A Fair Opportunistic Relaying Algorithm Using an Adaptive Selection Region in Cooperative Networks

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***Abstract*—This work proposes a new relay selection algorithm in an opportunistic cooperative network, which aims to establish fairness among the users. Our approach provides the same overall outage probability for users at different locations. To this end, we first define a selection region containing Amplify-and-Forward (AF) relays with a superior channel quality. Then, opportunistic relay selection from the region is applied. The fairness is achieved by adapting the size of the selection region according to the user location. Our analytical result provides a guideline to implement the proposed relay selection algorithm at each user.**

***Index Terms*—Amplify-and-Forward (AF), fairness, opportunistic relaying, outage probability, Poisson point process (PPP)**

I. Introduction

Cooperative relaying schemes can provide diversity gains by exploiting the fading conditions of various locations, without requiring multiple antenna arrays [1]. Various practical cooperation protocols have been proposed in the literature and their benefits in terms of an improved outage performance have been demonstrated [2], [3]. Of particular interest is the low-complexity of the Amplify-and-Forward (AF) relaying scheme where the relay amplifies the received noisy version of the source signal [3].

Several strategies have been proposed to select the relay(s) from a collection of candidate nodes for cooperation [3]–[8]. An effective method is opportunistic relaying which uses the bandwidth more efficiently [3] and reaches the near-optimal outage performance [6]. This method has received special attention of the researchers as only requires the channel state information of the link corresponding to the largest SNR, and the SNR ranking of the other links. The outage probability of opportunistic relaying has been widely studied in the literature, e.g. [6]–[10]. In particular, the outage performance of the networks with randomly distributed relays, which is considered in our system model, is investigated in [7], [8], [10], [11] and shown that the outage probability of the system exponentially decreases as the density of the relays increases.

Although being an attractive scheme, opportunistic relaying suffers from poor fairness due to variable outage performance for users at different locations [12] and uneven power consumption among the relays [13]–[15]. In [12], a probabilistic relay assignment strategy for coded cooperation has been proposed in order to provide equal outage probability for the users at different locations. In this method, when a source node takes the chance to request a relay, the best relay is chosen by the exhaustive search over the network. However, identifying the best relay among a large number of candidates is not desirable due to the tremendous cost of energy, the complexity imposed on the network, and the associated delay which increases with the number of relays.

To tackle these issues, in this paper we employ a method to restrict the search only among qualified candidate relays, while ensuring the same outage performance for users at different locations. Our main contributions are:

1) to characterize the region of AF relays with superior fading properties, and
2) to propose a relay selection algorithm that provides a fair service to users at different locations.

A key feature of our proposed algorithm is to adapt the selection region according to the system characteristics and requirements.

The rest of the paper is structured as follows. Section II presents the system model. Then, the fair relay selection strategy is proposed in Section III. Numerical results are provided in Section IV, and final remarks are presented in Section V.

II. System Model

Our system model focuses in the case of a wireless network composed by a source node, S, a destination node, D, and a number of relay nodes placed on the S–D plane. The network area is in the form of a disc of radius r and centered at D. The relay nodes are assumed to be distributed according to a two dimensional homogeneous Poisson point process (PPP) with constant density λ. The source node sends the data to the destination either directly or with the help of a relay node. We assume that each node is equipped with a GPS module and hence the locations of all nodes are known.

The wireless link between any pair of nodes is assumed to suffer of Rayleigh fading and a deterministic large scale path

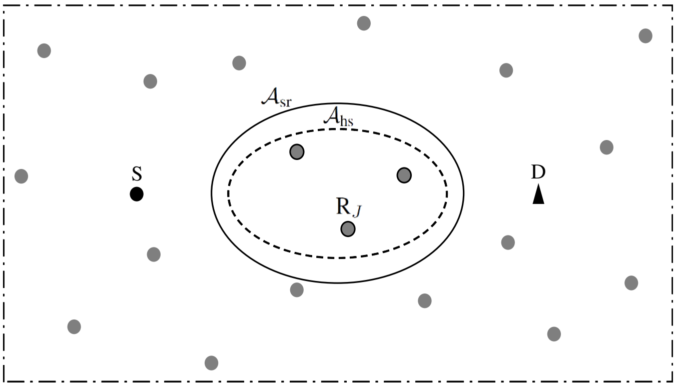

Fig. 1. A wireless cooperative network with a source node, S, a destination node, D, and a group of relay nodes.

loss. Therefore, the instantaneous SNR between node S and D, Γ_{SD}, can be written as

$$\Gamma_{SD} = \frac{P_S}{N_0 {\ell_{SD}}^{\alpha}} \Omega_{SD}, \tag{1}$$

where ℓ_{SD} denotes the source to destination distance, α is the path loss exponent, P_S is the transmit power at the source, N_0 is the noise power, and Ω_{SD} indicates the fading power with $E\{\Omega_{SD}\} = 1$. For simplicity, it is assumed that propagation parameters like antenna gains are included in the P_S term.

