On Lower Bounding the Information Capacity of Amplify and Forward Wireless Relay Channels with Channel Estimation Errors

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***Abstract*—We formulate a capacity lower bound for the dual-hop wireless relay channel which employs an amplify-and-forward (AF) protocol at the relay node. In AF relaying, even when the fading channel in both hops is complex Gaussian distributed, the overall dual-hop channel is non-Gaussian. We highlight that there is a fundamental difference between Gaussian and non-Gaussian channels in terms of deriving their capacity lower bound. Specifically for non-Gaussian channels, the channel estimation error variance depends on the received pilot signal and is, in general, different from the average error variance. Whereas for Gaussian distributed channels, which have been predominantly studied in the literature, the channel estimation error variance conditioned on the observed pilot signal coincides with the average error variance. We provide an example using the AF dual-hop channel to exhibit the numerical difference between the true capacity lower bound and that obtained by using the average instead of the conditional error variance.**

***Index Terms*—Amplify-and-forward relaying, dual-hop channel, channel estimation, capacity lower bound, Non-Gaussian channels, pilot-symbol-assisted modulation.**

I. Introduction

The use of relayed transmission increases the communication range and reduces the need for high power levels at the transmitter [1]. Studies on cooperative transmission also show that the use of relays provides spatial diversity gains in wireless communication systems [2]. A summary of relaying strategies was provided in [3], among which the amplify-and-forward (AF) and decode-and-forward (DF) schemes have been extensively studied in the past few years, especially in resource-constrained scenarios [4]. However, early studies on relayed transmissions have assumed that perfect channel state information (CSI) is available, at least at the destination node.

Recently, the design of channel estimation methods for AF relaying systems has drawn considerable attention. For example, [5]–[7] studied the bit and symbol error performance of the dual-hop (source-relay-destination) relayed channel, as well as cooperative transmission, under various imperfect CSI conditions. With AF relaying, the overall dual-hop channel $g = g_1 g_2$ contributes to a non-Gaussian channel, even when the individual channels (the source-relay channel g_1 and the relay-destination channel g_2) are assumed to be complex Gaussian distributed. In the absence of perfect CSI at the destination node about g_1 and g_2 individually, the dual-hop channel g can be estimated using pilot symbols that are periodically sent by the source, amplified by the relay and hence, travel the whole source-relay-destination link. The imperfect CSI at the destination will undoubtedly impact the achievable information rates in the system.

Since the acclaimed work [8] for lower bounding the capacity of direct-link (non-relayed) wireless channels with imperfect CSI, there has been considerable activity in extending and employing such bounds for system design and performance evaluation (*e.g.*, [9], [10]). With the growing interest in relayed wireless transmission, it is interesting to determine what information rates are achievable using AF relaying with channel estimation errors. In this context, the main contributions of this correspondence are summarized as follows:

- We provide a step-by-step derivation of a capacity lower bound for the AF dual-hop relay channel, which explicitly takes into account the non-Gaussianity of the channel and its estimate and the fact that in general, the channel estimation minimum mean-square error (MMSE) variance does depend on the received pilot signal.
- As a result of this derivation, we are able to draw a clear and general distinction between lower bounding the capacity of Gaussian channels with that of non-Gaussian channels. In the former case, the average and conditional channel estimation MMSE variances coincide. This is not necessarily true for non-Gaussian channels. As a result, extra conditioning and averaging on the received pilot signal is required in the latter. Such conditioning and averaging has not been explicitly made in the literature before.

The results obtained in this paper will serve as a basis for bounding the information capacity of other non-Gaussian channels with channel estimation errors.

II. System Model

We consider a dual-hop wireless communication system where a source node (SN) communicates with a destination node (DN) via a relay node (RN). Each node is equipped with a single transmit and receive antenna. The relay node uses amplify-and-forward (AF) protocol [11]. At the DN, the cascade channel SN $\rightarrow$ RN $\rightarrow$ DN is estimated using the pilot symbol transmission in the first time slot of each transmission

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block.[1] We assume that RN does not estimate the channel and only amplifies and forwards the received signal to the DN.

