu W) V = \nu \cdot N W = W \nu H\nu = \overline{q}(\nu W) \nu \quad \text{So } \overline{q}(\text{grad}\, h \nu) W = \overline{q}(\text{grad}\, h \nu) V, \text{ so self-adjoint}
\]

13.3 (a) \( \Rightarrow \) If \( L \) is posDef, then \( V \) eigenvector \( V \), \( \nu(V) = \lambda V \), \( \nu(V) \cdot V = \lambda V \) as \( L \) is posDef

{}={} We know that the eigenvectors \( V_i \) of \( V \) make up an orthonormal basis on \( Sp \). \( \forall \nu \in Sp \),

\[
\nu = \frac{1}{\sqrt{a}} a_i \nu_i \quad \nu(V) = 2 \left( \frac{a_i \nu_i \cdot V_i}{\sqrt{a}} \right) \frac{a_i \nu_i \cdot V_i}{\sqrt{a}} \nu_i = \left( \frac{a_i}{\sqrt{a}} \nu_i \cdot V_i \right) \left( \frac{a_i}{\sqrt{a}} \nu_i \cdot V_i \right)
\]

\[
= \frac{a_i}{\sqrt{a}} \lambda_i \nu_i \quad \text{because } \lambda_i > 0
\]

It is equal to 0 iff \( a_i \nu_i \nu = 0 \) i.e. \( v = 0 \).

(b) Since \( L \) is self-adjoint linear transformation, its associated matrix is symmetric, so it has two real valued eigenvalues \( \lambda_1, \lambda_2 \). def \( L > 0 \Rightarrow \lambda_1, \lambda_2 > 0 \). But if \( \lambda_1 < 0, \lambda_2 < 0 \), then \( L \) is negative definite, i.e., there can't be any \( v : \nu(V) > 0 \).

Thus \( \lambda_1 > 0, \lambda_2 > 0 \).