How Much Training is Needed Against Smart Jamming?

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***Abstract*—This paper studies training-based transmissions over multiple-input multiple-output (MIMO) fading channels in the presence of jamming. Each transmission block consists of a training phase and a data transmission phase. From an information-theoretic viewpoint, we formulate a max-min problem on the energy allocation between the two phases. The legitimate user of the channel aims to design a robust energy allocation strategy which maximizes its data rate under the worst case scenario assuming that the jammer is able to optimize its jamming energy allocation between the training phase and the data transmission phase. For a fixed training length, we derive an analytical solution to the robust energy allocation. When the training length is allowed to vary, we show that a robust design of the training length is generally larger than the number of transmit antennas and approaches half of the block length at low signal to jamming and noise ratio (SJNR). Our numerical results demonstrate a potential of 20% - 40% performance gain by using the proposed robust designs in various scenarios.**

I. Introduction

The problem of jamming in wireless communications has drawn considerable attention due to the nature of the wireless medium. Since the jammer and the legitimate users of the communication channel have the opposite objectives, theoretical studies often model the problem of jamming as zero-sum games [1–3]. The optimal transmission strategies of the transmitter and the jammer were studied in [1, 2] from an information-theoretic viewpoint, assuming the jammer has knowledge of the transmitted message (i.e., correlated jamming). Considering long-term energy constraints on block-wise transmissions, the optimal transmit and jamming energy allocations among multiple blocks were found as min-max and max-min solutions in [3].

One of the main assumptions in the above mentioned works is the perfect knowledge of the channel state information (CSI) at the receiver. In practical scenarios, however, the CSI needs to be estimated. One common approach to enable channel estimation is training-based transmission, in which known pilot symbols are periodically inserted into data transmission blocks [4]. When a jammer is present, jamming during the pilot transmission (or training) can result in poor channel estimation and, hence, impair the data detection. To suppress the effect of jamming, an improved channel estimation scheme was proposed in [5] for orthogonal frequency-division multiplexing (OFDM) systems, which detects and removes jammed pilot subcarriers. The effect of jamming on the information capacity in wideband regime was studied in [6, 7], and their results showed the benefit of having impulsive training that randomly changes its position in a transmission block. While the studies in [5–7] focused on the design from the legitimate user's point of view, very few existing results look at the design of smart jamming in training-based transmissions. Recently, jamming strategies, which make use of the legitimate user's CSI, were proposed in [8] to attack the channel estimation in singular value decomposition based MIMO systems as well as systems using space-time block codes.

In this paper, we study training-based MIMO systems in fading channels with jamming. For energy constrained communications, the energy allocation between training and data transmission is an important design parameter. Many works have been devoted to analyzing the trade-off in the transmit time and energy allocation between training and data transmission in jamming-free systems, e.g., in [9, 10]. To the best of the authors' knowledge, no such study has been carried out for systems with jamming. In this work, we consider systems with both the legitimate user and the jammer having their own energy constraints. The legitimate user aims to find robust choices of energy allocation and training length (i.e., the duration of the training phase) which maximize its data rate under the worst case scenario, assuming that the jammer is able to optimize its jamming energy allocation between the training phase and the data transmission phase. The main contributions of this work are summarized as follows:

- We derive a closed-form solution of the optimal (rate-minimizing) jamming energy allocation for any given legitimate user's strategy. Our result reveals that imbalance in the signal to jamming and noise ratio (SJNR) between the two phases is desirable for the jammer.
- For any fixed training length, we derive an analytical solution of the robust energy allocation for the legitimate user. Our numerical results demonstrate a significant improvement in the data rate by using the proposed robust design.
- If the training length is allowed to vary, we show that a robust choice of the training length is generally larger than the number of transmit antennas and approaches half of the block length at low SJNR.

The following notations will be used in the paper: Boldface

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upper and lower case letters denote matrices and column vectors, respectively. $\boldsymbol{I}$ is the identity matrix. $[\cdot]^{\dagger}$ denotes the complex conjugate transpose operation. $\mathbb{E}\{\cdot\}$ denotes the mathematical expectation. $|\cdot|$ denotes the matrix determinant.

