Enhancing CRDSA With Transmit Power Diversity for Machine-Type Communication

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***Abstract*—Contention resolution diversity slotted ALOHA (CRDSA) is a promising solution to meet the challenge of designing efficient random access in future wireless networks. In this paper, we consider CRDSA with transmit power diversity where each packet copy from a device is transmitted at a randomly selected power level. This results in interslot received power diversity, which is exploited by employing a signal-to-interference-plus-noise ratio based successive interference cancellation (SIC) receiver. Leveraging edge-weighted bipartite graph representation, we propose a novel graph-based message passing algorithm to model the SIC decoding. We derive an expression to characterize the recovery-error probability of the scheme. We also formulate and solve an optimization problem to determine the optimal transmit power distribution. The results show that by enhancing the capture effect, the optimal transmit power distribution leads to considerable performance improvement.**

***Index Terms*—Random access, CRDSA, machine-type communication, capture effect, SIC.**

I. Introduction

Motivated by the critical need to better support machine type communication (MTC) in future wireless networks [1], contention resolution diversity slotted ALOHA (CRDSA) [2] and its variants [3]–[5] have been proposed to enhance the performance of random access schemes [6]. The well known idea of CRDSA is to allow devices to transmit multiple copies (burst) of the packet in randomly selected slots within a frame and perform iterative successive interference cancellation (SIC) when attempting to resolve collisions. However, most of these variants are based on the clean packet model [2], [5] in which only interference-free packets are recoverable.

Very recently, efforts have been made to improve the performance of CRDSA by enhancing and exploiting the capture effect [7]–[10], i.e., a packet is recoverable if its signal-to-interference-plus-noise ratio (SINR) is above a predefined threshold. For instance, [8], [9] proposed to exploit the randomness in the channel gain to benefit from the capture effect. This is shown to support system load generally exceeding 1 packet/slot. Simulations are used in [8] to show that transmit power diversity (i.e., packet transmit power is randomly chosen by devices) is able to substantially enhance CRDSA performance. While the idea of transmit power diversity is common in wireless communications [11], to the best of our knowledge, no analytical framework has been proposed in the literature to date to analyse the impact of transmit power diversity on CRDSA.

Contributions: In this correspondence, we consider transmit power diversity with packet diversity in CRDSA which results in inter-slot received power diversity. We analyse the impact of transmit power diversity on CRDSA with SINR based iterative SIC decoding, in the asymptotic frame length regime. The novel contributions of this work are, (i) We describe the proposed system by an edge-weighted bipartite graph and develop a novel graph-based message passing algorithm to perform the iterative SIC based decoding process. (ii) We employ the AND-OR tree analysis to derive an expression to characterize the system performance. Simulation results confirm the accuracy of the derived expression. (iii) We formulate and solve an optimization problem to determine the optimal transmit power probability distribution. The results show that by maximizing the probability of the gap between the power levels of two copies transmitted in a given slot, the optimal transmit power distribution leads to considerable performance improvement. For instance, for only 2 copies per device per frame with only 5 power levels to choose from, the proposed scheme is shown to achieve a system load of 1.68 packets/slot, which shows its superiority to existing SIC based diversity slotted ALOHA methods.

II. Proposed Random Access Mechanism

System Model: We consider an uncoded slotted random access system with M machine-type communication devices (MTDs), which contend to access a single base station (BS). The time is divided into frames and each frame is divided into N equal duration slots. The MTDs are frame and slot synchronized, e.g., using global positioning system (GPS) or using periodic beacons transmitted by the BS [3]. Similar to existing works, we assume that (i) both the large-scale fading and small-scale fading coefficients are known perfectly to the BS. Hence, we do not consider fading in this work, (ii) the BS knows the number of MTDs M [2], [5] and (iii) each MTD generates a single packet, which fits in exactly one slot, for transmission in each frame.

Transmission Scheme: Similar to previous works on CRDSA, we assume that each MTD transmits d copies (i.e., bursts) of its single packet in randomly and uniformly selected slots within one frame, where $1 \leq d \leq D$ and D is the maximum number of allowable copies.