In the pilot transmission mode, the signal received at the DN can be written as

$$\begin{aligned} y_p &= A_p g_1 g_2 p + A_p g_2 n_1 + n_2 \\ &= A_p g p + A_p g_2 n_1 + n_2, \end{aligned} \tag{1}$$

where p is the pilot symbol transmitted from the SN with the power $P_{P,S} = |p|^2$, g_1 and g_2 are the SN $\to$ RN and RN $\to$ DN channel gains which are modeled as zero-mean complex Gaussian random variables with variance σ_1^2 and σ_2^2, respectively, n_1 and n_2 are the zero mean complex additive white Gaussian noise (AWGN) random processes at the RN and the DN each with variance σ_n^2, A_p is the gain of the RN during the pilot transmission and $g = g_1 g_2$ is the cascade gain of the dual-hop channel. Throughout the paper we assume that g_1, g_2, n_1 and n_2 are independent of each other. It is noted that the cascade channel g is non-Gaussian [12] and its statistical properties are quite different from a single-hop complex Gaussian channel.

In the data transmission mode, the received signal at the DN can be written as

$$y = A_d g x + A_d g_2 n_1 + n_2, \tag{2}$$

where x is the transmitted data symbol from the SN with an average power of $P_{D,S} = E\{|x|^2\}$ and A_d is the gain of the RN during the data transmission. In this paper, we assume that the RN employs fixed gain amplification and the gains for the two transmission modes are given by

$$A_p = \sqrt{\frac{P_{P,R}}{P_{P,S}\sigma_1^2 + \sigma_n^2}} \quad \text{and} \quad A_d = \sqrt{\frac{P_{D,R}}{P_{D,S}\sigma_1^2 + \sigma_n^2}}, \tag{3}$$

where $P_{P,R}$ and $P_{D,R}$ are the average power transmitted by the RN during pilot and data transmission, respectively. It is noted that with the fixed gain implementation, the relay gain does not depend on the instantaneous channel state of the SN $\to$ RN link, but on its variance σ_1^2. At the DN, the cascade channel g is estimated using the received pilot signal y_p and the known pilot symbol p. The estimate of g is denoted by $\hat{g} = f(y_p, p)$ with the estimation error given by $\tilde{g} = g - \hat{g}$. The proper type of estimation for obtaining a valid capacity lower bound will be discussed in the next section.

III. A Capacity Lower Bound

The exact capacity expressions for direct link (non-relayed) communication systems with channel estimation errors are not available. In the case of direct link Gaussian channels, a commonly-used lower bound on the capacity was given in [10] for multiple-antenna systems which uses pilot symbols for channel estimation. We wish to further investigate how to apply these bounding techniques to non-Gaussian channels, *e.g.*, the cascade channel in dual-hop communications. In the following, we provide a step-by-step derivation of the lower bound under the system model explained in Section II.

The average mutual information between the channel input and its output conditioned on the received pilot signal y_p for a given transmitted pilot symbol p is written as

$$I(Y; X|Y_p) = h(X|Y_p) - h(X|Y, Y_p), \tag{4}$$

where $h(\cdot)$ denotes the differential entropy. Here we assume that the input data symbols are drawn from a zero-mean complex Gaussian distribution with variance $P_{D,S}$ independent of the received pilot signal.[2] Therefore, $h(X|Y_p) = \ln(2\pi e P_{D,S})$. Using a similar approach to that in [8], we obtain an upper bound on $h(X|Y, Y_p)$ as

$$\begin{aligned} h(X|Y, Y_p) &= h(X - \alpha Y|Y, Y_p) \\ &\leq h(X - \alpha Y|Y_p) \qquad (5) \\ &\leq E_{|Y_p}\Big\{ \ln(2\pi e \mathrm{Var}\{X - \alpha Y|Y_p\} \Big\}, \qquad (6) \end{aligned}$$

where $\mathrm{Var}\{\cdot\}$ denotes variance of a random variable, expectation $E_{|Y_p}\{\cdot\}$ is over all realizations of y_p and α can be any constant or a function of y_p. Eq. (5) is obtained using the fact that conditioning reduces the entropy, and (6) is obtained from the fact that among all distributions with a given variance, Gaussian distribution maximizes the differential entropy function. Note that (6) also holds when we minimize the RHS over α.

