Optimizing Antenna Configuration for MIMO Systems with Imperfect Channel Estimation

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***Abstract*—We study the optimal antenna configuration (i.e. number of transmit and receive antennas) for multiple-input multiple-output systems in pilot-symbol-assisted modulation schemes with imperfect channel estimation. We assume block flat-fading channels and focus on a practical range of high signal-to-noise ratio. An ergodic capacity lower bound is used as the objective function to be maximized. We analytically study the capacity gain from adding extra antennas to the transmitter or to the receiver in two different scenarios. Our numerical results show that the optimal antenna configuration under imperfect channel estimation can be significantly different from that under perfect channel estimation assumption. In addition, we investigate the capacity gain from optimizing antenna configuration and find that the gain can be larger than that achieved by optimizing transmit power over pilot and data symbols, particularly for large block lengths.**

***Index Terms*—Information capacity, multiple-input multiple-output, pilot-symbol-assisted modulation, channel estimation errors.**

I. INTRODUCTION

THE use of multiple transmit and receive antennas dramatically increases the information capacity in wireless communication systems [1, 2]. The optimal number of transmit and receive antennas has been studied in [3–5] assuming perfect channel state information at the receiver (CSIR). In particular, the authors in [4] studied the situation where one extra antenna is available to be allocated at either end of a multiple-input multiple-output (MIMO) system. Their results show that one should always allocate the extra antenna to the side with less number of antennas at high signal-to-noise ratio (SNR). However, the optimal antenna configuration in the perfect CSIR case may not be applicable to systems with imperfect CSIR.

In pilot-symbol-assisted modulation (PSAM) schemes, training symbols are inserted into data blocks to enable channel estimation [6–8] at the receiver. The authors in [8] find that the capacity decreases as the number of transmit antennas increases beyond the number of receive antennas at sufficiently high SNR. However, it is interesting to investigate the applicability of this asymptotic result at a practical range of high SNR, e.g. from 20 dB to 30 dB.

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In this letter, we study the optimal antenna configuration at the aforementioned practical range of high SNR in PSAM schemes from capacity maximization viewpoint. In particular, we extend the analysis in [4] and [8] to investigate the following two important problems for systems with imperfect CSIR and practical antenna sizes.

- Problem 1: If an extra antenna is available to be added at either end of a MIMO system, should one add it to the transmitter or to the receiver?
- Problem 2: If it is only practical to change the number of transmit antennas, what is the optimal number of transmit antennas?

We show that under imperfect channel estimation in PSAM schemes, the solution to Problem 1 heavily depends on the block length, and is particularly different from the solution in [4][1] for small block lengths. When studying Problem 2, we also investigate the capacity gain from optimizing antenna configuration. Our results show that optimizing antenna configuration could result in a noticeable capacity improvement and it is more beneficial than power optimization over pilot and data symbols at large block lengths.

II. SYSTEM MODEL

We consider MIMO systems with N_t transmit antennas and N_r receive antennas in block flat-fading channels. The $N_r \times 1$ received symbol vector at time ℓ is given by $\boldsymbol{y}_\ell = \boldsymbol{H}\boldsymbol{x}_\ell + \boldsymbol{w}_\ell$, where $\boldsymbol{x}_\ell$ is the $N_t \times 1$ transmitted symbol vector, $\boldsymbol{w}_\ell$ is the $N_r \times 1$ noise vector. The noise at each receive antenna is independent, identically distributed (i.i.d.) and zero-mean circularly symmetric complex Gaussian (ZMCSCG), each with variance $\sigma_{\boldsymbol{n}}^2$. $\boldsymbol{H}$ is the $N_r \times N_t$ channel gain matrix with i.i.d. and unit variance ZMCSCG entries. Furthermore, each transmission block of T symbols consists of T_p pilot symbols followed by $T - T_p$ data symbols. The receiver performs pilot-assisted channel estimation using linear minimum mean square error estimator. The channel estimate and the estimation error are denoted by $\hat{\boldsymbol{H}}$ and $\tilde{\boldsymbol{H}}$, respectively.