Different from existing works on CRDSA, in addition to the packet diversity due to the copies, we allow *transmit power diversity*. Hence, each MTD randomly chooses a transmit power level, E_l, for each of its copies from the set of equally spaced L available power levels, denoted by $\mathcal{E} = \{E_1, E_2, \ldots, E_L\}$, where $1 \leq l \leq L$. We allow selection of transmit power levels with replacement, i.e., two copies from a MTD may be transmitted on the same power level.

Due to slot-level synchronization, copies of packets from different MTDs either completely overlap (i.e., collide) or not at all. Therefore, a single slot may contain collided copies of packets from k MTDs, where $1 \leq k \leq M$. A collision free slot is called a singleton slot and the packet is called a clean packet.

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Let Λ_d denote the probability that a MTD transmits d copies, Ψ_k denote the probability that k MTDs choose to transmit a copy on any given single slot and Γ_l denote the probability that a MTD chooses power level E_l for a given copy. According to [5], the polynomial representations for the probabilities Ψ_k, Λ_d, and Γ_l are

$$\Psi(z) \triangleq \sum_k \Psi_k z^k,\ \Lambda(z) \triangleq \sum_d \Lambda_d z^d,\ \Gamma(z) \triangleq \sum_l \Gamma_l z^l. \quad (1)$$

Recovery Mechanism: The received signal at the BS in the nth slot is given by

$$y_n = \mathbf{e_n} \mathbf{x_n} + \omega_n, \quad (2)$$

where $n = 1, 2, \ldots, N$ is the slot index, $m = 1, 2, \ldots, M$ is the user index, $\mathbf{e_n} = [\sqrt{e_{1,n}}, \ldots, \sqrt{e_{m,n}}, \ldots, \sqrt{e_{M,n}}]$ is the $1 \times M$ vector of transmit power levels and $e_{m,n} \in \mathcal{E}$, $\mathbf{x_n} = [x_{1,n}, \ldots, x_{m,n}, \ldots, x_{M,n}]^T$ is the $M \times 1$ vector of transmitted signals, where $x_{m,n}$ refers to one of the copies of the packet transmitted by the mth MTD on the nth slot, and ω_n is the additive white gaussian noise (AWGN) with zero mean and variance σ^2.

We assume that the BS employs SIC to iteratively recover each MTD's packet. For a given iteration of the SIC process, the BS first checks to see if any packet in any slot can be successfully decoded. In this work, we consider that a packet is successfully decoded if, at least in one of its copies, the received signal to interference ratio (SINR) is above a threshold γ^{th}. We assume that each copy contains the slot indices to all its copies so that once a packet is successfully decoded, full information about the location of its copies is available. Hence, once a MTD's packet is decoded at the BS, the interference caused by that packet can be completely removed from all slots. This in turn increases the SINR of the remaining packets in the following iterations. The process is repeated until all MTD packets are recovered or a maximum number of iterations $i_{\max}$ is reached.

Let $\mathcal{M}_n$ denote the set of MTDs transmitting on the nth slot. Let $\mathcal{N}_m$ denote the set of slots selected by the mth MTD for transmitting of its copies. We are interested in the recovery-error probability, which is defined as the probability that the BS fails to recover the mth MTD's packet in the ith iteration of the SIC process. It can be expressed as

$$q_m^{(i)} = 1 - \mathbb{P}\left\{\max\left\{\gamma_{m,n}^{(i)}\right\} \geqslant \gamma^{\text{th}}\right\} \quad (3)$$

where $\mathbb{P}\{\cdot\}$ denotes the probability, $\max\{\cdot\}$ denotes the max operation, $n \in \mathcal{N}_m$, $|\mathcal{N}_m| = d$, and $\gamma_{m,n}^{(i)}$ is the SINR of the mth MTD in the ith iteration for its copy transmitted with power level E_l in the nth slot, which is given by

$$\gamma_{m,n}^{(i)}(E_l, u) = \frac{E_l}{\Theta_{m,n}^{(i)}(u) + \sigma^2}, \quad (4)$$

where σ^2 is the noise power and $\Theta_{m,n}^{(i)}(u)$ represents the cumulative interference power from u unresolved MTDs. Note that $q_m^{(i)}$ in (3) is the same for each MTD device since the probabilities of selecting a transmission slot, power level and number of copies, respectively, are the same for all nodes.

