

# Energy-Efficient Resource Allocation in SWIPT Cooperative Wireless Networks

Shengjie Guo, Xiangwei Zhou, and Xiangyun Zhou

**Abstract**—In this paper, energy-efficient resource allocation in *simultaneous wireless information and power transfer* (SWIPT) cooperative wireless networks is studied. With the transferred power, the SWIPT relays are adopted to improve the energy efficiency at the destination node. Two typical relay structures, *decode-and-forward* (DF) and *amplify-and-forward* (AF), are exploited for the optimal relay selection and power allocation with a power splitting SWIPT architecture. Non-convex energy efficiency optimization problems are formulated for both DF and AF relay types. Based on the signal-to-noise ratios at the destination node, closed-form expressions of the optimal power splitting ratios are provided for DF and AF relays, respectively. With the optimal power splitting ratio, relay selection schemes based on full and partial knowledge of the channel state information are derived. Moreover, a novel power allocation scheme is proposed and illustrated based on the property of the simplified optimization problem with the power and quality of service constraints. Simulation results demonstrate that the proposed resource allocation scheme achieves the maximum energy efficiency with low computational complexity, in which the proposed relay selection outperforms the typical relay selection schemes in terms of energy efficiency.

**Index Terms**—Cooperative wireless networks, energy efficiency, power allocation, relay selection, simultaneous wireless information and power transfer.

## I. INTRODUCTION

### A. Motivation

With the fast growth of wireless data traffic and mobile devices, energy consumption in wireless networks has experienced a dramatic increase in the last decade. As reported, telecommunication technologies have contributed to the world's greenhouse gas emissions with more than 2% share [2]. Facing such environmental issues, wireless communications with high energy efficiency has drawn great attention [3]–[5].

Recently, there has been a new trend to use the *radio frequency* (RF) signal for transferring power and transmitting

information simultaneously [6], [7], which is a more reliable and predictable way to harvest energy to support low-power devices such as wireless sensors. The concept of *simultaneous wireless information and power transfer* (SWIPT) has been proposed in [8]. A SWIPT system transfers the power from the signals to support its own operations and is less dependent on external power supplies. For passive receivers, they can simultaneously receive information and transfer power from the signals to support information processing, which prolongs the usage time of such receivers with limited energy storage. With the growth of attention to SWIPT networks, SWIPT has been studied in different systems [8]–[10]. A SWIPT *multiple-input multiple-output* (MIMO) wireless broadcast system has been studied in [8]. Then the SWIPT concept has been extended to the *orthogonal frequency-division multiple access* (OFDMA) system and studied in [9]. Furthermore, an energy efficiency maximization problem in SWIPT mobile wireless sensor networks has been studied in [10].

Moreover, cooperative relaying is effective for system performance improvement in terms of energy efficiency as well as reliability. However, the locations of conventional relay power supply usually limit the performance of cooperative networks. In ubiquitous RF environments, the SWIPT relays provide more flexibility for deployment and physical maintenance, which conquers the limitations of typical relays. Therefore, the SWIPT relays are quite suitable for cooperative relay networks [11]. Recently, SWIPT cooperative networks have been extensively studied [12]–[20]. In [12], the energy efficiency of SWIPT AF MIMO relay networks has been maximized, where the source and relay precoding matrices are designed jointly. In [19], a robust energy efficiency optimization problem has been formulated for SWIPT MIMO two-way relay, where imperfect *channel state information* (CSI) is considered and the worst-case energy efficiency is maximized. The outage probability and throughput of SWIPT cognitive cooperative networks have been analyzed in [13], where the outage probabilities of all receiving nodes are derived. With a focus on time splitting based SWIPT relay systems, the power allocation and flexible time splitting rules to maximize the average system throughput have been proposed in [14]. Meanwhile, power allocation schemes have also been studied in different SWIPT relay networks to improve the throughput [15], [16]. Furthermore, relay selection has been explored in multiple-relay scenarios [17], [18]. In consideration of the available CSI, different relay selection schemes, i.e., first-link relay selection (FLRS), random relay selection (RRS), and distance-based relay selection (DBRS), lead to various performance.

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In [21], non-orthogonal multiple access has been adopted in SWIPT cooperative networks to improve the system energy efficiency.

### B. Related Work

Although SWIPT cooperative transmission is a promising technology for wireless communications, the existing study of SWIPT relays introduced above focuses more on the flexible utilization of spectral resources rather than energy efficiency. In [22], a locally optimal solution and a simplified equal power allocation scheme have been provided to improve the energy efficiency in SWIPT AF relay networks. In [23], a cooperative SWIPT scheme has been proposed to maximize the energy efficiency of wireless sensor networks. The formulated optimization problem is transformed to a convex one and solved by a heuristic algorithm. In [24], an optimal resource allocation scheme has been developed to maximize the energy efficiency of SWIPT two-way DF relays, where the transmit power and power splitting ratios are jointly optimized. However, relay selection has not been considered in [22]–[24]. In [25], energy-efficient relay selection and power allocation have been studied in SWIPT cooperative wireless networks, where DF relay nodes with batteries are considered. However, the maintenance and replacement of batteries are required for such relays. Therefore, SWIPT relays with batteries are limited to certain environments only. In [26], a relay selection scheme for full-duplex SWIPT relay networks without batteries has been proposed. The transmit powers of relays are no longer restricted by the volumes of batteries and thus the deployment of relays is more flexible without the need of battery maintenance and replacement. The energy efficiency optimization problem in such a scenario is interesting and has not been studied yet. Recently, joint relay selection and resource allocation for time-switching SWIPT AF relay networks has been studied in [27], where the energy efficiency maximization problem is formulated and solved via an iterative algorithm. However, the proposed scheme cannot be applied to power splitting SWIPT relays.

### C. Contributions

As mentioned above, recent studies focus on specific aspects of optimization in SWIPT cooperative wireless networks but joint relay selection and resource allocation optimization has not been studied. Besides, most of the aforementioned studies assume batteries with SWIPT relays, which in fact limits the flexibility of relay placement due to the maintenance and replacement of the equipped batteries. In our previous work [1], we have studied the optimal energy-efficient power allocation for SWIPT cooperative networks with a single DF relay node. In this paper, we extend our previous work to the case with multiple relays without external power supplies or batteries. Without the direct link, the selected relay utilizes the transferred power to help the destination node. Due to different relay locations, the performance of the relays varies. Therefore, selecting the best relay among multiple available relays would further improve the energy efficiency of the entire cooperative network. The relay selection and power allocation

are jointly considered for energy efficiency maximization. Different from the approach in [28], we do not sum up the energy efficiencies of individual nodes who cannot directly exchange their powers. In future Internet of Things (IoT) based sensor networks, our approach can significantly improve the performance of ubiquitous sensors at any locations. According to the principle of relaying, the AF relay simply amplifies the received signal including both information and noise then forwards it to the destination while the DF relay would decode the received signal then only amplifies and forwards the information to the destination node. However, the structure and signal processing unit of the DF relay would be more complicated. Therefore, in addition to DF relays, we also investigate the energy-efficient resource allocation with AF relays and give the performance comparison between DF and AF relays. The contributions of our work are summarized below.

From the energy efficiency perspective, we formulate joint relay selection and power allocation problems in both DF and AF relay types for power splitting SWIPT cooperative wireless networks without batteries.

According to the *signal-to-noise ratios* (SNRs) at the destination node, we derive the closed-form expressions of the optimal power splitting ratios for both DF and AF relay types to achieve the best energy efficiency.

Given full knowledge of CSI, we design the optimal relay selection schemes for both DF and AF relay types. With partial knowledge of CSI, we propose the suboptimal relay selection scheme in consideration of overhead.