II. System Model

We consider a flat-fading MIMO system in the presence of jamming. The legitimate user has N_t transmit antennas and N_r receive antennas. The received signal is given by

$$\boldsymbol{y} \;=\; \boldsymbol{H}\boldsymbol{x}+\boldsymbol{w}+\boldsymbol{n}, \qquad (1)$$

where $\boldsymbol{x}$ and $\boldsymbol{y}$ are the $N_t\times 1$ transmitted symbol vector and the $N_r\times 1$ received symbol vector, respectively. $\boldsymbol{H}$ is the $N_r\times N_t$ channel gain matrix, $\boldsymbol{w}$ is the $N_r\times 1$ received jamming noise, and $\boldsymbol{n}$ is the $N_r\times 1$ additive white Gaussian noise (AWGN) at the receiver. We assume that $\boldsymbol{H}$ is circularly symmetric whose entries are independent and identically distributed (i.i.d.) zero-mean complex Gaussian random variables. The same assumption is made to $\boldsymbol{n}$ and $\boldsymbol{w}$. Although the distribution of the jamming signal $\boldsymbol{w}$ can be arbitrary in general, the Gaussian distribution is usually found to be the optimal choice from the jammer's design point of view [2]. Without loss of generality, we normalize the variances of the entries in $\boldsymbol{H}$ and $\boldsymbol{n}$ to one.

A. Training-Based Transmission

We assume that the channel gains $\boldsymbol{H}$ remain constant during one transmission block of L symbols, and change to some independent values in the next block. Each transmission block consists of a training phase and a data transmission phase. During training, the transmitter sends L_p pilot symbols which are used by the receiver to estimate $\boldsymbol{H}$. We assume $L_p\geq N_t$ so that there are at least as many measurements as unknowns for estimation. During data transmission, the transmitter sends L_d data symbols and the receiver performs coherent detection using the estimated channel gains. Therefore, we have $L=L_p+L_d$. The signal power (per symbol) during training and data transmission is given by $\mathcal{P}_p$ and $\mathcal{P}_d$, respectively.

Apart from the legitimate user's transmission, the jammer also injects artificial noise signals. Similar to [6], we assume that the jammer knows the training-based block-wise transmission and uses different power levels to jam training and data transmission. The jamming signal power during the training phase and data phase is denoted by $\mathcal{P}_{wp}$ and $\mathcal{P}_{wd}$, respectively.

B. Channel Estimation

Collecting the received symbol vectors during training into a $N_r\times L_p$ matrix, we have

$$\boldsymbol{Y}_p \;=\; \boldsymbol{H}\boldsymbol{X}_p+\boldsymbol{W}_p+\boldsymbol{N}_p, \qquad (2)$$

where $\boldsymbol{X}_p$ is the $N_t\times L_p$ pilot matrix, $\boldsymbol{W}_p$ and $\boldsymbol{N}_p$ are the jamming and noise matrices, respectively.

We consider the linear minimum mean square error (LMMSE) estimator [11], which is given by

$$\hat{\boldsymbol{H}}=\boldsymbol{Y}_p\Big({\boldsymbol{X}_p}^{\dagger}\boldsymbol{X}_p+(\mathcal{P}_{wp}+1)\boldsymbol{I}\Big)^{-1}{\boldsymbol{X}_p}^{\dagger}, \qquad (3)$$

and the estimation error is given by $\tilde{\boldsymbol{H}}=\boldsymbol{H}-\hat{\boldsymbol{H}}$. The optimal $\boldsymbol{X}_p$ that minimizes the channel estimation error has an orthogonal structure such that $\boldsymbol{X}_p{\boldsymbol{X}_p}^{\dagger}=(\mathcal{P}_pL_p/N_t)\boldsymbol{I}$ [9]. With the orthogonal pilots, the variance of each element of $\tilde{\boldsymbol{H}}$ is then given by [11]

$$\sigma_{\tilde{h}}^2=\left(1+\frac{\mathcal{P}_pL_p}{N_t(\mathcal{P}_{wp}+1)}\right)^{-1}=\frac{\mathcal{P}_{wp}+1}{\mathcal{P}_{wp}+1+\mathcal{P}_pL_p/N_t}. \qquad (4)$$

III. Problem Formulation

We consider an energy constrained scenario, where the total energy for a transmission block is fixed for both the legitimate user and the jammer. This can also be interpreted as a constraint on the average transmit or jamming power in each block [9]. We denote the fixed energy budget of the legitimate user and the jammer as $\mathcal{E}$ and $\mathcal{E}_w$, respectively, which are assumed to be known to each other.[1] Hence, we have

$$\mathcal{E}=\mathcal{P}_pL_p+\mathcal{P}_dL_d,\qquad \mathcal{E}_w=\mathcal{P}_{wp}L_p+\mathcal{P}_{wd}L_d. \qquad (5)$$

We also define the average power budget for the legitimate user and the jammer as $\mathcal{P}\triangleq\mathcal{E}/L$ and $\mathcal{P}_w\triangleq\mathcal{E}_w/L$, respectively.