Due to the $\max\{\cdot\}$ operation, (3) is not tractable across iterations of the SIC process. Hence, we employ graph based analysis to analyze $\gamma^{(i)}$. In this regard, by defining an interference threshold $\varphi^{\text{th}}(E_l) = \frac{E_l}{\gamma^{\text{th}}} - \sigma^2$, we can get the following relationship which will be used in the subsequent analysis,

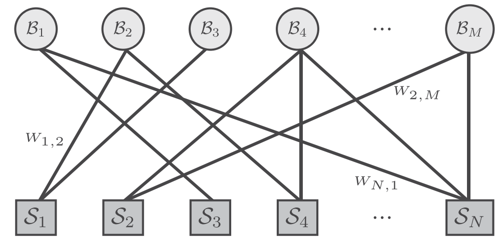

Fig. 1. Edge-weighted bipartite graph representation of the proposed scheme.

$$\mathbb{P}\left\{\gamma_{m,n}^{(i)}(E_l, u) \geqslant \gamma^{\text{th}}\right\} = \mathbb{P}\left\{\Theta_{m,n}^{(i)}(u) \leqslant \varphi^{\text{th}}(E_l)\right\}. \quad (5)$$

III. Recovery-Error Probability Analysis

Graph representation: The system under consideration can be represented as an edge-weighted bipartite graph and can be analysed using the theory of codes on graph [5], [12]. This is illustrated in Fig. 1, where the MTDs and slots are shown by circles and rectangles, representing burst nodes and sum nodes, respectively.

An edge-weighted bipartite graph is defined by $\mathcal{G} = \{\mathcal{B},\ \mathcal{S},\ \mathcal{E}_W\}$, where $\mathcal{B}$, $\mathcal{S}$, and $\mathcal{E}_W$ represent the sets of burst nodes (MTDs), sum nodes (slots), and edges with weights, respectively. The edge weight $W_{m,n}$ corresponds to the transmit power level $e_{m,n} \in \mathcal{E}$. The number of edges connected to a node represents the node degree.

Let λ_d denote the probability that an edge is connected to a degree-d burst node and ρ_k denote the probability that an edge is connected to a degree-k sum node. According to [5], the definitions of λ_d and ρ_k can be given as

$$\lambda_d \triangleq \frac{\Lambda_d d}{\sum_r \Lambda_r r}, \qquad \rho_k \triangleq \frac{\Psi_k k}{\sum_j \Psi_j j} \quad (6)$$

where the probabilities Λ_d and Ψ_k are represented in (1).

Message passing algorithm: We develop a graph-based message passing algorithm representation to track the iterative SIC process in the proposed scheme.

The set of burst nodes transmitting on sum node $\mathcal{S}_n$ is equal to $\mathcal{M}_n$ and the set of sum nodes selected by burst node $\mathcal{B}_m$ for burst transmission is equal to $\mathcal{N}_m$. The message $\mathbf{S}_{n\to m}^{(i)}$ is sent by the sum node $\mathcal{S}_n$ to burst node $\mathcal{B}_m$ to advertise the cumulative interference power $\Theta_{m,n}^{(i)}$ [defined below (4)] experienced in the ith iteration due to the unresolved edges connected to it. On the graph, this cumulative interference power is the sum of weights of all unresolved edges on a given sum node. In response to the message $\mathbf{S}_{n\to m}^{(i)}$, a burst node $\mathcal{B}_m$ replies with a message $\mathbf{B}_{m\to n}^{(i)}$. This message tells the sum node $\mathcal{S}_n$ whether the edge connected to the burst node $\mathcal{B}_m$ should be removed or not. If the interference power is below the interference threshold $\varphi^{\text{th}}(W_{m,n})$ (defined above (5)), which implies that the edge can be removed, then the message $\mathbf{B}_{m\to n}^{(i)}$ equals the interference power contributed by $\mathcal{B}_m$ on sum node $\mathcal{S}_n$, i.e., the weight $W_{m,n}$ is subtracted (edge is removed) from the cumulative interference power on $\mathcal{S}_n$ and the updated interference power is reported to the remaining unresolved burst nodes in the next iteration. Otherwise, if the message $\mathbf{B}_{m\to n}^{(i)}$ is zero then it implies