By analyzing the properties of the simplified power allocation problems, we propose an optimal *one-time power allocation* (OPA) scheme to achieve the maximum energy efficiency in both DF and AF cases.

The rest of our paper is organized as follows. A SWIPT cooperative wireless network model and energy efficiency optimization problems for both DF and AF relay types are presented in Section II. In Sections III and IV, the decomposed resource allocation schemes for DF and AF relay types are derived, respectively. In Section V, the performance comparison between DF and AF relay types is considered. Numerical results are given in Section VI. Finally, conclusions are drawn in Section VII.

## II. SYSTEM MODEL

In this section, we present the cooperative wireless network model and power splitting SWIPT relay architecture, based on which we formulate the energy efficiency optimization problems for both the DF and AF relay types.

As shown in Fig. 1, a cooperative wireless network with one source node, one destination node, and  $M$  SWIPT relays, each equipped with one antenna, is considered. We assume that the direct link between the source and destination nodes is blocked. A SWIPT relay would receive and forward the signal in the same slot of duration  $T$ . With half-duplex nodes and relays, the signal is transmitted in the first half slot from the source node and forwarded in the second half one by the selected SWIPT relay. We assume that the transmission only

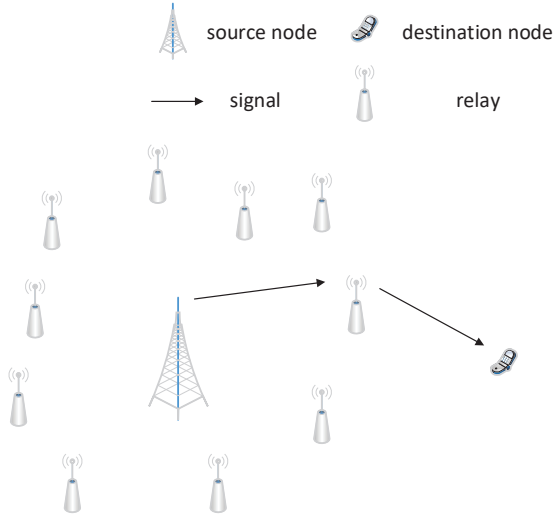


Fig. 1: System model.

occupies one frequency band during each time slot, which can effectively avoid the interference. Besides, due to the limited amount of transferred power at SWIPT relays, such an assumption can help achieve higher signal-to-noise ratios and thus support more complicated and practical applications. Subscripts  $S$ ,  $r_i$ , and  $d$  denote the source node, the  $i$ -th relay, and the destination node, respectively.

At the source node, the power amplifier efficiency factor is  $\eta_s$ . Because the source node is always processing and exchanging data with the core network, a total static power consumption  $P_c$  is considered. We assume that no batteries are installed at the SWIPT relays to accumulate the transferred power, which provides more flexibility for their deployment [29]. The super-capacitor is used for temporary power holding. As illustrated in Fig. 2, the SWIPT relay architecture adopts a power splitting structure, for better tradeoff between the information rate and transferred power [9], [30]. With the power splitting structure, the  $i$ -th relay splits the received signal into two streams with a power splitting ratio  $\rho$ . Specifically,  $\rho$  of the received signal power is used for power transfer and  $(1 - \rho)$  of the received signal power is used for information processing. An efficiency of  $\eta_r$ ,  $0 < \eta_r < 1$ , for  $\rho$  of the transferred power is considered, which is determined by the SWIPT circuit. At the SWIPT relay, due to the power amplifier inefficiency,  $\eta_r$  of the effectively transferred power is used to forward the signal. Since SWIPT relays only work when the signals are received with their power transferred, no static circuit power consumption at SWIPT relays is considered.

Let  $P_s$  and  $P_{r_i}$  be the transmit powers of the source node and the  $i$ -th relay, respectively. Denote  $l_{sr_i}$  and  $l_{r_i d}$  as the distances from the source node to the  $i$ -th relay and from the  $i$ -th relay to the destination node, respectively. Let  $g_{s,r_i}$  be the channel coefficient from the source node to the  $i$ -th relay and  $g_{r_i,d}$  be the channel coefficient from the  $i$ -th relay to the destination node, both of which are assumed Rayleigh fading. Antenna and signal processing noises are modeled as *additive white Gaussian noise* (AWGN) with zero mean. Note that the CSI of each link can be estimated through training symbols

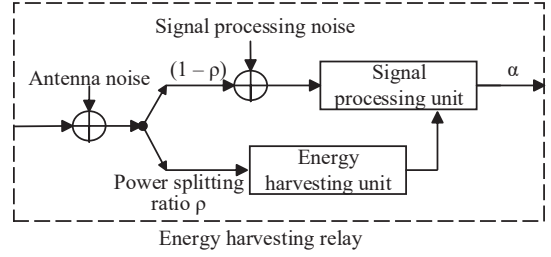


Fig. 2: SWIPT relay architecture.

without error. For the links from the source node to the relays, called the first links, the energy transfer unit is activated when the training signals are transmitted from the source and the transferred power is used to process them. For the links from the relays to the destination node, called the second links, the relays forward the training signals to the destination node, collect the CSIs from the destination node. After the CSIs are obtained at the relay, the CSIs of both links will be sent back to the source node.

#### A. Signal Model and Problem Formulation for DF Relay

We first consider the DF relay type where the selected relay decodes the received signal then forwards the signal to the destination node. The received signal at the  $i$ -th DF relay,  $y_{r_i}^{DF}$ , is given by

$$y_{r_i}^{DF} = \sqrt{\rho} \frac{g_{sr_i}}{\sqrt{k l_{sr_i}}} x_s + \sqrt{1 - \rho} n_{r_i}^{[a]} + n_{r_i}^{[s]}, \quad (1)$$

where  $x_s$  is the transmitted signal from the source node,  $n_{r_i}^{[a]}$  and  $n_{r_i}^{[s]}$  are the antenna and signal processing noises at the  $i$ -th DF relay, respectively, and  $g_{sr_i} = \frac{g_{sr_i}}{\sqrt{k l_{sr_i}}}$  denotes the equivalent channel coefficient from the source node to the  $i$ -th relay, where  $k$  is the path loss constant and  $l_{sr_i}$  is the path loss coefficient. Similarly, the received signal at the destination node in the DF relay case with the  $i$ -th relay,  $y_d^{DF}$ , is given by

$$y_d^{DF} = \sqrt{\rho} \frac{g_{r_i d}}{\sqrt{k l_{r_i d}}} x_s + n_d^{[a]} + n_d^{[s]}, \quad (2)$$

where  $g_{r_i d} = \frac{g_{r_i d}}{\sqrt{k l_{r_i d}}}$  denotes the equivalent channel coefficient from the  $i$ -th DF relay to the destination node,  $n_d^{[a]}$  is the antenna noise at the destination node, and  $n_d^{[s]}$  is the signal processing noise at the destination node.