A. A Capacity Lower Bound

For MIMO systems with imperfect CSIR, the exact capacity expression is still unavailable. We consider a lower bound for

[1] Under perfect channel estimation, the solution in [4] does not depend on the block length.

the ergodic capacity per transmission, given by [8,9]

$$\begin{aligned} C_{\mathrm{LB},\ell}(N_t,N_r) &= E_{\hat{\boldsymbol{H}}}\Big\{\log_2\Big|\boldsymbol{I}+\frac{\mathcal{P}_d}{\sigma_e^2\mathcal{P}_d+\sigma_{\boldsymbol{n}}^2}\frac{\hat{\boldsymbol{H}}\hat{\boldsymbol{H}}^\dagger}{N_t}\Big|\Big\},\\ &= E_{\boldsymbol{H}}\Big\{\log_2\Big|\boldsymbol{I}+\frac{\mathcal{P}_d(1-\sigma_e^2)}{\sigma_e^2\mathcal{P}_d+\sigma_{\boldsymbol{n}}^2}\frac{\boldsymbol{H}\boldsymbol{H}^\dagger}{N_t}\Big|\Big\}, \quad (1)\end{aligned}$$

where $T_p+1 \le \ell \le T$, $\mathcal{P}_d$ is the total power per data transmission which is uniformly divided into N_t antennas, the estimation error variance is $\sigma_e^2 = E\{\mathrm{tr}\{\tilde{\boldsymbol{H}}\tilde{\boldsymbol{H}}^\dagger\}\}/(N_tN_r)$, and $\rho_{\mathrm{eff}} \triangleq \mathcal{P}_d(1-\sigma_e^2)/(\sigma_e^2\mathcal{P}_d+\sigma_{\boldsymbol{n}}^2)$ is referred to as the effective SNR [8]. The corresponding lower bound for the average ergodic capacity per transmission block is given by

$$C_{\mathrm{LB}}(N_t,N_r) = \frac{T-T_p}{T}C_{\mathrm{LB},\ell}(N_t,N_r). \quad (2)$$

B. PSAM Transmission Scheme

In this subsection, we provide a summary of the important PSAM design results in [8], which will be used in our analysis. For a given block length T, the main PSAM design parameters are the pilot sequence, the ratio of power allocated to pilots and data, and the number of pilot symbols per block.

The optimal pilot sequence has the orthogonality property among transmit antennas. With this choice of pilots, it was shown that the channel estimation error variance can be reduced to $\sigma_e^2 = 1/(1+\mathcal{P}_pT_p/N_t)$, where $\mathcal{P}_p$ is the total power per pilot transmission which is uniformly divided into N_t antennas. We will assume this optimal pilot sequence throughout our analysis.

Two power allocation schemes are considered. The first scheme, called equal power allocation, transmits each pilot and data symbol with equal power. At high SNR, the effective SNR in the equal power allocation scheme is given by $\rho_{\mathrm{eff}} = \rho/(1+N_t/T_p)$ where ρ denotes the expected SNR at each receive antenna. The second scheme, called optimal power allocation, aims to maximize the average ergodic capacity lower bound in (2). At high SNR, the effective SNR in the optimal power allocation scheme is given by $\rho_{\mathrm{eff}} = \frac{\rho T}{(\sqrt{T-T_p}+\sqrt{N_t})^2}$.

The optimal number of pilots per block, denoted by $T_{p,\mathrm{opt}}$, depends on the power allocation scheme used. For equal power allocation, $T_{p,\mathrm{opt}}$ ranges from $T/2$ at $\rho=-\infty$ to N_t at $\rho=\infty$. For optimal power allocation, $T_{p,\mathrm{opt}} = N_t$ for all values of T and ρ.

III. Optimal Antenna Configuration at High SNR

In this section, we investigate the optimal antenna configuration at practical high SNR regime for MIMO systems with imperfect CSIR. Firstly, we obtain a closed-form approximation of (1) and (2) to ease our analysis.