Algorithm 1: Iterative SIC as a message passing algorithm.

Initialization: $i = 0$, $\mathbf{B}^{(0)}_{n\to m} = 0$, $\mathbf{S}^{(0)}_{n\to m} = \Theta^{(0)}_{m,n}\ \forall m, n.$
for $i \geqslant 0$ && $i \leqslant i_{\max}$ && $\mathbf{S}^{(i)}_{n\to m} = 0$ **do**
  for $n \geqslant 1$ && $n \leqslant N$ **do**
    for $\mathcal{M}_n \neq \{\}$ **do**
      In ith iteration sum node $\mathcal{S}_n$ sends a message $\mathbf{S}^{(i)}_{n\to m}$ to burst node $\mathcal{B}_m$, $\forall m \in \mathcal{M}_n$

$$\mathbf{S}^{(i)}_{n\to m} = \mathbf{S}^{(i-1)}_{n\to m} - \sum_{m'\in\mathcal{M}_n\setminus\mathcal{B}_m} \mathbf{B}^{(i-1)}_{m'\to n}$$

      Within ith iteration, burst node $\mathcal{B}_m$ sends a message $\mathbf{B}^{(i)}_{m\to n}$ to sum node $\mathcal{S}_n$, $\forall n \in \mathcal{N}_m$

$$\mathbf{B}^{(i)}_{m\to n} = \begin{cases} W_{m,n}, & \text{if } \mathbf{S}^{(i)}_{n\to m} - W_{m,n} \leqslant \varphi^{\text{th}} \\ 0, & \text{otherwise.} \end{cases}$$

    end for
  end for
end for

that the edge cannot be removed in the current iteration. This process is repeated until the maximum number of iterations $i_{\max}$ is reached or all burst nodes are recovered. Algorithm 1 summarizes the proposed graph-based message passing algorithm representation of the iterative SIC-based decoding process. Note that the developed message passing algorithm is different from [7] due to the consideration of transmit power diversity and slot-wise SINR based recovery in our work.

Derivation of recovery-error probability: We derive the recovery-error probability using ANR-OR tree analysis technique. The basic principle of this approach is to represent the graph as a tree and then formulate $q^{(i)}_m$. In the ANR-OR process, an unknown edge of a burst node is recovered if at least one of its edges are revealed (*OR rule*). Similarly, for a sum node an edge is recovered in the given iteration if all other edges have been recovered in prior iterations (*AND rule*).

We construct a tree using the edge-weighted bipartite graph $\mathcal{G}$. The depth of the tree is twice the maximum allowed number of iterations $i_{\max}$. A burst node is the root of the tree at depth 0. The children of a burst node are those sum nodes which were chosen for the burst transmissions. Similarly, the sum nodes have those burst nodes as children who chose them for burst transmissions. A node at depth ζ has children in depth $\zeta + 1$. Thereby, for the proposed random access mechanism, burst nodes and sum nodes are located at depths $0, 2, 4, .., 2\,i_{\max}$ and $1, 3, 5, \ldots, 2\,i_{\max} - 1$, respectively. According to Algorithm 1, in each iteration the burst nodes at depth ζ send the messages to their parent sum nodes at depth $\zeta - 1$ and the sum nodes send messages to their parent burst nodes at depth $\zeta - 2$. *Note that a packet is lost if in the final iteration ($i_{\max}$) the message received by the root burst node does not satisfy the SINR criterion. The probability of this event is denoted by $q^{(i_{\max})}$.*

The following proposition gives the main result for $q^{(i)}_m$.