Denote the ratio of energy allocated to training as ϕ and ζ for legitimate user and jammer, respectively. Hence, for the legitimate user, we have

$$\mathcal{P}_p=\frac{\phi\mathcal{E}}{L_p},\qquad \mathcal{P}_d=\frac{(1-\phi)\mathcal{E}}{L_d}. \qquad (6)$$

For the jammer, we have

$$\mathcal{P}_{wp}=\frac{\zeta\mathcal{E}_w}{L_p},\qquad \mathcal{P}_{wd}=\frac{(1-\zeta)\mathcal{E}_w}{L_d}. \qquad (7)$$

A max-min problem on the energy allocation is formulated as follows: We consider the scenario that the communication is in the presence of a smart jammer. Therefore, the legitimate user aims to design a robust energy allocation strategy which maximizes its data rate, assuming that the smart jammer can always optimize its jamming energy allocation.

The objective function for the max-min problem is the achievable data rate, which can be characterized by the ergodic capacity of the legitimate user. Although the exact expression of the ergodic capacity is unknown, one commonly used lower bound can be found by following [9] as

$$C_{\text{LB}}=\frac{L_d}{L}\mathbb{E}\Big\{\log_2\Big|\boldsymbol{I}+\rho_{\text{eff}}\frac{\boldsymbol{H}_0{\boldsymbol{H}_0}^{\dagger}}{N_t}\Big|\Big\}, \qquad (8)$$

where $\boldsymbol{H}_0$ is statistically identical to $\boldsymbol{H}$, the fraction $\frac{L_d}{L}$ accounts for the capacity loss due to training overhead and ρ_{eff} is referred to as the effective SNR given by

$$\begin{aligned}\rho_{\text{eff}} \;&=\; \frac{(1-\sigma_{\tilde{h}}^2)\mathcal{P}_d}{\mathcal{P}_{wd}+1+\sigma_{\tilde{h}}^2\mathcal{P}_d}\\ &=\; \frac{\mathcal{P}_d\mathcal{P}_pL_p/N_t}{(\mathcal{P}_{wd}+1)(\mathcal{P}_{wp}+1+\mathcal{P}_pL_p/N_t)+(\mathcal{P}_{wp}+1)\mathcal{P}_d}. \qquad (9)\end{aligned}$$

[1] We assume that the locations of the transmitter and the receiver as well as the path loss exponent are known to the jammer. Hence, the jammer can infer $\mathcal{E}$ from its measurement and the distances between the three terminals. The receiver can also obtain $\mathcal{E}_w$ from its measurement and feed it back to the transmitter.

This capacity lower bound was shown to be a very accurate approximation of the true capacity for jamming-free systems with LMMSE channel estimation [12]. Hence, we use $C_{\rm LB}$ as the objective function for the max-min problem. When the training length is fixed, the max-min problem is given by

$$\max_{\phi} \min_{\zeta} \ \rho_{\rm eff}(\phi, \zeta), \tag{10}$$

since optimizing $C_{\rm LB}$ is the same as optimizing $\rho_{\rm eff}$ for any given L_p. When the training length is allowed to vary, the max-min problem is given by

$$\max_{L_p, \phi} \min_{\zeta} \ C_{\rm LB}(L_p, \phi, \zeta). \tag{11}$$

Note that $\zeta \in [0, 1]$, $\phi \in [0, 1]$, and $L_p \geq N_t$.

IV. Optimal Jamming Energy Allocation

We first solve the inner problem in the max-min formulation, i.e., finding the jamming energy allocation strategy ζ that solves $\min_\zeta \rho_{\rm eff}$ for any given legitimate user's energy allocation strategy ϕ and training length L_p. We denote the optimal value of ζ by ζ^*.

