Interference Suppression Using Generalized Inverse Precoder for Downlink Heterogeneous Networks

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***Abstract*—This letter proposes an optimal precoder design using a generalized inverse structure for downlink crosstier interference suppression between macro base station (MBS) and femtocell access point (FAP) transmission in heterogeneous networks. Using Tikhonov regularization, we derive a closed form relationship between given MBS transmit power or interference constraints and the regularization parameter. Determining a Pareto optimal regularization parameter requires extensive computation. As an alternative, we propose an algorithm to find a suitable regularization parameter to achieve fairness in the system, i.e., equal FAP and MBS user rates. Simulation results show that the proposed precoder greatly improves FAP user rates at the expense of a small yet tolerable decrease in average MBS user rate.**

***Index Terms*—Heterogeneous networks, precoder, Tikhonov regularization, interference management.**

I. Introduction

HETEROGENEOUS networks (HetNets), which allow co-existence of macro base stations (MBS) and small cells such as femtocell access points (FAPs), are seen by network operators as a promising solution to provide high data rates and seamless coverage in fifth generation (5G) wireless communication systems [1]. The successful deployment of such networks relies on the management of cross-tier interference, e.g., a user equipment (UE) is offloaded to an FAP (which may also occur during cell range expansion) but is suffering from downlink MBS interference [2]. Thus, it is important to investigate solutions which allow an MBS to suppress its interference to HetNet UEs without compromising service to macro UEs (MUEs).

Recently, many papers have focused on interference management in HetNets [3]. Conventional interference management techniques such as resource allocation or scheduling approaches [4] do not allow multiple users to be served simultaneously in shared spectrum environments. Advanced methods such as coordinated multipoint (CoMP), almost blank subframes (ABS), enhanced intercell interference coordination (eICIC), and even cognitive radio based approaches [5], [6] require extensive cooperation and reliable backhaul which may not be always practical in HetNet scenarios. Power control, i.e., increasing the FAP transmit power, can combat crosstier interference, but in dense networks this will in turn cause significant interference to nearby small cells.

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Another possible approach for HetNet interference management is the use of transmit precoding. For traditional cellular networks, well known methods such as zero-forcing (ZF) [7], regularized and vector inverses are available [8], but these have disadvantages such as lack of design flexibility, signal leakage and complexity. Interference alignment can also be employed to completely cancel inter-cell interference under certain conditions [9], [10]. However, complete cancellation may not be desirable as the decodability of the interfering signals limits the data rate of the other users [9]. In [11], an approach for crosstier interference mitigation using precoder codebooks is presented, but is more about precoder *selection* rather than precoder *design*. A precoder design for HetNets is studied in [12], but its focus is energy efficiency rather than interference management. To the best of our knowledge, precoder designs for crosstier interference management in multi-user downlink HetNet systems have not been presented.

In this letter, we formulate the optimal precoder design problem using the generalized inverse structure for suppressing downlink interference from the MBS to femtocell UEs (FUEs) subject to given power or interference constraints. Our contributions are threefold: (i) We show that a generalized inverse precoder can suppress interference to a desired user without adversely compromising service to other users. (ii) Using Tikhonov regularization, we obtain a closed form relationship between the constraints and the precoder regularization parameter. (iii) Since a Pareto optimal regularization parameter is difficult to determine, and does not allow the MBS to target a specific FUE rate, we present a linear approximation to find a suitable regularization parameter to ensure fairness in the system, which is defined as equal FUE and average MUE user rates. Our results show that a small compromise in the average MUE rate greatly improves the FUEs' rates.

Notation: We use conventional mathematical notations for matrix operations, including $(\cdot)^T$ and $(\cdot)^H$ for matrix transpose and conjugate transpose respectively, $(\cdot)^{-1}$ for matrix inverse, $(\cdot)^+$ for matrix psuedoinverse, $\det(\cdot)$ for determinant, $\|\cdot\|$ for Frobenius norm, $\mathrm{vec}(\cdot)$ for matrix vectorization, $\mathrm{tr}(\cdot)$ for matrix trace, $\mathrm{diag}(\cdot)$ for diagonal matrix, $\mathbf{I}_N$ for identity matrix of dimension N, and $\mathbf{0}_{M,N}$ for an $M \times N$ zero matrix. The complex normal Gaussian distribution is denoted as $\mathcal{CN}(a, b)$ with mean a and variance b.