Proposition 1: In the asymptotic setting, as the number of slots $N \to \infty$, the recovery-error probability for a burst node $\mathcal{B}_m$ in the ith iteration, denoted by $q^{(i)}_m$, is given by

$$q^{(i)}_m = \sum_{d=1}^{D} \Lambda_d \left(1 - \sum_{l=1}^{L}\sum_{u=0}^{M-1} p^{(i)}(u)\,\mathbb{P}\left\{\Theta^{(i)}_m(u) \leqslant \varphi^{\text{th}}(E_l)\right\}\Gamma_l\right)^{d} \tag{7}$$

where Λ_d and Γ_l are defined in (1), $\mathbb{P}\{\Theta^{(i)}_m(u) \leqslant \varphi^{\text{th}}(E_l)\}$ is defined in (5), $\Theta^{(i)}_m = \Theta^{(i)}_{m,n}$, for $n \in \mathcal{N}_m$, since experiencing given interference power on any sum node is equally probable, and $p^{(i)}(u)$ is given by

$$p^{(i)}(u) = \sum_{k=u}^{M-1} \Psi_k \binom{k}{u} \left(1 - q^{(i-1)}_m(k,u)\right)^{k-u} \left(q^{(i-1)}_m(k,u)\right)^{u} \tag{8}$$

where Ψ_k is defined in (1) and $q^{(i-1)}_m = 1$ for $i \leqslant 1$.

Proof: The proof is provided in Appendix A. Note that (5) cannot be expressed in closed form but it can be found numerically using (13), as discussed in the Appendix. ■

IV. Performance Optimization

We define the system load G as the normalised number of packets per slots. Since each device generates only one packet, we have $G = \frac{M}{N}$. Based on the condition $q^{(i)} < q^{(i-1)}$ (note that $q^{(i)} = q^{(i)}_m$ since the recovery-error probability is the same for each device), we can define the maximum achievable load G^* such that recovery-error probability $q^{(i_{\max})}$ after $i_{\max}$ iterations will be almost zero for system load $G \leqslant G^*$ in the asymptotic setting when $N \to \infty$. Mathematically,

$$G^* = \max\{G\} \qquad \text{such that} \quad q^{(i_{\max})} = 0. \tag{9}$$

In this work, we are interested in finding the optimal transmit power probability distribution which will maximize G^* for a given degree distribution. This optimization problem can be formulated as follows.

$$\begin{aligned} \underset{\Gamma_l}{\text{maximize}} \quad & G^* \\ \text{subject to} \quad & E_1 = \sigma^2\gamma^{\text{th}}, \\ & \bar{E}\bar{\Lambda} = E_{\text{tot}}, \\ & \sum_l \Gamma_l = 1,\ \forall\, l, \\ & 0 \leqslant \Gamma_l \leqslant 1,\ \forall\, l, \end{aligned} \tag{10}$$

where the first constraint defines the criterion for the minimum power level, the second constraint mandates that the average power consumption must be equal to the power budget per frame per device denoted by E_{tot}, where $\bar{E} = \sum_l \Gamma_l E_l$ represents the average power level and $\bar{\Lambda}$ is the average degree of repetition given as $\bar{\Lambda} = \sum_d \Lambda_d d$, the third constraint mandates that the sum of all transmit power probability levels is equal to one and the last constraint mandates that each probability level needs to be between zero and one.

V. Results

In this section, first we present the results to illustrate the advantage of employing transmit power diversity in conjunction with SINR based packet recovery in CRDSA, referred to as the *proposed scheme*. For comparison, we consider a *benchmark scheme* in which the random access transmission scheme does not employ transmit power diversity, however the rest of the transmission strategy and the recovery mechanism is the same as in Section III.