Intuitively, it is more efficient to jam the training phase since a short burst of strong jamming signal can effectively increase the channel estimation error and, hence, harm the data detection of the legitimate user. Therefore, one might expect that the optimal jamming strategy is to concentrate all energy in the training phase, i.e., $\zeta^* = 1$. The following lemma shows that this expectation is usually incorrect.

Lemma 1: The optimal jamming energy allocation strategy against a given legitimate user's strategy is given as

$$\zeta^* = \begin{cases} 0, & \text{for } \mathcal{E}_w < -\kappa, \\ 1, & \text{for } \mathcal{E}_w < \kappa, \\ \frac{\mathcal{E}_w + \kappa}{2\mathcal{E}_w}, & \text{otherwise,} \end{cases} \tag{12}$$

where

$$\begin{aligned} \kappa &= L_d - L_p + \mathcal{P}_d L_d - \mathcal{P}_p L_p^2 / N_t & (13) \\ &= L_d - L_p + \mathcal{E} - \mathcal{E}\phi(1 + L_p/N_t), & (14) \end{aligned}$$

where (14) is obtained by substituting (6) into (13).

Proof: Substituting (7) into (9), the optimal jamming energy allocation is given by

$$\begin{aligned} &\arg\max_{\zeta} \rho_{\rm eff}^{-1} \\ =&\arg\max_{\zeta} \left\{ \frac{\zeta(1-\zeta)\mathcal{E}_w}{L_d L_p} + \frac{1-\zeta}{L_d} + \frac{\zeta}{L_p} + \frac{(1-\zeta)\mathcal{P}_p L_p}{L_d N_t} + \frac{\zeta \mathcal{P}_d}{L_p} \right\} & (15) \\ =& \ \frac{\mathcal{E}_w + \kappa}{2\mathcal{E}_w}, & (16) \end{aligned}$$

where κ is defined in (13). Since $\zeta \in [0, 1]$, ζ^* is given by (16) if it is within this range. From (15), one can show that this objective function is concave in ζ. Therefore, $\zeta^* = 1$ if (16) is greater than 1, and $\zeta^* = 0$ if (16) is smaller than 0. ■

Corollary 1: The optimal jamming energy allocation ζ^* is a continuous and non-increasing function of ϕ.

Corollary 1 implies that the jammer should increase its power during the training (data) phase if the legitimate user decreases the training (data) power. In other words, imbalance in the SJNR between the two transmission phases is desired for the jammer.

In the case of $\mathcal{E}_w \gg \mathcal{E}$ and assuming that the jamming power is at least comparable to the receiver noise power, i.e., $\mathcal{P}_w \geq 1$ or $\mathcal{E}_w \geq L$, the optimal jamming energy allocation is given by

$$\zeta^* = \frac{\mathcal{E}_w + \kappa}{2\mathcal{E}_w} \approx \frac{\mathcal{E}_w + L_d - L_p}{2\mathcal{E}_w}, \tag{17}$$

which is independent of the legitimate user's strategy. In this case, it is never optimal to jam the training (or data) phase only. Furthermore, $\zeta^* \to \frac{1}{2}$ as $\mathcal{E}_w \to \infty$.

V. Robust Design of the Legitimate User

We now solve the outer problem of the max-min formulation. Depending on the degrees of freedom that the legitimate user has, it can adjust only the transmit power or both the transmit power and the block transmission structure. We denote the robust design of the energy allocation and training length as ϕ^o and L_p^o, respectively. We focus on the scenario in which the jamming power is at least comparable to the receiver noise power, i.e., $\mathcal{P}_w \geq 1$ or $\mathcal{E}_w \geq L$.