II. System Models and Problem Statement

Consider an MBS with N antennas serving $N - k$ MUEs and an FAP with N_f antennas serving k FUEs. All MUEs and FUEs employ single antennas. The FUEs are within the MBS cell radius and are receiving interference from the MBS. We assume the FAP has no initial users, but our system can be extended without loss of generality to include any initial users. The system with $k = 1$ is illustrated in Fig. 1.

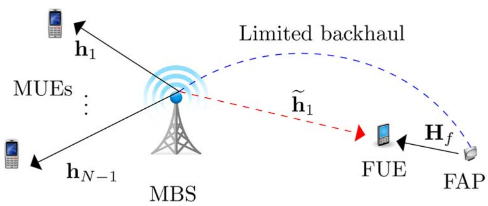

Fig. 1. System model comprising of MBS, MUEs, FUE, and FAP. Interference from MBS to one FUE is illustrated.

We make the following channel assumptions: (i) the MBS has perfect channel state information (CSI) of its $N-k$ MUEs, (ii) the FAP has perfect CSI of its k FUEs and (iii) the MBS may have imperfect CSI of the MBS-FUE channels due to either imperfect feedback from the FUEs themselves or feedback via limited backhaul from the FAP.

Let $\mathbf{H} = (\mathbf{h}_1 \ \dots \ \mathbf{h}_{N-k})$ denote the channels of the $N-k$ MUEs, where each column $\mathbf{h}_i$ is the $N \times 1$ channel vector for the ith user and each element $\sim\mathcal{CN}(0,1)$. The k MBS-FUEs' channels are similarly defined as $\widetilde{\mathbf{H}} = (\widetilde{\mathbf{h}}_1 \ \dots \ \widetilde{\mathbf{h}}_k)$ whose entries are also $\sim\mathcal{CN}(0,1)$. We denote the MBS precoder as $\mathbf{W}$, which will be defined in the next section.

Taking into account individual MUEs' power allocations, let $\mathbf{Q} = \mathrm{diag}(q_1, q_2, \dots, q_{N-k}), q_i \geq 0, \Sigma_{i=1}^{N-k} q_i = N-k$ denote the random MBS power allocation matrix. Using $\mathbf{Q}$, the equivalent precoder is $\mathbf{W} = \mathbf{W}\mathbf{Q}^{\frac{1}{2}}$. Let $\mathbf{x}$ be the vector of $N-k$ independent data streams (one for each MUE) with unit power, and $\mathbf{n} \sim \mathcal{CN}(0, \sigma^2)$ be the independent additive white Gaussian noise (AWGN) vector of dimension $N \times 1$. Thus, using a normalized $\mathbf{W}$ in accordance with our power constraint, the received signals from all MBS transmissions form the vector

$$\mathbf{y} = \sqrt{p_m} \begin{pmatrix} \mathbf{D}^{\frac{1}{2}} \mathbf{H}^H \\ \widetilde{\mathbf{D}}^{\frac{1}{2}} \widetilde{\mathbf{H}}^H \end{pmatrix} \mathbf{W}\mathbf{x} + \mathbf{n}, \tag{1}$$

where p_m is the MBS transmit power, and $\mathbf{D} = \mathrm{diag}(\delta_1, \dots, \delta_{N-k})$ and $\widetilde{\mathbf{D}} = \mathrm{diag}(\widetilde{\delta}_1, \dots, \widetilde{\delta}_k)$ are the diagonal pathloss matrices for the MBS-MUE and MBS-FUE channels respectively. The pathloss elements can be determined using well known free-space or industry standard path loss models.

We assume the FAP uses any suitable scheduling scheme to serve its k users during its downlink transmission. Thus, the received signals at the FUEs are denoted as

$$\mathbf{y}_F = \underbrace{\sqrt{p_f} \mathbf{D}_f^{\frac{1}{2}} \mathbf{H}_F^H \mathbf{x}_F}_{\text{desired}} + \underbrace{\sqrt{p_m} \widetilde{\mathbf{D}}^{\frac{1}{2}} \widetilde{\mathbf{H}}^H \mathbf{W}\mathbf{x}}_{\text{interference}} + \mathbf{n}_F, \tag{2}$$

where p_f is the FAP transmit power, $\mathbf{H}_F = \mathrm{diag}(\mathbf{h}_{f,1}, \dots, \mathbf{h}_{f,k})$ is the $k \times k$ equivalent diagonal Rayleigh fading channel matrix from FAP to FUEs, $\mathbf{D}_f = \mathrm{diag}(\delta_{f,1}, \dots, \delta_{f,k})$ is the diagonal FAP-FUE pathloss matrix, $\mathbf{x}_F$ is the data $k \times 1$ vector transmitted from the FAP with unit power and $\mathbf{n}_F$ is the $k \times 1$ AWGN vector whose independent elements follow $\sim\mathcal{CN}(0, \sigma^2)$.