The conditional variance in (6) can be written explicitly as a function of y_p

$$\begin{aligned} \mathrm{Var}\{X - \alpha Y|Y_p = y_p\} &= E\{|X - \alpha Y|^2|Y_p = y_p\} \\ &\quad - |E\{X - \alpha Y|Y_p = y_p\}|^2 \\ &= E\{|X|^2|Y_p = y_p\} \\ &\quad - \alpha^* E\{XY^*|Y_p = y_p\} \\ &\quad - \alpha E\{X^* Y|Y_p = y_p\} \\ &\quad + |\alpha|^2 E\{|Y|^2|Y_p = y_p\}. \end{aligned} \tag{7}$$

From (7) we observe that in order to compute a valid capacity lower bound, one needs to compute $E\{|Y|^2|Y_p = y_p\}$ which is the power of the received signal during data transmission conditioned on the received pilot signal y_p. Expanding $E\{|Y|^2|Y_p = y_p\}$ and using the short form $G = G_1 G_2$ yields

$$\begin{aligned} E\{|Y|^2|Y_p = y_p\} =& A_d^2 P_{D,S} E\{|G|^2|Y_p = y_p\} \\ &+ A_d^2 \sigma_n^2 E\{|G_2|^2|Y_p = y_p\} + \sigma_n^2. \end{aligned}$$

This shows that computation of $E\{|G|^2|Y_p = y_p\}$ and $E\{|G_2|^2|Y_p = y_p\}$ is an essential part of the lower bounding technique. Furthermore, for a given relay gain A_d and received pilot signal y_p, one can optimize α to minimize the conditional variance in (7) (and hence obtain the tightest lower bound) as follows

$$\begin{aligned} \alpha^\star =& \frac{E\{XY^*|Y_p = y_p\}}{E\{|Y|^2|Y_p = y_p\}} \\ =& \frac{A_d P_{D,S} E\{G|Y_p = y_p\}}{A_d^2 P_{D,S} E\{|G|^2|Y_p = y_p\} + A_d^2 \sigma_n^2 E\{|G_2|^2|Y_p = y_p\} + \sigma_n^2}. \end{aligned} \tag{8}$$

[1]For simplicity, we assume that there is no direct link between the SN and the DN and the channel in each link follows an independent block fading model in which the realization of the fading channel is constant during a transmission block of L symbols and changes to an independent value in the next transmission block. However, the derivation can be modified easily to include other cases as well.

[2]The optimal input distribution is generally unknown. However, Gaussian input distribution will prove to be convenient for deriving the capacity lower bound.

$$\begin{aligned} I(Y;X|Y_p) &\geq E_{|Y_p}\left\{\ln\left(\frac{P_{D,S}}{\mathrm{Var}\{X-\alpha Y|Y_p\}}\right)\right\} \\ &= E_{|Y_p}\left\{\ln\left(1+\frac{A_d^2 P_{D,S}|E\{G|Y_p=y_p\}|^2}{A_d^2 P_{D,S}\mathrm{Var}\{\tilde{G}|Y_p=y_p\}+A_d^2\sigma_n^2 E\{|G_2|^2|Y_p=y_p\}+\sigma_n^2}\right)\right\} \\ &= \int_{\mathbb{C}} \ln\left(1+\frac{A_d^2 P_{D,S}|E\{G|Y_p=y_p\}|^2}{A_d^2 P_{D,S}\mathrm{Var}\{\tilde{G}|Y_p=y_p\}+A_d^2\sigma_n^2 E\{|G_2|^2|Y_p=y_p\}+\sigma_n^2}\right) f_{Y_p}(y_p)dy_p, \end{aligned} \tag{10}$$