Impact of transmit power diversity: We compare the performance of the proposed scheme with the benchmark scheme by considering a simple transmit power diversity model, i.e., only two equally probable power levels. For both schemes the SINR based recovery model is

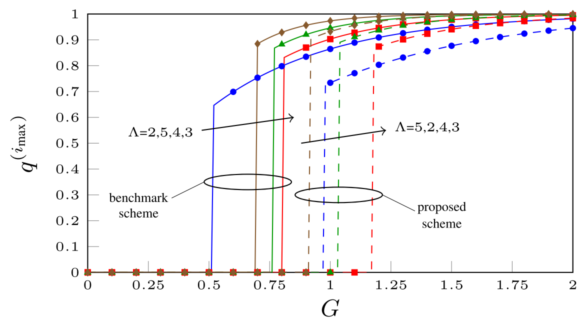

Fig. 2. Recovery-error probability $q^{(i_{\max})}$ vs. system load G, for the clean packet model and SINR based model, in asymptotic setting, with $\Lambda = 2,3,4,5$.

TABLE I
MAXIMUM ACHIEVABLE LOAD, G^*

Λ	2	3	4	5
Benchmark scheme	0.52	0.81	0.77	0.70
Proposed scheme	0.98	1.18	1.04	0.82

used which employs SIC. The parameter values are set as follows: $N = 1000, i_{\max} = 10\,000, L = 2, \Gamma_1 = \Gamma_2 = 0.5, E_1 = 0$ dB, $E_2 = 6.02$ dB, $\gamma^{\text{th}} = 0$ dB, $\sigma^2 = 0$ dB, $E_{\text{tot}} = 3.52$ dB. The transmit power level for each packet copy for the benchmark scheme is the same, i.e., $E_2 = 6.02$ dB.

Fig. 2 plots the recovery-error probability $q^{(i_{\max})}$ using Proposition 1 for both the proposed scheme and the benchmark scheme with repetition degrees $\Lambda = 2, 3, 4, 5$. We can see that for both schemes, the recovery-error probability is zero until a specific load value (G^*). As the load is further increased beyond G^*, the recovery-error probability abruptly escalates. As expected, the proposed scheme significantly outperforms the benchmark scheme for different repetition rates. The maximum achievable load G^* is tabulated in Table I. This performance improvement of the proposed scheme is owing to the enhanced capture effect as a result of the transmit power diversity. We can see that, for the given system parameters, repetition degree 3 can achieve the highest load. This can be explained as follows. Transmitting multiple packets copies increases the probability of success but it also increases the level of interference. This tradeoff results in the best performance for a certain repetition degree (3 in this case) and the performance degrades after that.

Convergence analysis: We investigate the convergence of the iterative SIC process at the maximum load G^*. Fig. 3 plots the evolution of the recovery-error probability in the asymptotic setting, when the system is operated at the maximum load G^*. It can be seen from Fig. 3 that the convergence condition $q^{(i)} < q^{(i-1)}$, defined above (9), is fulfilled.

Finally, we investigate the tightness of the result in Proposition 1 when the number of slots N is finite. Fig. 4 plots the recovery-error probability for $\Lambda = 3$ using Proposition 1 and simulations for $N = 100, 500, 1000, 5000$, respectively. We can see that as N increases, the simulation results quickly approach the asymptotic performance.

Optimal transmit power distribution: We solve (10) to determine the optimal transmit power distribution and investigate the improvement in performance. (10) can be efficiently solved using differential evolution optimization technique [13]. The parameter values used are as follows:

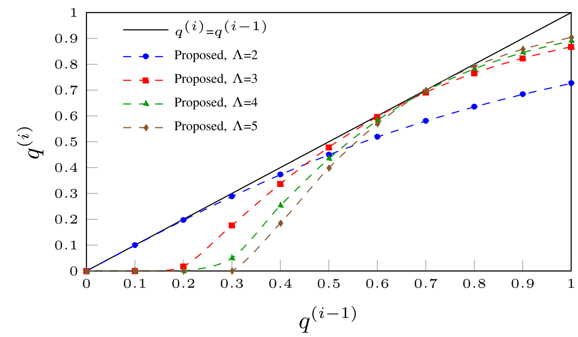

Fig. 3. Evolution of error probability $q^{(i)}$ when operating at maximum achievable load $G^* = \{0.98, 1.18, 1.04, 0.92\}$ for $\Lambda = 2, 3, 4, 5$.