The sum rates for the $N-k$ MUEs and any particular FUE are respectively defined as

$$C_{MUE} = \log_2 \left(\det(\mathbf{I}_{N-k} + p_m \mathbf{D}\mathbf{H}^H \mathbf{W}\mathbf{W}^H \mathbf{H}) \right), \tag{3}$$

$$C_{FUE} = \log_2(1+\gamma), \tag{4}$$

where γ is the signal-to-interference-plus-noise ratio at any particular kth FUE with FAP-FUE channel $\mathbf{h}_{f,k}$, defined as

$$\gamma = \frac{p_f \delta_{f,k} \|\mathbf{h}_{f,k}\|^2}{p_m \widetilde{\delta}_k \left\| \widetilde{\mathbf{h}}_k^H \mathbf{W} \right\|^2 + \sigma^2}, \tag{5}$$

where $\delta_{f,k}$ denotes the pathloss between FAP and kth FUE and $\widetilde{\delta}_k$ denotes the pathloss between MBS and kth FUE.

III. Precoder Design

In this section we address the precoder design to raise FUEs' rates by suppressing the MBS interference to FUEs, but maintaining interference-free transmission to the MUEs. We design a new precoding matrix for the $N-k$ MUEs using the generalized inverse structure [7]

$$\mathbf{W} = \mathbf{G} + \mathbf{U}\mathbf{B}, \tag{6}$$

where $\mathbf{G} = (\mathbf{H}^H)^+ = \mathbf{H}(\mathbf{H}^H\mathbf{H})^{-1}$ is the psuedoinverse of $\mathbf{H}^H$, $\mathbf{U}$ is the $(N-k) \times k$ nullspace of $\mathbf{H}^H$ [13], i.e., $\mathbf{H}^H\mathbf{U} = \mathbf{0}_{N,k}$, and $\mathbf{B}$ is a $k \times (N-k)$ matrix of variable coefficients. The intuition behind using the structure in (6) is that the elements of $\mathbf{B}$ can be appropriately chosen to achieve a desired level of interference suppression. In this regard, we will first formulate the problem and then show how to determine $\mathbf{B}$ (and hence the precoder in (6)) given the constraints. Finally, we will define a new fairness criteria and show how to determine the precoder accordingly.

Problem Formulation: We can formulate the MBS precoder design problem either in terms of an MBS power constraint α as

$$\min \left\| \widetilde{\mathbf{H}}^H (\mathbf{G} + \mathbf{U}\mathbf{B}) \right\|^2 \ \text{s.t.} \ \|\mathbf{G} + \mathbf{U}\mathbf{B}\|^2 \leq \alpha^2, \tag{7}$$

or acceptable MBS-FUE interference limit β as

$$\min \|\mathbf{G} + \mathbf{U}\mathbf{B}\|^2 \ \text{s.t.} \ \left\| \widetilde{\mathbf{H}}^H (\mathbf{G} + \mathbf{U}\mathbf{B}) \right\|^2 \leq \beta^2, \tag{8}$$

which can be shown to be equivalent [14]. Using a least squares approach, (7) and (8) are also equivalent to

$$\min \left\| \begin{pmatrix} \mathbf{U} \\ \lambda \widetilde{\mathbf{H}}^H \mathbf{U} \end{pmatrix} \mathbf{B} + \begin{pmatrix} \mathbf{G} \\ \lambda \widetilde{\mathbf{H}}^H \mathbf{G} \end{pmatrix} \right\|, \tag{9}$$

with the solution

$$\mathbf{B} = - \begin{pmatrix} \mathbf{U} \\ \lambda \widetilde{\mathbf{H}}^H \mathbf{U} \end{pmatrix}^+ \begin{pmatrix} \mathbf{G} \\ \lambda \widetilde{\mathbf{H}}^H \mathbf{G} \end{pmatrix}, \tag{10}$$

where $\lambda \geq 0$ is the regularization parameter, which refers to the amount of weighting given to interference suppression, i.e., a larger λ gives more preference to interference suppression. Substituting (10) into (6), we have

$$\mathbf{W} = \left(\mathbf{I}_N - \mathbf{U} \begin{pmatrix} \mathbf{U} \\ \lambda \widetilde{\mathbf{H}}^H \mathbf{U} \end{pmatrix}^+ \begin{pmatrix} \mathbf{I}_N \\ \lambda \widetilde{\mathbf{H}}^H \end{pmatrix} \right) \mathbf{G}. \tag{11}$$