According to (8) in order to obtain the tightest capacity lower bound, one should perform MMSE estimation of the channel during pilot mode, which is $\hat{g} = E\{G|Y_p = y_p\}$, and not any other estimation method. The corresponding channel estimation error variance conditioned on the received pilot signal is given by

$$\begin{aligned} \mathrm{Var}\{\tilde{G}|Y_p=y_p\} =& E\{|G|^2|Y_p=y_p\} - |E\{G|Y_p=y_p\}|^2 \\ =& E\{|G|^2|Y_p=y_p\} - |\hat{g}|^2. \end{aligned}$$

In the next subsection, we will see that the fundamental difference between lower bounding Gaussian and non-Gaussian channels arises from the requirement for MMSE channel estimation. For Gaussian channels, MMSE becomes identical to linear MMSE (LMMSE) and the channel estimation error variance becomes independent of the actual realization of y_p. This is generally not the case for non-Gaussian channels.

By substituting (8) in (7), we obtain

$$\begin{aligned} \mathrm{Var}\{X-\alpha Y|Y_p=y_p\} \geq& \mathrm{Var}\{X-\alpha^\star Y|Y_p=y_p\} \\ =& P_{D,S} - \alpha^\star E\{X^* Y|Y_p=y_p\} \\ =& P_{D,S} - \alpha^\star A_d P_{D,S} E\{G|Y_p=y_p\} \\ =& P_{D,S} - \frac{A_d^2 P_{D,S}^2 |E\{G|Y_p=y_p\}|^2}{\Gamma_{y_p}}, \end{aligned} \tag{9}$$

where

$$\begin{aligned} \Gamma_{y_p} =& A_d^2 P_{D,S} E\{|G|^2|Y_p=y_p\} \\ &+ A_d^2\sigma_n^2 E\{|G_2|^2|Y_p=y_p\} + \sigma_n^2. \end{aligned}$$

Finally, substituting (9) in (6) and then the result in (4), we have the capacity lower bound (10) at the top of the page, where $\mathrm{Var}\{\tilde{G}|Y_p = y_p\} = E\{|G|^2|Y_p = y_p\} - |E\{G|Y_p = y_p\}|^2$ is the variance of the channel estimation error conditioned on pilot transmission and $f_{Y_p}(y_p)$ is the probability distribution function of y_p.

Recently, the authors in [13] have derived capacity lower bounds of AF relay channels with channel estimation errors. One key distinction between our work and [13] is that in [13] it is assumed that the destination knows estimates of the SN $\rightarrow$ RN channel $\hat{g}_1$ and the RN $\rightarrow$ DN channel $\hat{g}_2$ individually, where the former is somehow forwarded reliably from the relay to the destination. Since each individual component of channel is assumed to be Gaussian and is estimated separately, the authors do not consider deriving the capacity lower bound for the overall non-Gaussian channel $g = g_1 g_2$. We, however, assume that no estimate about g_1 is available at the destination and the relay does not send its own pilot for separately estimating g_2 either, both of which would require transmit time and power. To the best of the authors' knowledge, this is the first proved capacity lower bound for dual-hop non-Gaussian channels with imperfect channel state information. Our derived capacity lower bound requires computation of the following terms and can only be numerically evaluated.