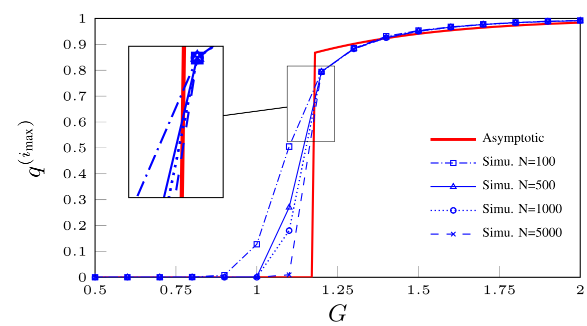

Fig. 4. Recovery-error probability $q^{(i_{\max})}$ vs. system load G, for the proposed scheme with asymptotic setting, compared with simulations for $\Lambda = 3$.

TABLE II
G^* FOR OPTIMAL PROBABILITY DISTRIBUTION VECTOR $\{\Gamma^*\}$

Λ	G^* Uniform	G^* Optimal	$\{\Gamma^*\} = \{\Gamma_1\ \Gamma_2\ \Gamma_3\ \Gamma_4\ \Gamma_5\}$
2	1.62	1.68	{ 0.35 0.20 0.15 0.15 0.15 }
3	1.46	1.58	{ 0.55 0.05 0.10 0.05 0.25 }
4	1.07	1.33	{ 0.72 0.00 0.00 0.00 0.28 }
5	0.92	1.06	{ 0.55 0.00 0.00 0.00 0.45 }

$N = 1000, i_{\max} = 10\,000, \Lambda = 3, L = 5, \gamma^{\text{th}} = 0$ dB, $\sigma^2 = 0$ dB, $E_1 = 0$ dB, $E_2 = 3.77$ dB, $E_3 = 7.53$ dB, $E_4 = 11.30$ dB, $E_5 = 15.06$ dB, $E_{\text{tot}} = 20$ dB. For comparison, two probability models are considered:

- An uniform probability model in which each power level has equal probability of being selected.
- An optimized probability model in which each power level is assigned an optimal probability.

The maximum achievable load G^* for both models is presented in Table II. It can be seen that the optimized probability distribution achieves better performance as compared to uniform probability distribution. This is due to the fact that the optimized transmit power distributions maximize the probability of the gap between the power levels of two copies transmitted in a given slot. This has a greater impact on maximizing the capture effect. Hence, for the considered system parameters, as the number of copies Λ increases, from 2 to 5, intermediate power levels may not necessarily be used. Accordingly, their probability is zero in the optimized transmit power distribution.

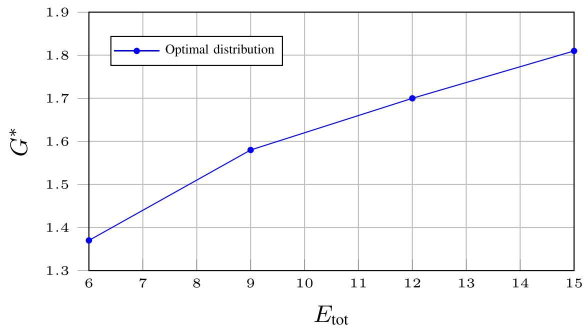

Fig. 5. Maximum achievable load of CRDSA with transmit power diversity when the frame budget is varied.