For the MBS to calculate (11), $\widetilde{\mathbf{H}}^H$ refers to the MBS estimate of the true MBS-FUE channel. Perfect CSI and $\lambda = \infty$ leads to interference nulling, while imperfect CSI leads to suppression.

Precoder for Given Power or Interference Constraints: From (11) we can see that we need to determine λ to find $\mathbf{B}$. Hence, we derive the closed form relationship between λ and the constraints α or β.

Proposition 1: The relationship between λ and α is

$$\lambda^2 = \frac{1}{\sum_i \mu_i^2}\left(\frac{\sum_i \omega_i}{\alpha + \sum_i \psi_i} - \sum_i \sigma_i^2\right), \quad (12)$$

where σ_i and μ_i are the generalized singular values of $\widetilde{\mathbf{H}}^H\mathbf{U} = \mathbf{L}_1\mathbf{\Sigma}\mathbf{R}^{-1}$ and $\mathbf{U} = \mathbf{L}_2\mathbf{M}\mathbf{R}^{-1}$ respectively, $\mathbf{\Omega} = \mathbf{L}_2\mathbf{M}(\mathbf{\Sigma}^H\mathbf{L}_1^H(-\widetilde{\mathbf{H}}^H\mathbf{G}) + \mathbf{M}^H\mathbf{L}_2^H(-\mathbf{G}))$ and $\mathbf{\Psi} = \mathbf{L}_2^H(-\mathbf{G})$ are both $N \times (N-k)$ matrices, ω_i denotes the elements of $\mathrm{vec}(\mathbf{\Omega})$ and ψ_i denotes the elements of $\mathrm{vec}(\mathbf{\Psi})$. Similarly, the relationship between λ and β is the same as (12) but with $\mathbf{\Omega} = \mathbf{L}_1\mathbf{\Sigma}(\mathbf{\Sigma}^H\mathbf{L}_1^H(-\widetilde{\mathbf{H}}^H\mathbf{G}) + \mathbf{M}^H\mathbf{L}_2^H(-\mathbf{G}))$, $\mathbf{\Psi} = \mathbf{L}_1^H(-\widetilde{\mathbf{H}}^H\mathbf{G})$ and replacing α with β.

Proof: See Appendix A.

Remark 1: Proposition 1 gives the value of λ^2 that minimizes the objective for a given power or interference constraint known to the MBS. However, this λ^2 may not be a Pareto optimal value which gives the best 'balance' of objective and constraint. For example, a predetermined interference constraint may require too much transmit power, so the corresponding λ^2 determined using (12) may not be suitable. Calculating such a Pareto optimal λ^2 can be done using methods such as L-curve curvature [14], but these are often very challenging to compute and hence not pursued in this work. Alternatively, the MBS may have a target FUE rate in mind to ensure user fairness. This is investigated below.

Precoder for User Fairness: Suppose we aim to find a λ^2 which achieves user fairness, defined as

$$f(\lambda^2) \triangleq \frac{C_{MUE}}{N-k} - C_{FUE} = 0, \quad (13)$$

i.e., when the average MUE rates and FUE rate are equal. Note that other definitions of fairness such as max-min and proportional fair exist in the literature [7]. Max-min fairness is not suitable since the FUE may not initially have the lowest rate, or its rate may not need to be completely maximized at the expense of large MUE rate degradation. Proportional fair is used for scheduling and is also not suitable since it requires past knowledge of user requirements. Hence, a more suitable fairness definition for our setup is when the FUE's rate is equal to the average MUE rates.

A λ^2 that achieves fairness is the root of (13), but an exact analytical expression for this is difficult to obtain. Root-finding algorithms can be used to find an approximate solution, but common algorithms such as Newton's method rely on the derivatives of (3) and (4) with respect to λ^2. Such extensive computation may not be practical, and thus we describe a simpler linear approximation.