A. Fixed Training Length

For a fixed training length, the design problem is described in (10). In order to find the solution, we proceed as follows: The optimal jamming energy allocation given in (12) can be rewritten as

$$\zeta^* = \begin{cases} 0, & \text{for } 1 > \phi > \frac{1}{2} + \frac{1}{2}\frac{\alpha + 2\mathcal{E}_w + 2L - 4L_p}{\beta}, \\ 1, & \text{for } 0 < \phi < \frac{1}{2} + \frac{1}{2}\frac{\alpha - 2\mathcal{E}_w + 2L - 4L_p}{\beta}, \\ \frac{\mathcal{E}_w + \kappa}{2\mathcal{E}_w}, & \text{otherwise,} \end{cases} \tag{18}$$

where

$$\alpha = \mathcal{E}(1 - L_p/N_t), \ \ \beta = \mathcal{E}(1 + L_p/N_t).$$

We see that there are three different ranges of ϕ, each of which corresponds to a different case of ζ^* in (18). Therefore, we need to find the local optimal value of ϕ in each range and then select the one which gives the maximum $\rho_{\rm eff}$. Indeed, one can show that the optimal value of ϕ is greater than $\frac{1}{2}$ if $\zeta = 1$ (see proof in [13]). Therefore, there is no local optimal ϕ in the second case of (18).

When $\zeta = 0$, the solution to $\max_\phi \rho_{\rm eff}$ is given by

$$\phi_1 = \gamma + \sqrt{\gamma(\gamma - 1)}, \tag{19}$$

where

$$\gamma = \frac{\mathcal{E}N_t + L_d N_t (1 + \mathcal{E}_w / L_d)}{\mathcal{E}N_t - \mathcal{E}L_d(1 + \mathcal{E}_w/L_d)}.$$

Note that ϕ_1 is the local optimal value if it is within the range of ϕ in the first case of (18).

When $\zeta = \frac{\mathcal{E}_w + \kappa}{2\mathcal{E}_w}$, the solution to $\max_\phi \rho_{\rm eff}$ is given by

$$\phi_2 = \frac{1}{2} + \frac{1}{2}\frac{\alpha}{\beta + 2\mathcal{E}_w + 2L}. \tag{20}$$

Note that ϕ_2 is the local optimal value if it is within the range of ϕ in the third case of (18).

Now, the robust energy allocation strategy ϕ^o can be found using the following procedure:

Step 1: If $\phi_1 > \frac{1}{2} + \frac{1}{2}\frac{\alpha+2\mathcal{E}_w+2L-4L_p}{\beta}$, set $\rho_{\text{eff},1} = \rho_{\text{eff}}(\phi_1)$, where ϕ_1 is given in (19). Otherwise, set $\rho_{\text{eff},1} = 0$.
Step 2: If $\phi_2 < \frac{1}{2} + \frac{1}{2}\frac{\alpha+2\mathcal{E}_w+2L-4L_p}{\beta}$, set $\rho_{\text{eff},2} = \rho_{\text{eff}}(\phi_2)$, where ϕ_2 is given in (20). Otherwise, set $\rho_{\text{eff},2} = 0$.
Step 3: $\phi^o = \arg_\phi \max\{\rho_{\text{eff},1}, \rho_{\text{eff},2}\}$.

Since $\rho_{\text{eff}} = 0$ at both $\phi = 0$ and $\phi = 1$, there always exists a robust $\phi^o \in (0, 1)$ that solves the max-min problem in (10). Hence, $\rho_{\text{eff},1}$ and $\rho_{\text{eff},2}$ cannot equal zero at the same time.

The following lemma gives the robust design for the special case of $L_p = N_t$. Note that the choice of $L_p = N_t$ was shown to be the optimal training length in jamming-free systems [9].

Lemma 2: The robust energy allocation strategy of the legitimate user for $L_p = N_t$ is given by

$$\phi^o = \frac{1}{2}. \tag{21}$$

Proof: The value of ϕ^o for $L_p = N_t$ is found using the procedure described above. With $L_p = N_t$, we get $\alpha = 0$. Therefore, we need to show that ϕ^o is always given by ϕ_2. The key step is to show that the optimal value of ϕ is smaller than $\frac{1}{2}$, if $\zeta = 0$. The detailed proof is given in [13]. ■

B. Variable Training Length

When the legitimate user also has control over the training length L_p, the design problem is described in (11). Using the analytical results obtained on ζ^* in Section IV and ϕ^o in Section V-A for any given L_p, the problem in (11) can be easily solved numerically by a linear search on L_p, which takes integer values between N_t and L. The following two lemmas provide analytical solutions in the two limiting cases of $\mathcal{E}_w \gg \mathcal{E}$ and $\mathcal{E} \gg \mathcal{E}_w$.