Finally, we investigate the impact of the total power budget per device per frame (E_{tot}), which is an important constraint in (10). Fig. 5 plots the maximum achievable load G^* vs. E_{tot} for repetition rate $\Lambda = 2$ and other parameters same as before. It can be seen that a larger E_{tot} gives more room to exploit the transmit power diversity and thus boosts the performance. This leads to further significant performance improvement.

VI. Conclusion

In this correspondence, we have considered CRDSA with transmit power diversity and SINR based iterative SIC decoding, in the asymptotic frame length regime. We have proposed a framework for accurately predicting the performance of the system. The proposed framework is validated by simulation results. The results show promising improvements in maximum achievable system load, which is relevant to efficient random access for MTC scenario.

Appendix A
Proof of Proposition 1

The probability $\mathbb{P}(\max\{\gamma_{m,n}^{(i)}\} \geqslant \gamma^{\text{th}})$, defined in (3), is not tractable due to the max $\{\cdot\}$ function, thus we use bipartite graph to find $q_m^{(i)}$. First we formulate $p_{m,n}^{(i)}$ which denotes the probability that the burst node $\mathcal{B}_m$ selects edge weight equal to E_l and the cumulative weight of u other edges $\Theta_{m,n}^{(i)}(u)$, connected to the sum node $\mathcal{S}_n$, is less than or equal to $\varphi^{\text{th}}(E_l)$. It can be given as

$$p_{m,n}^{(i)} = \sum_{l=1}^{L} \sum_{u=0}^{M-1} p^{(i)}(u)\, \mathbb{P}\left\{\Theta_{m,n}^{(i)}(u) \leqslant \varphi^{\text{th}}(E_l)\right\} \Gamma_l, \tag{11}$$

where $p^{(i)}(u)$, defined in (8), represents the probability of k edges connected to a given sum node, out of which $k-u$ edges are resolved in the previous iterations, but u edges are still connected to burst node $\mathcal{B}_m$'s edge in the ith iteration. To compute $p^{(i)}(u)$, we need to know Ψ_k which in the asymptotic setting, when $N \to \infty$, can be given as [14]

$$\Psi_k = e^{-\frac{\alpha^{k+1}}{k!}}, \tag{12}$$

where $\alpha = \bar{\Lambda}\frac{M}{N}$ is the average degree of a sum node, and $\bar{\Lambda}$ is the average degree of a burst node given as $\bar{\Lambda} = \sum_d \Lambda_d d$.

Next, we need $\Theta_{m,n}^{(i)}(u)$, in order to know the cumulative weight of edges on a sum node $\mathcal{S}_n$, which may vary from iteration to iteration depending upon the number of connected edges u. It is given as

$$\Theta_{m,n}^{(i)}(u) = \sum_{c=0}^{u} W_{c,n}, \quad 0 \leqslant u \leqslant M-1, \tag{13}$$

where $W_{c,n}$ represents the edge weight of burst node $\mathcal{B}_c, \forall c \in \mathcal{M}_n \setminus \mathcal{B}_m$, connected with sum node $\mathcal{S}_n$. A burst node selects weight equal to E_l with probability Γ_l which is defined in (1). Using (13), the probability in (5) can be numerically evaluated.

Since, the selection of any sum node is equally probable, it implies that $p_{m,n}^{(i)} = p_{m,n'}^{(i)}$, for $n \neq n'$, where $n, n' \in \mathcal{N}_m$. Thus, $p_m^{(i)} = p_{m,n}^{(i)}$, which is given by (11). Recall, the *OR rule* for success and *AND rule* for error explained in Section III. Thus, the recovery-error probability for burst node $\mathcal{B}_m$ with node degree $d = |\mathcal{N}_m|$, in the ith iteration, is equal to $(1 - p_m^{(i)})^d$. By averaging it over the degree distribution represented by $\{\Lambda_d\}$, defined in (1), yields

$$q_m^{(i)} = \sum_d \Lambda_d \left(1 - p_m^{(i)}\right)^d \tag{14}$$

Substituting the value of $p_m^{(i)}$ from (11) in (14), yields (7).

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