For $N_t \ge N_r$, the ergodic capacity lower bound per transmission in (1) is approximated as

$$\begin{aligned} C_{\mathrm{LB},\ell}(N_t,N_r) &\approx E_{\boldsymbol{H}}\Big\{\log_2\Big|\rho_{\mathrm{eff}}\frac{\boldsymbol{H}\boldsymbol{H}^\dagger}{N_t}\Big|\Big\},\\ &= N_r\log_2\rho_{\mathrm{eff}} - N_r\log_2 N_t\\ &\quad +\frac{1}{\ln 2}\Big(\sum_{j=1}^{N_r}\sum_{k=1}^{N_t-j}\frac{1}{k}-N_r\gamma\Big), \quad (3)\end{aligned}$$

where $\gamma \approx 0.577$ is the Euler's constant and the last term in (3) equals $E_{\boldsymbol{H}}\{\log_2|\boldsymbol{H}\boldsymbol{H}^\dagger|\}$ (see [4]). The average capacity lower bound in (2) is therefore approximated by

$$\begin{aligned} C_{\mathrm{LB}}(N_t,N_r) \approx \frac{T-T_p}{T}\Big[&N_r\log_2\rho_{\mathrm{eff}} - N_r\log_2 N_t\\ &+\frac{1}{\ln 2}\Big(\sum_{j=1}^{N_r}\sum_{k=1}^{N_t-j}\frac{1}{k}-N_r\gamma\Big)\Big]. \quad (4)\end{aligned}$$

Similarly, for $N_t < N_r$, the average capacity lower bound in (2) can be approximated by

$$\begin{aligned} C_{\mathrm{LB}}(N_t,N_r) \approx \frac{T-T_p}{T}\Big[&N_t\log_2\rho_{\mathrm{eff}} - N_t\log_2 N_t\\ &+\frac{1}{\ln 2}\Big(\sum_{j=1}^{N_t}\sum_{k=1}^{N_r-j}\frac{1}{k}-N_t\gamma\Big)\Big]. \quad (5)\end{aligned}$$

It was shown in [4] under perfect CSIR that the approximations are accurate for $\rho \ge 20$ dB, which is also confirmed by our simulations. For example, we have compared the average capacity lower bound in (2) with its approximation in (4) for a 4×4 MIMO system with optimal pilot length $T_{p,\mathrm{opt}}$ and block length $T=50$. For both equal power allocation and optimal power allocation, the difference between the bounds and their approximations is around 5% at 20 dB, and less than 1% at 30 dB. Therefore, the closed-form approximations in (4) and (5) are accurate for the SNR range of interest. Hence, they will be used in our analysis.

A. Solution to Problem 1

Now, we investigate the first problem: *If an extra antenna is available to be added on either end of a MIMO system, should one add it to the transmitter or to the receiver?* This question is relevant in the design of point-to-point MIMO wireless links with fixed total number of antennas [4]. It may also occur in on-the-fly link adaptation, e.g. IEEE802.11n. Due to space limitation, we will only present the analysis for the optimal power allocation. The result for the equal power allocation is similar, hence is omitted.

We start with the case where $N_t < N_r$. Using (5), we compute the capacity difference between the systems having (N_t, N_r+1) and (N_t+1, N_r), i.e., $\delta C_{\mathrm{LB}} \triangleq C_{\mathrm{LB}}(N_t,N_r+1) - C_{\mathrm{LB}}(N_t+1,N_r)$ as

$$\begin{aligned}\delta C_{\mathrm{LB}} \approx \frac{1}{T\ln 2}\Big[&N_t(T-N_t)\ln\rho_{\mathrm{eff}} - (N_t+1)(T-N_t-1)\ln\rho'_{\mathrm{eff}}\\ &+N_t(T-N_t)\ln(1+\frac{1}{N_t}) - (2N_t-T+1)\ln(N_t+1)\\ &+(T-N_t)\sum_{j=1}^{N_t}\frac{1}{N_r+1-j} - (T-N_t)\sum_{k=1}^{N_r-N_t-1}\frac{1}{k}\\ &+\sum_{j=1}^{N_t+1}\sum_{k=1}^{N_r-j}\frac{1}{k} - (2N_t-T+1)\gamma\Big], \quad (6)\end{aligned}$$