$$f(y_p) = \int_{\mathbb{C}} f(y_p|g_2)f(g_2)\mathrm{d}g_2$$

$$E\{|G_2|^2|Y_p=y_p\} = \frac{1}{f(y_p)}\int_{\mathbb{C}} |g_2|^2 f(y_p|g_2)f(g_2)\mathrm{d}g_2$$

$$\begin{aligned} E\{G|Y_p=y_p\} =& \frac{1}{f(y_p)} \\ &\times \int_{\mathbb{C}}\int_{\mathbb{C}} g_1 g_2 f(y_p|g_1,g_2)f(g_1)f(g_2)\mathrm{d}g_1\mathrm{d}g_2 \\ E\{|G|^2|Y_p=y_p\} =& \frac{1}{f(y_p)} \\ &\times \int_{\mathbb{C}}\int_{\mathbb{C}} |g_1|^2|g_2|^2 f(y_p|g_1,g_2)f(g_1)f(g_2)\mathrm{d}g_1\mathrm{d}g_2, \end{aligned}$$

where we have used the short form $f(x)$ for the probability distribution function $f_X(x)$ of the random variable X. $f(y_p|g_2)$, $f(g_1)$, $f(g_2)$ and $f(y_p|g_1, g_2)$ are all probability density functions of the complex Gaussian random variables defined in (1) with known means and variances.

A. *Capacity lower bound: single-hop non-Gaussian channel*

For a traditional single-hop wireless fading channel, we can write the observation equation as $y = gx + n$, where the channel g is randomly distributed (not necessarily Gaussian) with variance σ_1^2 and n is AWGN with variance σ_n^2. An example of single-hop wireless channel with non-Gaussian gain g can be found in two-way training systems [14]. Removing the unnecessary terms due to g_2 from (10), we obtain the average capacity lower bound for this wireless channel as

$$C_{LB} = \int_{\mathbb{C}} \ln\left(1+\frac{P_{D,S}|E\{G|Y_p=y_p\}|^2}{P_{D,S}\mathrm{Var}\{\tilde{G}|Y_p=y_p\}+\sigma_n^2}\right) f(y_p)dy_p. \tag{11}$$

One can show that for the special case of a Gaussian channel with pilot observation signal $y_p = gp + n$, the optimal MMSE estimate $\hat{g} = E\{G|Y_p = y_p\}$ reduces to $\hat{g} = K_p y_p$ which is the LMMSE estimate with the LMMSE factor K_p. Hence, there is a linear one-to-one correspondence between $\hat{g}$ and y_p. Moreover, the error variance conditioned on the received pilot signal $\mathrm{Var}\{\tilde{G}|Y_p = y_p\}$ can be written as

$$E\{|G - K_p y_p|^2 | Y_p = y_p\} = \frac{\sigma_n^2 \sigma_1^2}{\sigma_1^2 |p|^2 + \sigma_n^2} \triangleq \sigma_{\tilde{G}}^2,$$

which becomes *statistically independent*[3] of any realization of y_p. Therefore, the error variance $E\{|G - K_p y_p|^2\}$ averaged over all realizations of the received pilot signal equals the error variance for a given y_p. The capacity lower bound for the Gaussian channel reduces to

$$C_{LB} = \int_{\mathbb{C}} \ln\left(1 + \frac{P_{D,S}|\hat{g}|^2}{P_{D,S}\sigma_{\tilde{G}}^2 + \sigma_n^2}\right) f_{\hat{G}}(\hat{g}) d\hat{g}. \qquad (12)$$

Remark: We comment on an earlier derivation of the capacity lower bound for single-hop multiple-antenna channels [10]. In [10], it is stated that the derived capacity lower bound (equation (15) in [10]) does not require the channel to be Gaussian. Based on this, one might be led to think that one can apply the derived bounds in [10] to the AF channels in a straightforward manner. However, three important factors are not made explicit in the formulation of [10], which make that formulation effectively suitable for Gaussian channels only. The first is that the dependence of the channel estimate $\hat{g}$ on the received pilot signal through MMSE estimate $\hat{g} = E\{G|Y_p = y_p\}$ is not explicit in (15) in [10] (here we have slightly adapted notations in [10] to match those in this paper). Second, $\sigma_{\tilde{G}}^2$ appears in the lower bound as the MMSE error variance. Again, the dependence of $\sigma_{\tilde{G}}^2$ on the received pilot signal y_p in the form of $\mathrm{Var}\{\tilde{G}|Y_p = y_p\}$ is not explicit. Hence, one might be led to use the average error variance $E_{|Y_p}\{\mathrm{Var}\{\tilde{G}|Y_p = y_p\}\}$ instead of the true conditional variance $\mathrm{Var}\{\tilde{G}|Y_p = y_p\}$. Finally, the expectation for obtaining the ergodic capacity lower bound does not specify the random variable for expectation and seems to be on the only random variable present in the formulation, which is $\hat{g}$.