Since the antenna noise power is negligible in comparison with the signal processing noise power in practice [31], [32], we set both the signal processing noise powers at the  $i$ -th relay and destination node to be  $\sigma^2$  and the antenna noise powers to be 0. Therefore, the SNR at the  $i$ -th relay,  $\gamma_{r_i}^{DF}$ , can be expressed as

$$\gamma_{r_i}^{DF} = \frac{(1 - \rho) P_s |g_{sr_i}|^2}{2 \sigma^2}; \quad (3)$$

where  $| \cdot |$  is the absolute value operator, and the SNR at the destination node with the  $i$ -th relay,  $\gamma_d^{DF}$ , is

$$\gamma_d^{DF} = \frac{P_{r_i} |g_{r_i d}|^2}{2 \sigma^2}; \quad (4)$$

During the first half slot, the effectively transferred energy at the  $i$ -th relay,  $E_{r_i}$ , is

$$E_{r_i} = \frac{1}{2} \rho \frac{P_s}{1 - \alpha_i} g_{sr_i}^2 P_s T \quad (5)$$

Because only  $\frac{1}{2}$  of the effectively transferred energy is used to transmit the signal in the second half slot, the transmit power of the  $i$ -th relay is

$$P_{r_i} = \frac{E_{r_i}}{T/2} = \frac{1}{2} \rho \frac{P_s}{1 - \alpha_i} g_{sr_i}^2 P_s \quad (6)$$

After we substitute (6) into (4), the SNR at the destination node is expressed in terms of  $P_s$  as

$$\gamma_d^{DF} = \frac{\rho \frac{1}{2} g_{sr_i}^2 P_s j g_{r_i d}^2}{1 - \alpha_i} \quad (7)$$

In DF relay networks, the achievable transmit rate with the  $i$ -th relay,  $R_i^{DF}$ , is [33]

$$R_i^{DF} = \frac{1}{2} \log_2 (1 + \min \{ \gamma_{r_i}^{DF}; \gamma_d^{DF} \}) \quad (8)$$

where a factor  $\frac{1}{2}$  is applied for a half time slot due to half-duplex communication.

Accordingly, the energy efficiency for DF relay networks with the  $i$ -th relay,  $\eta_i^{DF}$ , is given by [34]

$$\eta_i^{DF} = \frac{R_i^{DF}}{\frac{1}{2} P_s + P_c} \quad (9)$$

where  $\frac{1}{2} P_s$  is the power consumption for signal transmission and  $P_c$  is the total static circuit power consumption at the source and destination nodes. Note that the power consumption at the relay for cooperative transmission is entirely compensated by the transferred power.

The energy efficiency optimization problem for DF relay networks can be formulated as

$$\begin{aligned} & \max_{i; P_s; \alpha_i} \eta_i^{DF} & (10) \\ \text{s.t. : } & \min \{ \gamma_{r_i}^{DF}; \gamma_d^{DF} \} \geq \gamma_{th}; \delta_i; & (10a) \\ & 0 \leq P_s \leq P_{max}; & (10b) \\ & 0 \leq \alpha_i \leq 1; \delta_i; & (10c) \end{aligned}$$

where  $\gamma_{th}$  is the minimum SNR requirement to guarantee the quality of service (QoS) and  $P_{max}$  is the maximum transmit power of the source node.

### B. Signal Model and Problem Formulation for AF Relay

In the AF relay case, the selected relay directly amplifies the received signal then forwards the signal to the destination node. The received signal at the  $i$ -th AF relay,  $y_{r_i}^{AF}$ , is the same as in the DF relay case. The received signal at the destination node through the  $i$ -th relay,  $y_d^{AF}$ , is given by

$$y_d^{AF} = \rho \frac{P_{r_i} g_{r_i d}}{1 - \alpha_i} \frac{y_{r_i}^{AF}}{P_s j g_{sr_i}^2 + \frac{1}{2}} + n_d^{[s]}; \quad (11)$$

where  $\frac{1}{1 - \alpha_i} \frac{P_s j g_{sr_i}^2 + \frac{1}{2}}$  is the normalizing factor to guarantee that the relay transmit power satisfies the power constraint. With some mathematical manipulation,  $y_d^{AF}$  can

be expressed as

$$\begin{aligned} y_d^{AF} = & \rho \frac{P_s P_{r_i} g_{sr_i} g_{r_i d} X_s}{(1 - \alpha_i) P_s j g_{sr_i}^2 + \frac{1}{2}} \\ & + \rho \frac{P_{r_i} g_{r_i d} n_{r_i}^{[s]}}{(1 - \alpha_i) P_s j g_{sr_i}^2 + \frac{1}{2}} + n_d^{[s]}; \quad (12) \end{aligned}$$

The first term in (12) is the desired signal and the rest two are noises. With (6), the SNR at the destination node in the AF relay case with the  $i$ -th relay,  $\gamma_d^{AF}$ , is

$$\begin{aligned} \gamma_d^{AF} = & \frac{(1 - \alpha_i) P_s P_{r_i} j g_{sr_i}^2 j g_{r_i d}^2}{(1 - \alpha_i) P_s j g_{sr_i}^2 + \frac{1}{2}} \\ & \frac{P_{r_i} j g_{r_i d}^2}{(1 - \alpha_i) P_s j g_{sr_i}^2 + \frac{1}{2}} \\ = & \frac{i(1 - \alpha_i) j g_{sr_i}^4 j g_{r_i d}^2 P_s^2}{i j g_{sr_i}^2 j g_{r_i d}^2 P_s^2 + (1 - \alpha_i) j g_{sr_i}^2 P_s^2 + \frac{1}{4}} \quad (13) \end{aligned}$$

In AF relay networks, the achievable transmit rate with the  $i$ -th relay,  $R_i^{AF}$ , is

$$R_i^{AF} = \frac{1}{2} \log_2 (1 + \gamma_d^{AF}); \quad (14)$$

The energy efficiency for AF relay networks with the  $i$ -th relay,  $\eta_i^{AF}$ , is expressed as

$$\eta_i^{AF} = \frac{R_i^{AF}}{\frac{1}{2} P_s + P_c} \quad (15)$$

and the energy efficiency optimization problem for AF relay networks is formulated as

$$\begin{aligned} & \max_{i; P_s; \alpha_i} \eta_i^{AF} & (16) \\ \text{s.t. : } & \gamma_d^{AF} \geq \gamma_{th}; \delta_i; & (16a) \\ & 0 \leq P_s \leq P_{max}; & (16b) \\ & 0 \leq \alpha_i \leq 1; \delta_i; & (16c) \end{aligned}$$

It is obvious that the energy efficiency optimization problems (10) and (16) are multi-variable and non-convex. Although the optimal solutions can be obtained through the computation of energy-efficient power allocation at each relay, the complexity is prohibitive in practice. Therefore, we analyze the problems and propose individual relay selection schemes and power allocation to simplify the problems. Firstly we derive the optimal power splitting ratios. With different levels of CSI knowledge, we propose different relay selection schemes for the DF and AF cases. Moreover, we efficiently solve the power allocation problems without iterative algorithms.

### III. RESOURCE ALLOCATION FOR DF RELAY

In this section, the energy efficiency optimization problem is simplified into relay selection and energy-efficient power allocation in the DF relay case. The corresponding relay selection scheme and power allocation method are derived and discussed.

To obtain the optimal solution, the SWIPT relays will report the forward link CSI to the source node for resource allocation. To simplify the joint relay selection and power allocation problems, we have the following proposition in DF relay case.

**Proposition 1:** For the joint SWIPT DF relay selection and power allocation problem (10), the relay selection can be decoupled from the power allocation without loss of optimality. The maximum energy efficiency,  $\eta^{DF}$ , can be obtained at the  $i$ -th DF relay with the highest equivalent channel-to-noise ratio (CNR),  $i = \arg \max_i f \frac{(1-i)jg_{sr_i}j^2}{2}; \frac{i jg_{sr_i}j^2 jg_{r_i}d^2}{2} g$ .

**Proof:** See Appendix A.

With Proposition 1, the DF relay selection can be carried out without consideration of the power allocation, which simplifies the original problem (10). Designing the relay selection scheme based on the CNR is called the best CNR principle in this paper.

### A. DF Relay Selection

1) *Relay selection with full CSI:* We start with the case that the full knowledge of CSI is available at the source node.