where $\rho_{\mathrm{eff}} = \frac{T\rho}{T+2\sqrt{(T-N_t)N_t}}$ is the effective SNR for a (N_t, N_r+1) system, and $\rho'_{\mathrm{eff}} = \frac{T\rho}{T+2\sqrt{(T-N_t-1)(N_t+1)}}$ is the effective SNR for a (N_t+1, N_r) system. We can see that the sign of $(2N_t-T+1)$ plays an important role in (6). In the case where $2N_t-T+1 \ge 0$ or $T \le 2N_t+1$, we find that

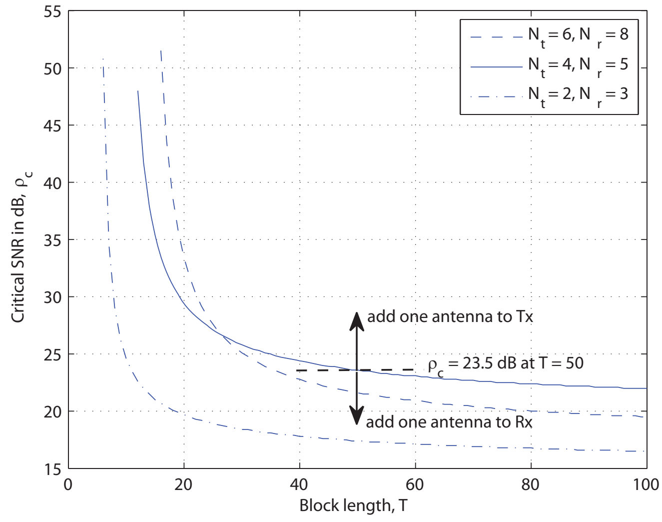

Fig. 1. The critical SNR found using (6) vs. block length in determining whether to add the extra antenna to the transmitter or to the receiver, for MIMO systems with different antenna configurations.

one should add the extra antenna to the receiver at moderate to high SNR. In the following, we focus on the more practical case where $T > 2N_t + 1$.

We call the SNR value at which $\delta C_{\text{LB}} = 0$ the critical SNR, denoted by ρ_c. It is the threshold SNR in determining at which end the extra antenna should be added. As the block length T approaches infinity in (6), we see that ρ_c approaches a limiting value, given by

$$\rho_{c,\infty}(N_t, N_r) = \exp\Big[N_t \ln(1 + \frac{1}{N_t}) + \ln(1 + N_t) + \sum_{j=1}^{N_t} \frac{1}{N_r + 1 - j} - \sum_{k=1}^{N_r - N_t - 1} \frac{1}{k} + \gamma\Big]. \quad (7)$$

<u>*Numerical Results*</u>: Fig. 1 shows the critical SNR ρ_c for a wide range of block length T, where $T > 2N_t + 1$. In this case, one should add the extra antenna to the transmitter if the operating SNR is above ρ_c for any given block length, and vice versa. For example, consider the case where ($N_t = 4, N_r = 5$) and $T = 50$. Fig. 1 suggests that one should add the extra antenna to the receiver when the operating SNR is below $\rho_c = 23.5$ dB, while one should add the antenna to the transmitter when the operating SNR is above $\rho_c = 23.5$ dB.

Furthermore, we see from Fig. 1 that $\rho_c \to \infty$ as $T \to 2N_t + 1$. This suggests that one should always add the extra antenna to the receiver when T is close to $2N_t + 1$. We also see that ρ_c decreases as T increases and approaches $\rho_{c,\infty}$ as $T \to \infty$. Therefore, $\rho_{c,\infty}$ serves as the infimum of ρ_c. This implies that one should always add the extra antenna to the receiver if the operating SNR is below $\rho_{c,\infty}$, regardless of the block length. For example, $\rho_{c,\infty} = 19$ dB for a ($N_t = 4, N_r = 5$) system. These important trends of the critical SNR are not attained from the analysis in [4] under the perfect CSIR assumption. Therefore, the result for perfect CSIR case cannot be directly applied to the systems with imperfect CSIR, particularly at small block lengths.