Lemma 3: In the case of $\mathcal{E}_w \gg \mathcal{E}$, the robust strategy of the legitimate user is given by

$$L_p^o = \frac{L}{2}, \qquad \phi^o = \frac{1}{2}. \tag{22}$$

Proof: In the case of $\mathcal{E}_w \gg \mathcal{E}$, the optimal jamming energy allocation is approximated by (17). Therefore, the jamming power $\mathcal{P}_{wp}$ and $\mathcal{P}_{wd}$ are independent of ϕ. Moveover, the effective SNR in (9) can be approximated as

$$\rho_{\text{eff}} \approx \frac{(1-\phi)\phi\mathcal{E}^2}{(\mathcal{P}_{wd}+1)(\mathcal{P}_{wp}+1)L_d N_t}. \tag{23}$$

The value of ϕ that maximizes ρ_{eff} is given by $\phi^o = \frac{1}{2}$.

Let σ be an arbitrary non-zero eigenvalue of $\frac{\boldsymbol{H}_0\boldsymbol{H}_0{}^\dagger}{N_t}$ and n be the rank of $\boldsymbol{H}_0\boldsymbol{H}_0{}^\dagger$. The ergodic capacity lower bound in (8) can be rewritten as

$$C_{\text{LB}} = \frac{nL_d}{L\ln 2}\mathbb{E}\Big\{\ln(1+\rho_{\text{eff}}\sigma)\Big\}. \tag{24}$$

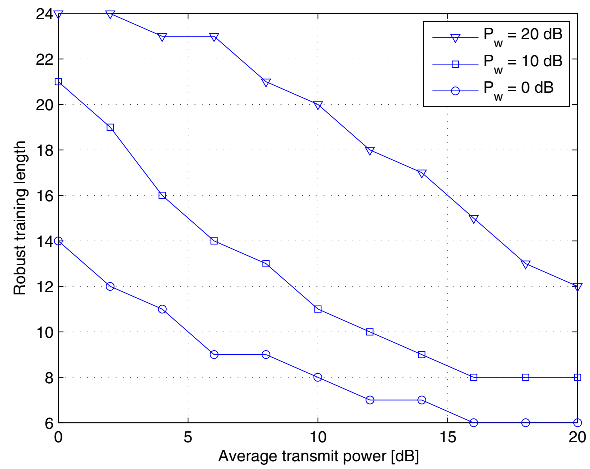

Fig. 1. The legitimate user's robust design of training length L_p^o versus the average transmit power $\mathcal{P}$ for $N_t \times N_r = 4 \times 4$ systems with a block length of $L = 50$.

Since $\rho_{\text{eff}} \to 0$ as $\mathcal{E}_w/\mathcal{E} \to \infty$, C_{LB} in (24) can be approximated as

$$\begin{aligned} C_{\text{LB}} &\approx \frac{nL_d}{L\ln 2}\rho_{\text{eff}}\mathbb{E}\{\sigma\} \\ &\approx \frac{n\mathbb{E}\{\sigma\}}{L\ln 2}\cdot\frac{\mathcal{E}^2/4}{(\mathcal{P}_{wd}+1)(\mathcal{P}_{wp}+1)N_t} && (25) \\ &\approx \frac{n\mathbb{E}\{\sigma\}\mathcal{E}^2}{4LN_t\ln 2}\Big[\Big(\frac{\mathcal{E}_w}{2(L-L_p)}+\frac{L_p}{2(L-L_p)}+\frac{1}{2}\Big) \\ &\qquad \times\Big(\frac{\mathcal{E}_w}{2L_p}+\frac{L-L_p}{2L_p}+\frac{1}{2}\Big)\Big]^{-1}, && (26) \end{aligned}$$

where (25) is obtained by substituting $\phi^o = \frac{1}{2}$ into ρ_{eff} in (23), and (26) is obtained by using (17). The optimal training length is then given by

$$\begin{aligned} L_p^o &= \arg\min_{L_p}\Big(\frac{\mathcal{E}_w}{2(L-L_p)}+\frac{L_p}{2(L-L_p)}+\frac{1}{2}\Big) \\ &\qquad \times\Big(\frac{\mathcal{E}_w}{2L_p}+\frac{L-L_p}{2L_p}+\frac{1}{2}\Big), && (27) \end{aligned}$$

which can be directly solved to be $\frac{L}{2}$. ■

Lemma 4: In the case of $\mathcal{E} \gg \mathcal{E}_w$, the robust strategy of the legitimate user is given by

$$L_p^o = N_t, \qquad \text{and} \qquad \phi^o = \frac{1}{2}. \tag{28}$$

Proof: See [13] for the proof of $L_p^o = N_t$. With $L_p^o = N_t$, the value of ϕ^o is then given in Lemma 2. ■

In general, the optimal training length for the robust design is larger than the number of transmit antennas and, hence, is different from the result in jamming-free systems.