In DF relay networks, according to (8) and the best equivalent CNR principle, the index of the selected relay is

$$i = \arg \max_i f_i g = \arg \max_i f \min \left\{ f_{r_i}^{DF}; \frac{D^DF}{d} g g \right\}; \quad (17)$$

where  $f_{r_i}^{DF} = \frac{(1-i)jg_{sr_i}j^2}{2}$  is the CNR at the  $i$ -th DF relay and  $\frac{D^DF}{d} = \frac{i jg_{sr_i}j^2 jg_{r_i}d^2}{2}$  is the CNR at the destination node through the  $i$ -th DF relay. Since the splitting ratio  $i$  influences the values of  $f_{r_i}^{DF}$  and  $\frac{D^DF}{d}$ , the optimal splitting ratio  $i$  is determined according to the following proposition.

**Proposition 2:** The optimal splitting ratio  $i$  for DF relay selection is obtained when  $f_{r_i}^{DF} = \frac{D^DF}{d}$ , which is  $i = \frac{1}{1 + \frac{1}{jg_{r_i}d^2}}$ .

**Proof:** See Appendix B.

According to Proposition 2, the CNR at the  $i$ -th relay can be rewritten as

$$f_{r_i}^{DF} = \frac{jg_{sr_i}j^2 jg_{r_i}d^2}{2 \left( 1 + \frac{1}{jg_{r_i}d^2} \right)} \quad (18)$$

and the index of the selected DF relay is

$$i = \arg \max_i f \frac{jg_{sr_i}j^2 jg_{r_i}d^2}{1 + \frac{1}{jg_{r_i}d^2}} g; \quad (19)$$

With full knowledge of CSI, the selected DF relay based on (19) is optimal, which is called *full CSI relay selection* (FRS).

2) *Relay selection with first link CSI:* Note that obtaining full knowledge of CSI involves huge overhead associated with periodic reporting, especially for the CSI of the second link. Therefore, we consider the DF relay selection with partial knowledge of CSI in the following to reduce overhead. Since there are direct links between the source node and relays, the first link CSI is easy to be collected at the source node. In such a case, only the first link CSI, i.e.,  $jg_{sr_i}j^2; \delta i$ , with distance information is assumed to be known.

In practice, relays are commonly deployed and selected between the source and destination nodes. In such a case, the relays closer to the source are usually further away from the destination. Therefore, the distances  $l_{sr_i}$  and  $l_{r_i d}$  are usually negatively correlated, which results in the negative correlation between  $jg_{sr_i}j^2$  and  $jg_{r_i}d^2$ . According to (18), the CNR,  $f_{r_i}^{DF}$ , increases with the increasing  $jg_{sr_i}j^2$  and  $jg_{r_i}d^2$ . Therefore,

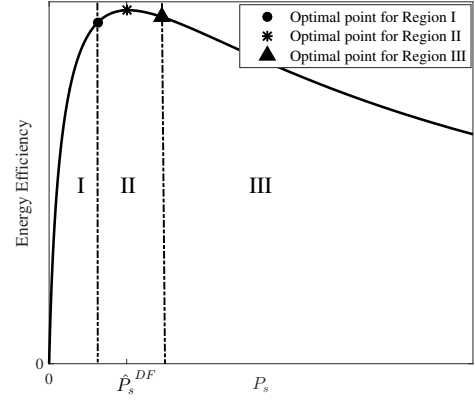


Fig. 3: Energy efficiency vs. transmit power within feasible regions.

when only  $jg_{sr_i}j^2; \delta i$  is known, a larger  $jg_{sr_i}j^2$  or a smaller  $jg_{sr_i}j^2$  may both improve  $f_{r_i}^{DF}$ . In such a case, how to select the best relay with only the information of  $jg_{sr_i}j^2; \delta i$ ; needs to be analyzed.

Given the distributions of  $jg_{sr_i}j^2$ , we analyze the expectation of CNR, based on which we derive the relay selection scheme for the case with partial knowledge of CSI.

In the Rayleigh fading channel model, when distance information is known,  $jg_{r_i}d^2$  follows the exponential distribution with rate parameter  $\lambda = kl_{sr_i}$ . With the known  $jg_{sr_i}j^2$ , we have the following proposition.

**Proposition 3:** The expectation of the CNR at the  $i$ -th relay approximates

$$E[f_{r_i}^{DF}] = \frac{jg_{sr_i}j^2}{\left( 1 + \frac{1}{jg_{r_i}d^2} \right)^2} + \frac{2 jg_{sr_i}j^2}{\left( 1 + \frac{1}{jg_{r_i}d^2} \right)^3}; \quad (20)$$

**Proof:** See Appendix C.

In (20),  $E[\cdot]$  is the expectation operator. Since  $E[f_{r_i}^{DF}]$  is an increasing function of  $jg_{sr_i}j^2$ . The maximum  $jg_{sr_i}j^2$  renders the best performance. Therefore, when  $jg_{sr_i}j^2$  is known, the index of the selected DF relay is

$$i = \arg \max_i f jg_{sr_i}j^2 g; \quad (21)$$

which is the FLRS.

### B. Energy-Efficient Power Allocation for DF Relay

With the selected DF relay  $i$  and corresponding optimal power splitting ratio,  $i$ , the energy efficiency optimization problem (10) is simplified as

$$\max_{P_s} \eta_i^{DF} = \frac{\frac{1}{2} \log_2 \left( 1 + \frac{D^DF}{r_i} P_s \right)}{1 P_s + P_c} \quad (22)$$

$$s.t. : \quad \frac{D^DF}{r_i} P_s \leq \theta; \quad (22a)$$

$$0 \leq P_s \leq P_{max}; \quad (22b)$$

From (22), it is obvious that the problem is a fractional non-convex optimization problem. Even though the Dinkelbach's method [35] can be used to transform the fractional problem

into a linear form, the number of iterations in the Dinkelbach's method can be large. Therefore, we derive a more straightforward OPA method by analyzing the property of the problem.

We define  $P_{min}$  as the minimum transmit power to ensure the QoS in (22a), which is  $P_{min} = \frac{th}{r_i^{DF}}$  in the DF relay case. Then we have the following proposition regarding the optimal solution to (22).

**Proposition 4:** The objective function,  $"_i^{DF}$ , with respect to (w.r.t.)  $P_s^{DF}$ , is a unimodal function and there exists a unique global maximizer  $\hat{P}_s^{DF}$ , which is the solution to the following equation

$$\frac{r_i^{DF}(P_c + \frac{1}{2}\hat{P}_s^{DF})}{1 + \frac{r_i^{DF}\hat{P}_s^{DF}}{P_s^{DF}}} = \ln(1 + \frac{r_i^{DF}\hat{P}_s^{DF}}{P_s^{DF}}); \quad (23)$$

**Proof:** See [36].

Since  $P_{min}$  and  $P_{max}$  define the feasible transmit power region for QoS guarantee and power limitation, the optimal transmit power in the feasible region  $[P_{min}; P_{max}]$  is [37]

$$P_s = \max\{P_{min}; \min\{P_{max}; \hat{P}_s^{DF}\}\}; \quad (24)$$

All possible cases are illustrated in Fig. 3, in which  $\hat{P}_s^{DF}$  is the solution to (23). If  $P_{max} < \hat{P}_s^{DF}$ , the feasible transmit power falls in Region I, in which  $"_i^{DF}$  is strictly increasing and  $P_s = P_{max}$  renders the maximum  $"_i^{DF}$ . If  $P_{min} < \hat{P}_s^{DF} < P_{max}$ , the feasible transmit power falls in Region II and the optimal transmit power  $P_s$  is equal to  $\hat{P}_s^{DF}$  that renders the maximum  $"_i^{DF}$ . If  $\hat{P}_s^{DF} < P_{min} < P_{max}$ , the feasible transmit power falls in Region III, in which  $"_i^{DF}$  is strictly decreasing and  $P_s = P_{min}$  renders the maximum  $"_i^{DF}$ . Therefore, through OPA method, the optimal transmit power is obtained by (24).