In short, the lower bound in (12) is only applicable to Gaussian channels. For non-Gaussian channels, however, one needs to explicitly deal with the formulation given in (11) with the expectation being over the distribution of the received pilot signal y_p and modify it accordingly for multi-hop or other types of more complicated channels, as seen by the AF relay channel example in (10).

IV. Numerical Results

As an example, we consider the single-input single-output (SISO) dual-hop wireless communications setup described in Section II. Here we assume g_i, $i = 1, 2$, is zero mean complex Gaussian distributed with unit variance. For simplicity, we further assume that equal power is allocated to both pilot and data symbols (i.e., $P_{D,S} = P_{P,S} = P$). Amplification gains A_p and A_d are calculated using (3) assuming noise variance $\sigma_n^2 = 1$.

Fig. 1 depicts the correct capacity lower bound in (10) and that obtained by using the average error variance *as if* the channel was Gaussian, versus the SNR budget P for the dual-hop channel described above. From Fig. (1), it can be clearly observed that the formulation based on the average error variance overestimates the one obtained using the correct capacity lower bound, which considers the conditional error variance. The gap is more noticeable at low to moderate SNR. This numerical example highlights that although the use of average error variance can greatly simplify dealing with the lower bound, the obtained results should be treated with care.

[3]In the Gaussian case, the MMSE variance depends on the received signal y_p through its power or the second moment $(\sigma_1^2|p|^2 + \sigma_n^2)$, but not through the actual realization of y_p.

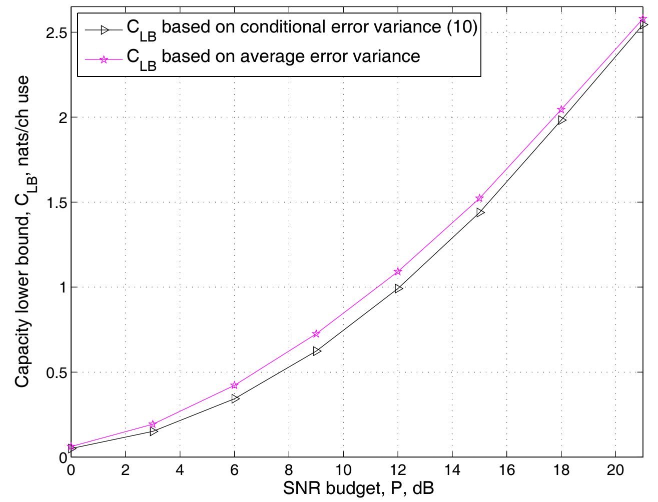

Fig. 1. Comparison of the proved capacity lower bound (10) for a dual-hop channel based on the conditional channel estimation error variance and that obtained based on average channel estimation error variance.

V. Conclusion

A lower bound on the capacity of dual-hop wireless relay channels with AF relaying was derived by taking into account the channel estimation errors at the destination node. Through a step-by-step derivation, we highlighted the impact of non-Gaussian distribution of the AF channel on the capacity lower bound and contrasted that with the case of widely-studied Gaussian channels. In particular, we showed that the MMSE channel estimation error variance conditioned on the observed pilot signal should be explicitly used in the capacity lower bound formulation of non-Gaussian channels. Using a numerical example, we showed that ignoring this dependence and using the average MMSE channel estimation error variance (as if the channel was Gaussian) can result in overestimation of the capacity lower bound.

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