The function (13) is a logarithmic function, which for some values of its domain exhibits linear behaviour. Through extensive simulations, we have determined that for parameter values in the practical range, the MBS can calculate a suitable λ^2 using a simple linear approximation with respect to λ^2, described in Algorithm 1. The initial guesses are arbitrary and chosen empirically, and affect only the approximation error.

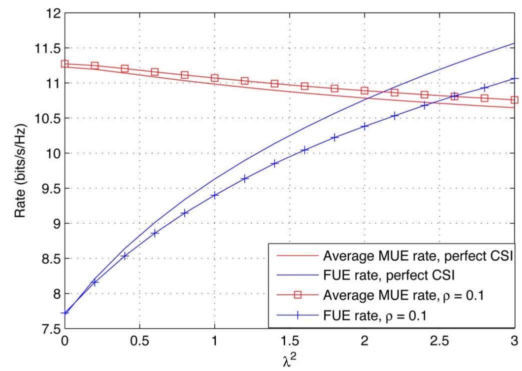

Fig. 2. Average MUE and FUE rates with perfect ($\rho = 0$) and imperfect ($\rho = 0.1$) MBS-FUE CSI.

Algorithm 1 Linear approximation to find root of $f(\lambda^2)$.

Initialize: Initial guesses $\lambda_0^2 = 0$ and $\lambda_1^2 = 2$ (arbitrary)
Approximation: Let $f(\lambda^2) \approx a\lambda^2 + b$
Calculate:
$f(\lambda_0^2) = f(0) = b$
If $b > 0$
  $f(\lambda_1^2) = a\lambda_1^2 + f(0)$
  $a = \frac{f(\lambda_1^2) - f(0)}{\lambda_1^2}$
  Solve: $f(\lambda^2) = 0$
  $\lambda^2 = -\frac{b}{a} = \frac{-\lambda_1^2 f(0)}{f(\lambda_1^2) - f(0)}$
End

Using this linear approximation to calculate the root of $f(\lambda^2)$ ensures that only three trials are needed. Further, if the first trial using $\lambda^2 = 0$, i.e., $\mathbf{W} = \mathbf{G}$ with no interference suppression, yields a negative value, the FUE is already experiencing better rates than the average MUE, and no additional design is necessary. The approximation error can be reduced if a higher degree polynomial approximation is used, but at the cost of additional computation. However, using a polynomial of degree five or higher may be exceptionally challenging since their roots cannot be found using rational formulas. The accuracy of the proposed linear approximation will be discussed in the next section.

IV. Simulation Results

We illustrate the performance of the proposed precoder for user fairness via simulation results averaged over 10,000 Monte Carlo realizations. We consider $N = 8$, $N_f = 2$, $k = 1$ FUE and $N-1$ MUEs. The MBS-FUE and FAP-FUE distances are fixed at $d_m = 500$ m and $d_f = 10$ m respectively. We set a random MUE power allocation matrix $\mathbf{Q}$, and fix MBS and FAP transmit powers to 20 W and 100 mW respectively. Standard pathloss models are considered for the MBS to FUE and FAP to FUE links, given by $15.3 + 37.6\log_{10}(d_m)$ dB and $38.5 + 20\log_{10}(d_f)$ dB [15]. The noise spectral density is set to -174 dBm/Hz and bandwidth is 20 MHz. We also consider the effect of imperfect FUE CSI at the MBS by modelling a particular MBS-FUE channel $\widetilde{\mathbf{h}}$ as $\widetilde{\mathbf{h}}_{\text{est}} = \widetilde{\mathbf{h}} + \rho\mathbf{e}$ where $\mathbf{e} \sim \mathcal{CN}(0, 1)$ is an independent random error vector, and $0 \leq \rho \leq 1$ is the error magnitude.

Fig. 2 shows the average MUE and FUE rates for a particular FAP transmit power with varying λ^2. The results show the benefit of using the proposed MBS generalized inverse precoder ($\lambda^2 > 0$) compared to conventional ZF with no interference

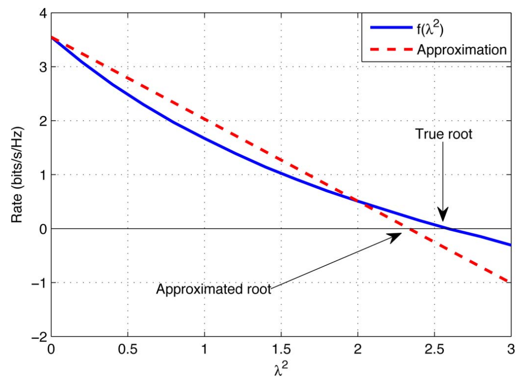

Fig. 3. Linear approximation of fairness function with $\rho = 0.1$.