According to Proposition 1, 2, and 4, the designed relay and power allocation can guarantee the optimality in single-carrier SWIPT cooperative networks.

### C. Complexity Analysis for DF Relay

Suppose the average number of iterations in Dinkelbach's method is  $K$ . Using Dinkelbach's method with exhaustive search requires  $KM$  calculations. However, through the proposed OPA method, each user only needs to calculate twice to obtain the optimal selected relay and transmit power. Therefore, with the proposed DF relay selection scheme and energy-efficient OPA method, the optimal solution to the energy efficiency problem (10) in DF relay networks can be obtained with low computational complexity.

## IV. RESOURCE ALLOCATION FOR AF RELAY

In this section, the AF relay energy-efficient resource allocation problem is decomposed into relay selection and energy-efficient power allocation. The relay selection scheme and energy-efficient OPA method are derived and discussed.

According to Proposition 1, we similarly have Lemma 1 for the AF relay case as follows.

**Lemma 1:** For the joint SWIPT AF relay selection and power allocation problem (16), the relay selection can be decoupled from the power allocation without loss of optimality. The maximum energy efficiency,  $"^{AF}$ , can be obtained at the AF relay with the highest CNR.

With Lemma 1, the AF relay selection can be carried out without consideration of the power allocation, which decomposes the original problem (16).

### A. AF Relay Selection

Similar to the DF relay case, the AF relay selection scheme is also based on the best CNR principle. With no signal decoding process at the AF relay, the AF relay selection scheme is only based on the CNR at the destination node, i.e.,

$$i = \arg \max_i f_d^{AF} g_i; \quad (25)$$

where  $f_d^{AF}$  is the CNR at the destination node through the  $i$ -th AF relay, which is

$$f_d^{AF} = \frac{i(1-i)jg_{sr_i}^A jg_{r_i d}^2 P_s}{ijg_{sr_i}^2 jg_{r_i d}^2 P_s^2 + (1-i)jg_{sr_i}^2 P_s^2 + i^4}; \quad (26)$$

Given that  $P_s$  in practice, (26) can be rewritten as

$$f_d^{AF} = \frac{i(1-i)jg_{sr_i}^2 jg_{r_i d}^2}{ijg_{r_i d}^2 + (1-i)^2}; \quad (27)$$

For (27), we have the following proposition.

**Proposition 5:** The optimal splitting ratio  $i$  for AF relay selection is

$$i = \frac{1}{1 + \frac{P_c}{jg_{r_i d}}}; \quad (28)$$

**Proof:** See Appendix D.

Substituting (28) into (27), we obtain the index of the selected AF relay as

$$i = \arg \max_i f_d \frac{jg_{sr_i}^2 jg_{r_i d}^2}{(1 + \frac{P_c}{jg_{r_i d}})^2} g_i; \quad (29)$$

With full knowledge of CSI, the FRS of the AF relay case is optimal based on (29).

With partial knowledge of CSI, similar to the DF relay case, the FLRS of the AF relay case is

$$i = \arg \max_i f_j g_{sr_i}^2 g_i \quad (30)$$

when only  $jg_{sr_i}^2$  is known.

### B. Energy-Efficient Power Allocation for AF Relay

With the selected AF relay  $i$  and corresponding optimal power splitting ratio  $i$ , the energy efficiency optimization problem for the AF relay case, similar to that of the DF relay case, can be simplified as

$$\max_{P_s} \quad "^{AF} = \frac{1}{2} \log_2 \left( 1 + \frac{f_d^{AF} P_s}{1 + P_c} \right) \quad (31)$$

$$s.t. : \quad f_d^{AF} P_s \leq th; \quad (31a)$$

$$0 \leq P_s \leq P_{max}; \quad (31b)$$

Therefore, the same methodology can be adopted to obtain the optimal transmit power,  $P_s$ , that is

$$P_s = \max \{P_{min}; \min \{P_{max}; \dot{P}_s^{AF} gg\} \} \quad (32)$$

where  $P_{min} = \frac{\eta P_c}{d^{AF}}$  is the minimum transmit power in the AF relay case and  $\dot{P}_s^{AF}$  is the solution to

$$\frac{d^{AF} (P_c + \dot{P}_s^{AF})}{1 + d^{AF} \dot{P}_s^{AF}} = \ln(1 + d^{AF} \dot{P}_s^{AF}) \quad (33)$$

With the proposed AF relay selection scheme and energy-efficient OPA method, the optimal relay and transmit power for the best energy efficiency in AF relay networks can be obtained with low computational complexity.

## V. PERFORMANCE ANALYSIS

In this section, we analyze and compare the performance of the SWIPT cooperative wireless networks with DF and AF relays. Besides, we extend the SWIPT cooperative wireless networks with DF or AF relays to a hybrid SWIPT cooperative wireless network with both DF and AF relays.

### A. Performance Comparison

According to the derived closed-form expressions of the optimal power splitting ratios in Propositions 2 and 4 for DF and AF relays, respectively, the corresponding CNR expressions for DF and AF relays are

$$d^{DF} = \frac{jg_{sr_i} j^2 g_{r_i} d^2}{2(1 + jg_{r_i} d^2)} \quad (34)$$

and

$$d^{AF} = \frac{jg_{sr_i} j^2 g_{r_i} d^2}{2(1 + \frac{P_c}{P_s} jg_{r_i} d^2)} \quad (35)$$

respectively.

It is obvious that the denominator of (35) is a little bit larger than that of (34), which is caused by the amplified noise from the source node to the AF relay. Therefore, the AF relay renders lower energy efficiency than the DF relay with the same power allocation. Since only the limited transferred power is used for cooperative transmission at the selected relay in the proposed SWIPT cooperative wireless networks due to power transfer efficiency and path loss, the amplification of noise is limited. Therefore, the performance difference between DF and AF relays is small. This is also verified in our simulation results that provide quantitative comparison. Therefore, in comparison with DF relays, AF relays are preferred in SWIPT wireless networks because of the much simpler structures of AF relays in practice.

### B. Hybrid SWIPT Cooperative Wireless Network Extension

In a hybrid SWIPT cooperative wireless network, both DF and AF relays exist. The relay selection between DF and AF relays cannot be determined through directly comparing their CNRs. To avoid calculating and comparing the energy efficiency with each relay, we can separate the SWIPT relays into DF and AF relay groups. In each group, we use the

TABLE I: Simulation Parameters

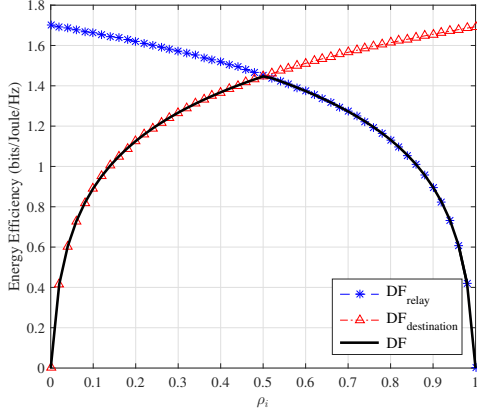
Parameter	Value
Channel bandwidth	10 kHz
Range of source node	60 m
Average number of relays	6 /cell
Relay power amplifier efficiency factor	0.9
Source node power amplifier efficiency factor	0.9
Efficiency coefficient	1
SINR threshold	0 dB
Maximum power $P_{max}$	46 dBm
Static circuit power $P_c$	10 dBm
Noise power density	-165 dBm/Hz

proposed resource allocation schemes to determine the best energy efficiency. Then the resource allocation resulting in a better energy efficiency among the two is chosen as the optimal solution.