In the case where $N_t \geq N_r$, the same analysis can be carried out. We find that it is generally better to place the extra antenna at the receiver side at moderate to high SNR when $N_t \geq N_r$, regardless of the block length. This observation agrees with the result for the perfect CSIR case in [4].

B. Solution to Problem 2

Now, we investigate the second problem for both equal and optimal power allocation: *If it is only practical to increase or reduce the number of transmit antennas, what is the optimal number of transmit antennas*[2]*?*

Firstly we consider the case where $N_t \geq N_r$. An example of this case would be the downlink in mobile cellular networks, where it is practical to alter the number of antennas at the base station to maximize the data rate for every mobile user, possibly without significant cost. From (2) we see that $C_{\text{LB},\ell} \to C_{\text{LB}}$ as $T \to \infty$. Therefore, we start with the analysis on the lower bound of the ergodic capacity per transmission for large block lengths to gain some insights into the answer.

Using (3), the gain in the ergodic capacity lower bound per transmission from adding an extra transmit antenna, i.e., $\triangle C_{\text{LB},\ell} \triangleq C_{\text{LB},\ell}(N_t + 1, N_r) - C_{\text{LB},\ell}(N_t, N_r)$, is given by

$$\begin{aligned}\triangle C_{\text{LB},\ell} &\approx \frac{1}{\ln 2}\Big(\sum_{j=1}^{N_r} \frac{1}{N_t+1-j} - N_r \ln \frac{N_t+1}{N_t} + N_r \ln \frac{\rho'_{\text{eff}}}{\rho_{\text{eff}}}\Big) \\ &\geq \frac{N_r}{\ln 2}\Big(\frac{1}{N_t} - \ln \frac{N_t+1}{N_t} + \ln \frac{\rho'_{\text{eff}}}{\rho_{\text{eff}}}\Big),\end{aligned}$$

where ρ_{eff} and ρ'_{eff} are the effective SNR for a (N_t, N_r) system and a $(N_t + 1, N_r)$ system, respectively (expressions for ρ_{eff} and ρ'_{eff} can be found in Section II-B). It was found in [8] that the optimal pilot length $T_{p,\text{opt}} = N_t$ for optimal power allocation at any values of T, and $T_{p,\text{opt}} \gg N_t$ for equal power allocation as $T \to \infty$. Therefore, one can show that $\rho'_{\text{eff}} \approx \rho_{\text{eff}}$ as $T \to \infty$. Also, it is easy to show that $\frac{1}{N_t} - \ln \frac{N_t+1}{N_t} > 0$. Hence, we have $\triangle C_{\text{LB}} \approx C_{\text{LB},\ell} > 0$. This implies adding more transmit antennas always results in higher average ergodic capacity for both power allocation schemes at sufficiently large T. However, this is not true for not so large block lengths.

Now, we present the general results on the average ergodic capacity for finite block lengths. The gain in the average ergodic capacity lower bound in (4) from adding an extra transmit antenna, i.e., $\triangle C_{\text{LB}} \triangleq C_{\text{LB}}(N_t + 1, N_r) - C_{\text{LB}}(N_t, N_r)$, is given by

$$\begin{aligned}\triangle C_{\text{LB}} \approx \frac{1}{T \ln 2}\Big(&N_r \ln(N_t+1) + (T-N_t)\sum_{j=1}^{N_r} \frac{1}{N_t+1-j} \\ &+ N_r\gamma - N_r(T-N_t)\ln\frac{N_t+1}{N_t} - \sum_{j=1}^{N_r}\sum_{k=1}^{N_t+1-j}\frac{1}{k} \\ &+ N_r(T-N_t-1)\ln\rho'_{\text{eff}} - N_r(T-N_t)\ln\rho_{\text{eff}}\Big). \quad (8)\end{aligned}$$

[2]From information-theoretic viewpoint, it is always beneficial to add extra antennas at the receiver as it increases the diversity of the system without the need to sacrifice information symbols for training symbols.