suppression ($\lambda^2 = 0$). We can see that increasing λ^2 gives more preference to interference suppression, and thus improves FUE rate. With an MBS transmit power constraint, i.e., normalized precoding, this compromises the average MUE rate. For our range of simulation parameters, the slopes of the curves indicate that the percentage increase in FUE's rate $(>40\%)$ is much greater than the percentage decrease in MUE rates $(<5\%)$. Thus, it is evident that significant benefits can be made to the FUE with a small but tolerable decrease in MUE rates. We also observe that more accurate CSI results in a smaller λ^2 to achieve fairness.

Fig. 3 illustrates how our linear approximation is used to estimate the root of $f(\lambda^2)$ when $\rho = 0.1$. In this realization, the approximation finds the root to be around $\lambda^2 = 2.3$, while the actual root is around $\lambda^2 = 2.6$. Through extensive simulations, we have observed that the percentage error of the average absolute difference between the rates given by the true fairness λ^2 and the approximated λ^2 is $\approx 5\%$ for practical range of system parameters.

V. Conclusion

We have proposed a generalized inverse MBS precoder which can suppress interference to an FUE in a HetNet scenario. We derive a closed form expression relating the regularization parameter to a given constraint, and also present an algorithm to achieve user fairness which requires only three trials. Compared with ZF, the FUE's rate can be significantly improved with a tolerable decrease in the average MUE rate.

Appendix A
Proof of Proposition 1

We prove Proposition 1 using Tikhonov regularization. (7) and (8) are equivalent to the Tikhonov regularization formulation with parameter $\lambda \geq 0$ [14], namely

$$\min \left\| \widetilde{\mathbf{H}}^H(\mathbf{G} + \mathbf{U}\mathbf{B}) \right\|^2 + \lambda^2 \|\mathbf{G} + \mathbf{U}\mathbf{B}\|^2. \quad (14)$$

The relationship between the constraints α, β and parameter λ relies on the generalized singular value decomposition (GSVD) of $\widetilde{\mathbf{H}}^H\mathbf{U}$ and $\mathbf{U}$. To show this, consider the expression for the solution to (14) [14], which is

$$\mathbf{B} = \left(\left(\widetilde{\mathbf{H}}^H\mathbf{U}\right)^H \widetilde{\mathbf{H}}^H\mathbf{U} + \lambda^2\mathbf{U}^H\mathbf{U} \right)^{-1} \times \left(\left(\widetilde{\mathbf{H}}^H\mathbf{U}\right)^H \left(-\widetilde{\mathbf{H}}^H\mathbf{G}\right) + \mathbf{U}^H(-\mathbf{G}) \right). \quad (15)$$

To first prove the relationship between α and λ, let the GSVD of $\widetilde{\mathbf{H}}^H\mathbf{U}$ and $\mathbf{U}$ be

$$\widetilde{\mathbf{H}}^H\mathbf{U} = \mathbf{L}_1\boldsymbol{\Sigma}\mathbf{R}^{-1} \quad \text{and} \quad \mathbf{U} = \mathbf{L}_2\mathbf{M}\mathbf{R}^{-1} \quad (16)$$

where $\boldsymbol{\Sigma} = \text{diag}(\sigma_1, \ldots, \sigma_k)$ and $\mathbf{M} = \text{diag}(\mu_1, \ldots, \mu_{N-k})$. Substituting the above into (15) and simplifying, we have

$$\mathbf{B} = \mathbf{R}\left(\boldsymbol{\Sigma}^H\boldsymbol{\Sigma} + \lambda^2\mathbf{M}^H\mathbf{M}\right)^{-1} \times \left(\boldsymbol{\Sigma}^H\mathbf{L}_1^H\left(-\widetilde{\mathbf{H}}^H\mathbf{G}\right) + \mathbf{M}^H\mathbf{L}_2^H(-\mathbf{G})\right). \quad (17)$$

Setting the equality constraint in (7) and substituting the above, after some tedious simplifying, we have

$$\alpha = \left\| \frac{\boldsymbol{\Omega}}{\sum_i \sigma_i^2 + \lambda^2 \sum_i \mu_i^2} - \boldsymbol{\Psi} \right\|. \quad (18)$$

Since a vectorized matrix has the same norm as the original matrix, we can simplify the above further to obtain (12).

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