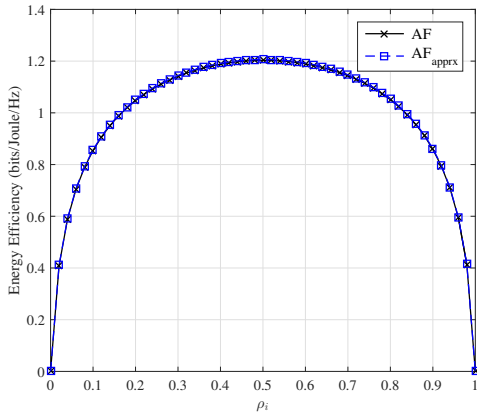
## VI. SIMULATION RESULTS

In this section, simulation results are presented to illustrate the improvement of the proposed schemes in a SWIPT cooperative wireless network. We use MATLAB in the entire simulation. The distance between the source node and destination node is 60 m. The SWIPT relays are randomly deployed following a homogeneous Poisson point process [5] with intensity  $\lambda = 0.0005/\text{m}^2$ , which implies the average number of available relays is 6 in the range of the source node of 60 m. Rayleigh fading is used as channel fast fading in the simulation. Since no direct link exists between the source and destination nodes, relays are not allowed to be too close to either the source or destination node, so as to avoid the case similar to the existence of direct link. The distances between the source node and relays are within a range of 15 m to 45 m. The path loss model is  $38.46 + 20 \log l$  dB [38]. The main simulation parameters are listed in Table I.

In Fig. 4, the energy efficiency performance is illustrated with different power splitting ratios in both the DF and AF relay cases. Similarly to [29], to reveal the relationship between the energy efficiency and power splitting ratio clearly, parameters are normalized in this simulation and only the selected relay  $i$  is considered. Only in this simulation, the transmit power  $P_s$  is 1 W, the static circuit power  $P_c$  is 0.01 W, the distances  $l_{sr_i}$  and  $l_{r_i d}$  are normalized to 1 m, and the noise power is scaled to be 0.01 W [29]. We normalize the path loss to be 1 so that the results illustrated purely reflect the relationship between energy efficiency and power splitting ratio with respect to the fast fading. In Fig. 4(a), the energy efficiency performance in the DF relay case is determined by the minimum performance of the first link that is from the source node to the relay and the second link that is from the relay to the destination node, which is consistent with (8). The DF curve is for the overall link energy efficiency, the  $DF_{relay}$  curve is for the energy efficiency from the base station to the



(a) DF relay case.



(b) AF relay case.

Fig. 4: Energy efficiency vs. power splitting ratio  $\rho_i$  with different relay types.

relay, and the  $DF_{destination}$  curve is for the energy efficiency from the relay to the destination. From Fig. 4(b), it is obvious that the energy efficiency with the approximation of SNR in the proposed relay selection is nearly the same as that with the exact SNR in the AF relay case. Due to the normalization of the path loss, the means of the channel gains of the first and the second links over 100 runs are 1.1032 and 0.9824, respectively. Therefore, the optimal splitting ratios of DF and AF relays are 0.5044 and 0.5022, respectively, according to Propositions 2 and 4, which are verified in the figures.

In Fig. 5, the relationship between the energy efficiency and the distance between the source node and the selected relay  $l_{sr_i}$ , is shown. Since no direct link between the source node and the destination node is assumed, the relay is deployed with 15 m  $l_{sr_i}$  45 m in this simulation. As shown in Fig. 5, the network achieves the minimum energy efficiency when the selected relay is in the middle of the source node and the destination node. When the selected relay approaches towards the source node or the destination node, the energy efficiency of the network increases gradually. This is because when the selected relay is close to the source node,  $|g_{sr_i}|$  is larger than  $|g_{r,d}|$  and thus the CNR of the first link is larger than that of the second link. In this case, the power splitting ratio is

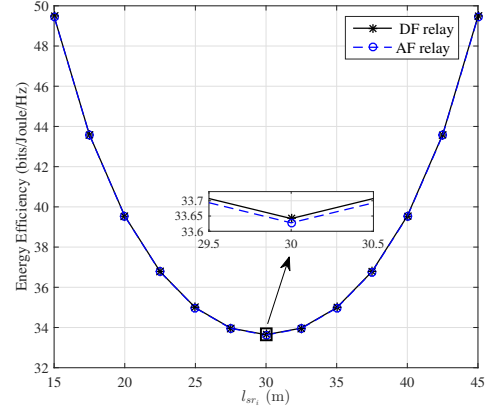


Fig. 5: Energy efficiency vs. distance between the source node and the relay in both DF and AF relay cases, where  $P_c = 10$  dBm,  $P_{max} = 46$  dBm, and  $\sigma^2 = -125$  dBm.

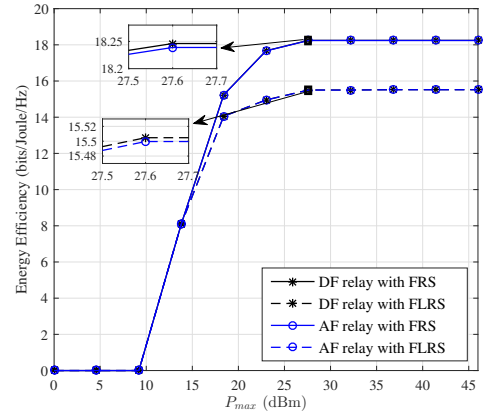


Fig. 6: The energy efficiency performance of different relay selection schemes with full and partial knowledge of CSI, where  $P_c = 10$  dBm and  $\sigma^2 = -125$  dBm.

adjusted towards 1 to transfer more power for the transmission of the second link. When the selected relay is close to the destination node,  $|g_{sr_i}|$  and the CNR of the first link are small. The power splitting ratio is adjusted towards 0 to increase the rate of the first link for the DF relay and reduce the power of the amplified noise for the AF relay. This phenomenon is similar to the throughput performance of the time splitting SWIPT relay networks in [14]. In Fig. 5, the energy efficiency in the DF relay case is slightly larger than that in the AF relay case, which is consistent with our analysis. In consideration of partial CSI, the energy efficiency of both AF and DF relays would be worse due to the channel estimation error. However, the minimum energy efficiency can still be reached when the selected relay is in the middle between the source and destination nodes.

In Fig. 6, the energy efficiency performance of different relay selection schemes is illustrated in both the DF and AF relay cases. It is obvious that the FRS is better than the FLRS scheme in both cases. Before  $P_{max}$  reaches 9 dBm with full



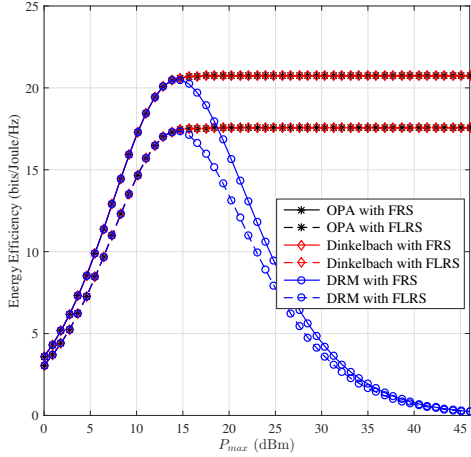


Fig. 7: Energy efficiency vs. maximum transmit power with different power allocation methods and relay selection schemes, where  $P_c = 10$  dBm and  $\gamma = -125$  dBm.