TABLE I
OPTIMAL NUMBER OF TRANSMIT ANTENNAS AND ITS CORRESPONDING RANGE OF BLOCK LENGTH FOR SYSTEMS WITH 4 RECEIVE ANTENNAS

	Optimal N_t	4	5	≥ 6
Equal power allocation (ρ = 20 dB)	Range of T	8 to 15	16 to 30	$\geq$ 31
Optimal power allocation (ρ = 20 dB)	Range of T	8 to 16	17 to 38	$\geq$ 39
Equal power allocation (ρ = 30 dB)	Range of T	8 to 22	23 to 46	$\geq$ 47
Optimal power allocation (ρ = 30 dB)	Range of T	8 to 26	27 to 60	$\geq$ 61

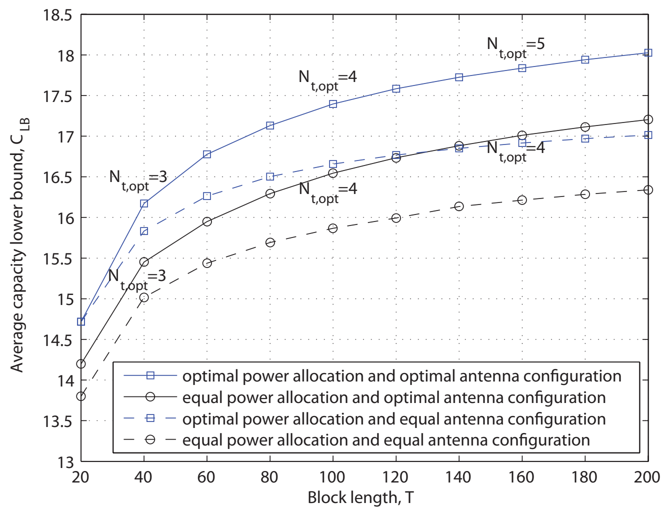

Fig. 2. The average capacity lower bound in (4) vs. block length for MIMO systems with 2 receive antennas at SNR $\rho = 30$ dB. Equal antenna configuration represents systems with 2 transmit and receive antennas.

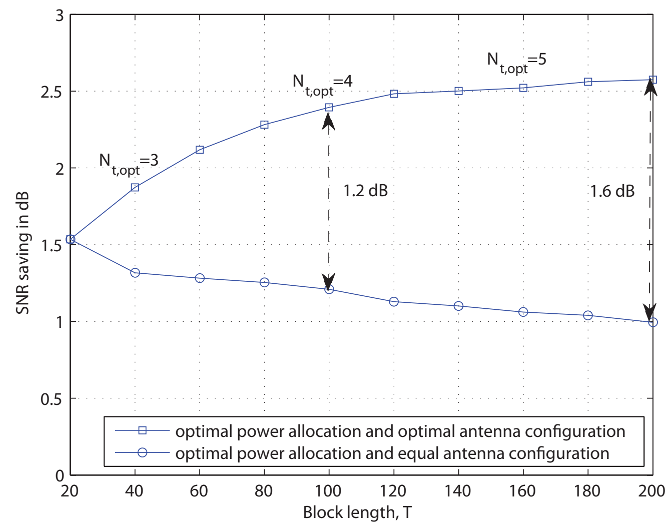

Fig. 3. The SNR saving computed from (4) vs. block length for MIMO systems with 2 receive antennas at SNR $\rho = 30$ dB. Equal antenna configuration represents systems with 2 transmit and receive antennas.

Numerical Results: Table I shows the optimal number of transmit antennas obtained from (8) by a linear search and its corresponding range of block length T for fixed SNR values, using $T_{p,\text{opt}}$ and $N_r = 4$. For optimal power allocation, $T_{p,\text{opt}} = N_t$. For equal power allocation, $T_{p,\text{opt}}$ is found numerically from (4). We have also restricted the minimum of T to be $2N_t$. We see that for both power allocation schemes the optimum value of N_t can exceed the value of N_r and increases with T at practical high SNR values. This result is not predicted in the asymptotic high SNR analysis in [8]. Therefore, the results presented here are more accurate and provide useful insights at practical high SNRs.