VI. Numerical Results

In this section, we present numerical results on the robust design of the energy allocation and the training length that solves the max-min problem described in (11).

Fig. 1 shows the legitimate user's robust design of training length L_p^o against smart jamming. The general trend is that L_p^o

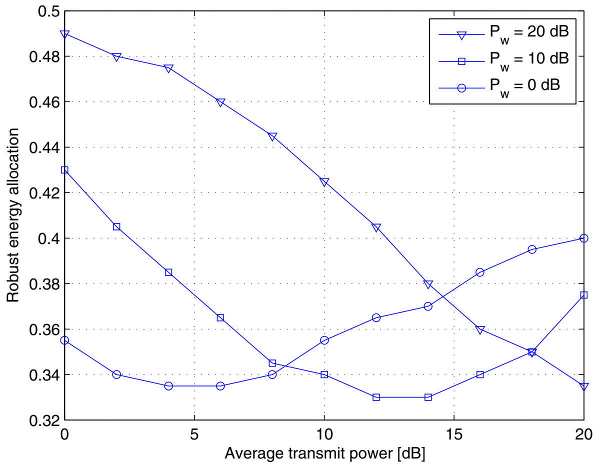

Fig. 2. The legitimate user's robust design of energy allocation ϕ^o versus the average transmit power $\mathcal{P}$ for $N_t \times N_r = 4 \times 4$ systems with a block length of $L = 50$.

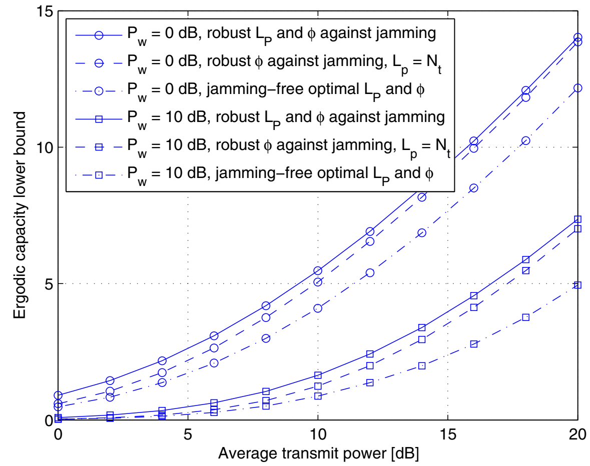

Fig. 3. The ergodic capacity lower bound C_{LB} in (8) versus the average transmit power $\mathcal{P}$ for $N_t \times N_r = 4 \times 4$ systems with a block length of $L = 50$. The jamming energy allocation is optimized to minimize C_{LB}. We show C_{LB} achieved by using the robust design of both training length and energy allocation (solid lines), the robust design of energy allocation only (dashed lines), as well as the optimal design for jamming-free systems as a suboptimal choice (dash-dotted lines). Note that the optimal training length for jamming-free systems is given by $L_p = N_t$.

reduces when the average transmit power $\mathcal{P}$ increases. When $\mathcal{P}$ is considerably lower than $\mathcal{P}_w$, L_p^o approaches $L/2 = 25$ from below. When $\mathcal{P}$ is considerably higher than $\mathcal{P}_w$, L_p^o approaches $N_t = 4$.[2] These observations agree with our analytical results in Section V-B. For a fixed $\mathcal{P}$, we see that L_p^o increases as the jamming power increases. That is to say, a longer training phase is needed when the jamming power increases.

Fig. 2 shows the legitimate user's robust design of energy allocation ϕ^o against smart jamming. We see that ϕ^o is not a monotonic function of the average transmit power $\mathcal{P}$. As $\mathcal{P}$ increases, the value of ϕ^o first decreases when $\mathcal{P}/\mathcal{P}_w$ is small and then increases when $\mathcal{P}/\mathcal{P}_w$ is large. We have also confirmed that ϕ^o converges to 0.5 as $\mathcal{P}/\mathcal{P}_w$ approaches zero or infinity, although the convergence as $\mathcal{P}/\mathcal{P}_w \to \infty$ is slow (plots are omitted for brevity).