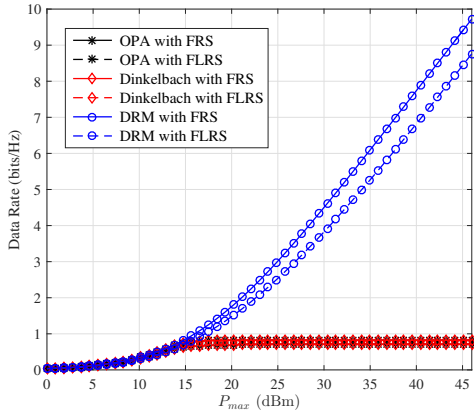


Fig. 8: Data rate vs. maximum transmit power with different power allocation methods and relay selection schemes, where  $P_c = 10$  dBm and  $\gamma = -125$  dBm.

and partial knowledge of CSI, the energy efficiency remains 0, since the requirement on SNR cannot be satisfied within the given  $P_{max}$ . After that, the energy efficiency increases with the increase of  $P_{max}$ . When  $P_{max}$  is larger than 27.6 dBm, the increase of  $P_{max}$  no longer provides energy efficiency benefit. As shown in Fig. 6, the DF relay achieves better performance than the AF relay.

In Fig. 7, the energy efficiency of the proposed OPA method is compared with that of the data rate maximization (DRM) method that allocates powers to maximize the data rate and the Dinkelbach's method in the DF and AF relay cases with different relay selection schemes. Different line colors are for different methods and different line types illustrate different CSI assumption. The tolerance of the Dinkelbach's method is  $10^{-3}$ . In Fig. 7, all the methods are shown to achieve the same energy efficiency, data rate, and power consumption when  $P_{max}$  is smaller than 15 dBm. In this case, the available power region for the proposed OPA method is Region I illustrated

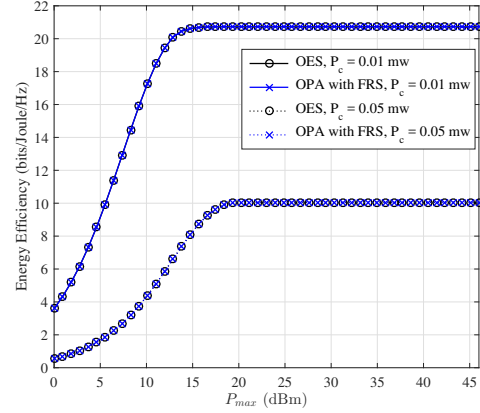


Fig. 9: Comparison of OES and OPA with FRS, where  $\gamma = -125$  dBm.

in Fig. 3, which results in  $P_{max}$  as the optimal power. Meanwhile, the DRM method also assigns  $P_{max}$  to achieve the maximum data rate. The Dinkelbach's method obtains the same performance through iterative calculations. When  $P_{max}$  keeps increasing, our proposed OPA method and the Dinkelbach's method still achieve the best energy efficiency but the energy efficiency of the DRM method decreases. This is because that the proposed OPA method and the Dinkelbach's method keep the optimal power but the DRM power allocation method still adopts  $P_{max}$  that results in increasing data rate but decreasing energy efficiency, as shown in Fig. 7 and 8. Therefore, the DRM method deteriorates the energy efficiency of the network. As shown in Fig. 7, our proposed OPA method renders the same performance as the Dinkelbach's method without iterative calculations and outperforms DRM method.

In Fig. 9, the energy efficiency of the optimal exhaustive search (OES) and that of our proposed decomposed OPA with FRS scheme are compared. The solid curves at the top are for 0.01mw total static power consumption and the dashed curves at the bottom are for 0.05mw total static power consumption. The OES scheme exhaustively calculates the energy efficiency with each SWIPT relay and choose the best result, which is the optimal solution to the formulated relay selection and power allocation problem. As shown in Fig. 9, when the full CSI is available, our proposed decomposed OPA with FRS scheme achieves the same performance as the optimal solution, which verifies the optimality of our scheme. With the different total static power consumptions of the source and destination nodes, the energy efficiency performance degrades with the increasing static power consumption, but the optimality of our proposed scheme still holds.

In Fig. 10, different relay selection schemes with partial knowledge of CSI are compared in the DF and AF relay cases. The results are averaged over  $10^3$  simulation trials. Different line types show different relay selection schemes and different line colors demonstrate different relay types. Since AF and DF have small performance difference as discussed in Section V, lines with different colors, i.e., for different relay types, almost overlap. However, different relay selection schemes

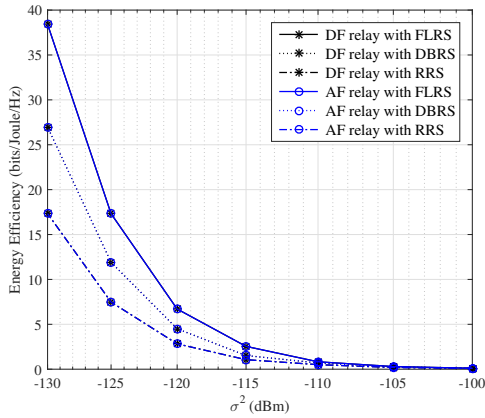


Fig. 10: Energy efficiency with different relay selection schemes with partial knowledge of CSI, where  $P_c = 10$  dBm and  $P_{max} = 46$  dBm.

render different energy efficiency performance. As shown in Fig. 10, the FLRS scheme renders better energy efficiency than the DBRS that selects the relay with the shortest source to relay distance, and the RRS that randomly selects the relay without any further information. The FLRS achieves the best energy efficiency performance and the RRS renders the lowest energy efficiency. Since only distance is considered, DBRS renders worse performance than the proposed FLRS scheme. As the noise power increases, all the relay selection schemes in both the DF and AF relay cases experience degraded energy efficiency performance.

## VII. CONCLUSIONS

In this paper, energy-efficient relay selection and power allocation are studied in SWIPT cooperative wireless networks with DF and AF relays. Non-convex energy efficiency optimization problems are formulated for both DF and AF relay cases. A decomposed relay selection and power allocation scheme is derived without loss of optimality. Given the CNRs at the destination node, the closed-form expressions of the optimal power splitting ratios are derived. With the optimal power splitting ratios, relay selection schemes with full and partial knowledge of CSI are developed, respectively. Based on the property of the simplified optimization problem, a simple closed-form power allocation scheme is adopted in both DF and AF relay networks. Furthermore, the performance difference between DF and AF relays is discussed. In addition, simulation results verify our analyses and demonstrate the improvement of our proposed resource allocation scheme in terms of the energy efficiency in SWIPT cooperative wireless networks. The proposed scheme achieves better energy efficiency in comparison with typical relay selection schemes and renders the same energy efficiency performance as the optimal exhaustive search scheme but with lower computational complexity.

## APPENDIX A

### PROOF OF PROPOSITION 1

*Proof:* According to (4)-(9), the energy efficiency with the  $i$ -th relay is equivalent to

$$\eta_i^{DF} = \frac{\frac{1}{2} \log_2(1 + i P_s)}{1 P_s + P_c},$$

where  $i$  is the equivalent CNR with the  $i$ -th relay, which is

$$i = \min \left\{ \frac{(1 - i) j g_{sr_i} j^2}{2}, \frac{i j g_{sr_i} j^2 g_{r_i} d^2}{2} g \right\}.$$

Note that different relays have the same transmit power source  $P_s$  due to the SWIPT capability.