Fig. 2 shows the average capacity lower bound computed using (4) vs. block length for MIMO systems with $N_r = 2$ and at $\rho = 30$ dB. We include the capacity lower bounds achieved using the optimal power allocation and/or optimal number of transmit antennas, as well as the non-optimized case (i.e. the equal power allocation and equal number of transmit and receive antennas). The optimal pilot length $T_{p,\text{opt}}$ is used in all computations. Comparing the two solid or dashed curves in Fig. 2, we see that the capacity percentage improvement by optimal power allocation generally decreases as the block length increases. On the other hand, the capacity improvement from optimal antenna configuration increases as the block length increases. Comparing the two curves with square (or circle) type markers in Fig. 2, we see that the capacity improvement from optimal antenna configuration reaches approximately 4.5% (or 4.3%) at $T = 100$ and around 6% (or 5.3%) at $T = 200$.

Fig. 3 shows the SNR saving calculated from (4). It indicates the amount of transmit power saved by using the optimal power allocation and/or optimal antenna configuration to achieve the same capacity as in the non-optimized case at $\rho = 30$ dB. The optimal pilot length is used in all computations. From Fig. 3, we see that the SNR saving by optimal power allocation decreases as the block length T increases, while the SNR saving by optimal antenna configuration increases with T. For large block lengths, optimal antenna configuration generally saves more power than optimal power allocation. At $T = 100$, the additional SNR saving by antenna optimization is 1.2 dB, which equals the SNR saving by power optimization alone. At $T = 200$, the additional SNR saving by antenna optimization increases to 1.6 dB, while the SNR saving by power optimization alone decreases to 1 dB. These results show that optimizing antenna configuration is more important than optimizing power allocation from an information-theoretic viewpoint.

When $N_t < N_r$, the analysis of the optimal number of transmit antennas can be carried out in the same manner as in the case where $N_t \geq N_r$. Assuming $T \gg 1$, the result in [8] suggests that adding extra transmit antennas always improves the capacity, provided N_t does not exceed N_r and $\rho \to \infty$. Our analysis confirms that this claim is also accurate at practical range of high SNR and practical antenna sizes.

The optimal antenna configuration at moderate SNR can be studied using the average capacity lower bound in (2). We find that the optimal antenna configuration obtained at high SNR can be different from that at low to moderate SNR. Particularly at sufficiently low SNR, (2) can be approximated as $C_{\text{LB}} = \frac{T\rho^2}{4\ln 2}\frac{N_r}{N_t}$ for both power allocation schemes [8], which implies that the optimal number of transmit antenna is 1. However, the trends observed at high SNR on the capacity gain and SNR saving by optimizing antenna configuration are also observed at moderate SNR (For brevity, the numerical results and detailed discussions are not presented.) That is to say, the capacity improvement from optimal antenna configuration increases as the block length increases, and optimizing antenna configuration is more beneficial than optimizing power allocation over pilot and data symbols at large block lengths.

IV. Summary of Results

In this letter, we studied the optimal antenna configuration for MIMO systems in PSAM schemes from an information-theoretical viewpoint. We focused on a practical range of high SNR values and answered the two problems raised in Section I.

When an extra antenna is available to be placed on either end of the system, one should always place it at the receiver at moderate to high SNR when the existing number of transmit antennas N_t is at least as large as that of the receive antennas N_r. When $N_t < N_r$, a critical SNR value needs to be considered, below which the extra antenna should be placed at the receiver. The critical SNR decreases from infinity to some limiting value as the block length increases from $2N_t + 1$ to infinity.

When it is only practical to change the number of transmit antennas, adding extra antennas generally improves the capacity at high SNR and large block lengths, provided $N_t < N_r$. More importantly, the optimal number of transmit antennas can exceed the number of receive antennas, and the capacity improvement by optimal antenna configuration is significant when the block length is large. We also showed that optimizing antenna configuration can be more beneficial than power optimization over pilot and data symbols, particularly at large block lengths.

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