Fig. 3 shows the legitimate user's capacity performance by using the robust design. The capacity gain from using the robust energy allocation alone is generally significant, which can be seen by comparing the dashed lines with the dash-dotted lines. For example, this gain at $\mathcal{P} = 10$ dB is 23% for $\mathcal{P}_w = 0$ dB and 41% for $\mathcal{P}_w = 10$ dB. Furthermore, the additional capacity gain from using the robust training length can be seen by comparing the solid lines with the dashed lines. This gain is significant when the average transmit power $\mathcal{P}$ is low and reduces as $\mathcal{P}$ increases.

VII. Conclusion

We considered a training-based MIMO system in the presence of a jammer and studied the trade-off in energy allocation between training and data transmission from an information-theoretic viewpoint. From the legitimate user's perspective, we obtained robust designs on the energy allocation and training length which maximize its data rate under the worst case jamming scenario. Numerical results demonstrated a 20% - 40% performance improvement by using the robust designs.

[2]The convergence of $L_p^o \to N_t$ is very slow and, hence, $L_p^o = N_t$ happens at very high $\mathcal{P}$, which is not shown in Fig. 1.

References

[1] M. Médard, "Capacity of correlated jamming channels," in *Proc. 35th Allerton Conf.*, 1997, pp. 1043–1052.

[2] A. Kashyap, T. Başar, and R. Srikant, "Correlated jamming on MIMO Gaussian fading channels," *IEEE Trans. Inform. Theory*, vol. 50, no. 9, pp. 2119–2123, Sep. 2004.

[3] G. T. Amariucai, S. Wei, and R. Kannan, "Gaussian jamming in block-fading channels under long term power constraints," in *Proc. IEEE Int. Symp. Inform. Theory (ISIT)*, Nice, France, Jun. 2007, pp. 1001–1005.

[4] J. K. Cavers, "An analysis of pilot symbol assisted modulation for Rayleigh fading channels," *IEEE Trans. Veh. Technol.*, vol. 40, no. 4, pp. 686–693, Nov. 1991.

[5] M. Han, T. Yu, J. Kim, K. Kwak, S. Lee, S. Han, and D. Hong, "OFDM channel estimation with jammed pilot detector under narrow-band jamming," *IEEE Trans. Veh. Tech.*, vol. 57, no. 3, pp. 1934–1939, May 2008.

[6] S. Ray, P. Moulin, and M. Médard, "On optimal signaling and jamming strategies in wideband fading channels," in *Proc. IEEE Workshop Signal Processing Advances Wireless Commun. (SPAWC)*, Cannes, France, Jul. 2006, pp. 1–5.

[7] ——, "On jamming in the wideband regime," in *Proc. IEEE Int. Symp. Inform. Theory (ISIT)*, Seattle, WA, Jul. 2006, pp. 2574–2577.

[8] R. Miller and W. Trappe, "Subverting MIMO wireless systems by jamming the channel estimation procedure," in *Proc. ACM Conf. Wireless Network Security (WiSec)*, Hoboken, NJ, 2010.

[9] B. Hassibi and B. M. Hochwald, "How much training is needed in multiple-antenna wireless links?" *IEEE Trans. Inform. Theory*, vol. 49, no. 4, pp. 951–963, Apr. 2003.

[10] V. Pohl, P. H. Nguyen, V. Jungnickel, and C. Helmolt, "Continuous flat-fading MIMO channels: Achievable rate and optimal length of the training and data phases," *IEEE Trans. Wireless Commun.*, vol. 4, no. 4, pp. 1889–1900, Jul. 2005.

[11] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Englewood Cliffs, NJ: Prentice Hall, 1993.

[12] X. Zhou, P. Sadeghi, T. A. Lamahewa, and S. Durrani, "Design guidelines for training-based MIMO systems with feedback," *IEEE Trans. Signal Processing*, vol. 57, no. 10, pp. 4014–4026, Oct. 2009.

[13] X. Zhou, D. Niyato, and A. Hjørungnes, "Optimizing training-based transmission against smart jamming," submitted.