Considering the  $i$ -th relay with equivalent CNR  $i$  and  $j$ -th relay with equivalent CNR  $j$ , where  $i > j$ , we denote  $\eta_i^{DF}$  and  $\eta_j^{DF}$  as the best energy efficiency with the  $i$ -th and  $j$ -th relays, respectively. Additionally, the corresponding optimal transmit powers of the  $i$ -th and  $j$ -th relays are  $P_{s;i}$  and  $P_{s;j}$ , respectively. Accordingly, we have

$$\eta_i^{DF} = \eta_i^{DF}(P_{s;i}) = \frac{\frac{1}{2} \log_2(1 + i P_{s;i})}{1 P_{s;i} + P_c}$$

and

$$\eta_j^{DF} = \eta_j^{DF}(P_{s;j}) = \frac{\frac{1}{2} \log_2(1 + j P_{s;j})}{1 P_{s;j} + P_c}.$$

Since  $P_{s;i}$  is the optimal power for the  $i$ -th relay and the ratio of the logarithmic and linear functions that compose  $\eta_i^{DF}$  is quasi-concave as illustrated in Fig. 3, we have

$$\eta_i^{DF}(P_{s;i}) > \eta_i^{DF}(P_{s;j}); \quad (36)$$

With  $i > j$ , we have

$$\eta_i^{DF}(P_{s;j}) > \eta_j^{DF}(P_{s;j}); \quad (37)$$

According to (36) and (37), we have

$$\eta_i^{DF}(P_{s;i}) > \eta_j^{DF}(P_{s;j}) \text{ when } i > j;$$

Therefore, the relay selection can be decoupled from the power allocation without loss of optimality and the DF relay with the highest equivalent CNR will render the maximum energy efficiency, which completes the proof. ■

## APPENDIX B

### PROOF OF PROPOSITION 2

*Proof:* Since the DF relay selection scheme is decided by  $\min \left\{ \frac{DF}{r_i}, \frac{DF}{d} \right\} g$ , there are two cases:  $\frac{DF}{r_i} < \frac{DF}{d}$  and  $\frac{DF}{r_i} > \frac{DF}{d}$ .

A.  $\frac{DF}{r_i} < \frac{DF}{d}$

To achieve the best CRN,  $\frac{DF}{d} = \frac{i j g_{sr_i} j^2 g_{r_i} d^2}{2}$  should be maximized. It is obvious that  $\frac{DF}{d}$  increases as  $i$  increases and  $\frac{DF}{r_i}$  increases as  $i$  decreases. Therefore, in this case, we increase  $i$  until

$$\frac{DF}{d} = \frac{DF}{r_i};$$

i.e.,

$$\frac{i j g_{sr_i} j^2 j g_{r_i} d^2}{2} = \frac{(1-i) j g_{sr_i} j^2}{2};$$

which results in the optimal splitting ratio  $i = \frac{1}{1 + \frac{1}{j g_{r_i} d^2}}$ .

B.  $\frac{DF}{r_i} < \frac{DF}{d}$

In this case,  $\frac{DF}{r_i} = \frac{(1-i) j g_{sr_i} j^2}{2}$  should be maximized. Since  $\frac{DF}{r_i}$  increases as  $i$  decreases and  $\frac{DF}{d}$  increases as  $i$  increase, we decrease  $i$  until  $\frac{DF}{d} = \frac{DF}{r_i}$  and obtain the same optimal splitting ratio  $i = \frac{1}{1 + \frac{1}{j g_{r_i} d^2}}$ .

Combining both cases, we complete the proof of Proposition 2. ■

## APPENDIX C

### PROOF OF PROPOSITION 3

*Proof:* According to [39], [40], and the Taylor expansion, the following is a good approximation to the expectation of the ratio of two random variables

$$E\left[\frac{X}{Y}\right] \approx \frac{E[X]}{E[Y]} - \frac{\text{cov}[X; Y]}{E[Y]^2} + \frac{E[X]}{E[Y]^3} \text{var}[Y]; \quad (38)$$

where  $X$  and  $Y$  are two random variables,  $\text{var}[\cdot]$  is the variance operator, and  $\text{cov}[\cdot]$  is the covariance operator. Let  $X = j g_{sr_i} j^2 j g_{r_i} d^2$  and  $Y = 1 + j g_{r_i} d^2$ . since  $j g_{r_i} d^2$  is an exponential random variable, we have  $E[X] = \frac{j g_{sr_i} j^2}{1}$ ,  $E[Y] = \frac{1}{1}$ ,  $\text{var}[X] = \frac{j g_{sr_i} j^4}{1}$ , and  $\text{var}[Y] = \left(\frac{1}{1}\right)^2$ . Through simple manipulation, we have  $E[XY] = \frac{j g_{sr_i} j^2}{1} + \frac{2 j g_{sr_i} j^2}{1}$  and  $E[X]E[Y] = \frac{j g_{sr_i} j^2}{1} + \frac{j g_{sr_i} j^2}{1}$ . Since  $\text{cov}[X, Y] = E[XY] - E[X]E[Y]$ , we can obtain  $\text{cov}[X, Y] = \frac{j g_{sr_i} j^2}{1}$ .

If we combine the above with (38);  $E\left[\frac{j g_{sr_i} j^2 j g_{r_i} d^2}{1 + j g_{r_i} d^2}\right]$  can be approximated as

$$E\left[\frac{j g_{sr_i} j^2 j g_{r_i} d^2}{1 + j g_{r_i} d^2}\right] \approx \frac{1 j g_{sr_i} j^2}{\left(1 + \frac{1}{1}\right)^2} + \frac{2 j g_{sr_i} j^2}{\left(1 + \frac{1}{1}\right)^3}; \quad (39)$$

which completes the proof of Proposition 3. ■

## APPENDIX D

### PROOF OF PROPOSITION 5

*Proof:* Rewrite the CNR at the destination node in AF relay networks,  $\frac{AF}{d}$ , as a function of power splitting ratio  $i$  as

$$\begin{aligned} y(i) &= \frac{AF}{d} = \frac{i(1-i) j g_{sr_i} j^2 j g_{r_i} d^2}{i j g_{r_i} d^2 + (1-i)^2} \\ &= \frac{a i(1-i)}{b i + c(1-i)}; \end{aligned} \quad (40)$$

where  $a = j g_{sr_i} j^2 j g_{r_i} d^2$ ,  $b = j g_{r_i} d^2 + 2$ , and  $c = 2$ .

Taking the derivative of (40) w.r.t.  $i$ , we have

$$\frac{\partial y}{\partial i} = \frac{a(b-c) i^2 - 2ac i + ac}{[(b-c) i + c]^2}; \quad (41)$$

Define a function  $y(i)$  representing the numerator of (41), i.e.,  $y(i) = a(b-c) i^2 - 2ac i + ac$ . Since  $[(b-c) i + c]^2 >$

0, the sign of (41) only depends on  $y(i)$ . It is clear that  $y(0) = ac > 0$ ,  $y(1) = ab < 0$ , and  $y(i)$  is a quadratic function. Therefore, there exists an optimal  $i$  satisfying  $y(i) = 0$ , which maximizes  $y(i)$ . According to the property of the quadratic function, there are two possible cases:  $b > c$  and  $b < c$ .

(a)  $b > c$

In this case, the greater root should be chosen. Therefore, the optimal splitting ratio is

$$\begin{aligned} i &= \frac{2ac}{2a(b-c)} \sqrt{\frac{4a^2c^2 + 4a(b-c)ac}{4a^2c^2}} \\ &= \frac{1}{1 + \frac{1}{j g_{r_i} d^2}}. \end{aligned}$$

(b)  $b < c$

In this case, the smaller root should be chosen. Since  $y(b) > 0$ , the smaller root actually is the same as the greater root in case a. Therefore, the expression for the optimal splitting ratio is the same.

Combining (a) and (b), we complete the proof of Proposition 5. ■

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