Analogously, for an arbitrary relay R_j, the instantaneous SNR of the links S–R_j and R_j–D in our system can be respectively expressed as

$$\Gamma_{Sj} = \frac{P_S}{N_0 {\ell_{Sj}}^{\alpha}} \Omega_{Sj}, \tag{2a}$$

$$\Gamma_{jD} = \frac{P_R}{N_0 {\ell_{jD}}^{\alpha}} \Omega_{jD}. \tag{2b}$$

Above, Ω_{Sj} and Ω_{jD} are fading powers of S–R_j and R_j–D links respectively with unit mean, P_R is the relay power including the antenna gain as well, and ℓ_{Sj} and ℓ_{jD} denote the source to R_j and R_j to the destination distances respectively. Furthermore, we assume that fading powers between any pair of nodes, including Ω_{SD}, Ω_{Sj} and Ω_{jD}, are independent random variables (RVs).

It has been shown that the overall SNR of AF-based relaying path, i.e. S–R_j–D, can be written as [10]

$$\Gamma_j = \frac{\Gamma_{Sj}\Gamma_{jD}}{\Gamma_{Sj} + \Gamma_{jD} + 1}. \tag{3}$$

For relaying we adopt an opportunistic approach [10], where the relay R_J with the highest instantaneous SNR Γ_J of the S–R_J–D link among the candidate relays is chosen. Obviously, this best relay might differ for the source nodes at different locations.

III. Fairness Approach in Relay Selection

In this section, we propose our relay selection algorithm which aims to provide a fair QoS measured in terms of the outage probability. The ultimate goal is to achieve the same target outage probability for the source node regardless of its location. In effect, if the source node is located near D then there is no need for relaying, while if the source node is located sufficiently away, direct transmission alone is not enough and hence relaying is needed. A geographic region is defined within which relay can be selected. Depending on how far away the source is located from D, the size of the selection region is adaptively chosen in order to meet the same target outage probability.

In addition, we assume that the transmit power of a relay is fixed to P_R, while the transmit power of a source can be adaptively chosen according to the location of the source. For example, the source can lower its transmit power when it is located close to D.

In the sequel, Section III-A defines a geographical region containing relays which are more likely to have a favorable SNR. The outage probability of the system is discussed in Section III-B. After this, our fair relay selection algorithm is presented in Section III-C.

A. *Selection Region Characterization*

A location $\mathcal{P}$ is a point in the S–D plane, parameterized by its distances to the source and to the destination, i.e. $\mathcal{P} = (\ell_{Sj}, \ell_{jD})$. A location is a *hot spot* if the SNR Γ_j of a relay R_j potentially located there would satisfy

$$\Pr\{\Gamma_j \le \Theta\} \le \delta, \tag{4}$$

where Θ is a SNR threshold, Γ_j is given by (3), and $0 < \delta < 1$ is a real number. The region formed by all the hot spots is a subset of the S–D plane denoted as $\mathcal{A}_{hs}$ (see Fig. 1).

In general, characterizing $\mathcal{A}_{hs}$ directly is intractable. As a simpler alternative, we propose to study a *selection region* $\mathcal{A}_{sr}$ defined by

$$\Pr\{\min(\Gamma_{Sj}, \Gamma_{jD}) \le \Theta\} \le \delta. \tag{5}$$

It can be shown from (3) that $\min(\Gamma_{Sj}, \Gamma_{jD}) > \Gamma_j$, and hence

$$\Pr\{\min(\Gamma_{Sj}, \Gamma_{jD}) \le \Theta\} \le \Pr\{\Gamma_j \le \Theta\}, \tag{6}$$

Therefore, $\mathcal{A}_{hs} \subseteq \mathcal{A}_{sr}$ (see Fig. 1), and hence all the hot spots are contained in $\mathcal{A}_{sr}$. For evaluating $\mathcal{A}_{sr}$, one can write

$$\begin{aligned}\Pr\{\min(\Gamma_{Sj},\Gamma_{jD}) \le \Theta\} &= 1 - \Pr\{\min(\Gamma_{Sj}, \Gamma_{jD}) > \Theta\} \\ &= 1 - \Pr\{\Gamma_{Sj} > \Theta\}\Pr\{\Gamma_{jD} > \Theta\},\end{aligned} \tag{7}$$

where the last equation follows from the fact that Γ_{Sj} and Γ_{jD} are independent RVs. Because of the Rayleigh fading assumption, the two probability terms in (7) can be expressed as

$$\begin{aligned}\Pr\{\Gamma_{Sj} > \Theta\} &= \exp\left(-\mu_1 \Theta {\ell_{Sj}}^{\alpha}\right), \\ \Pr\{\Gamma_{jD} > \Theta\} &= \exp\left(-\mu_2 \Theta {\ell_{jD}}^{\alpha}\right).\end{aligned} \tag{8}$$

Above, $\mu_1 = N_0/P_S$ and $\mu_2 = N_0/P_R$ have been introduced as shorthand notations. By using (7) and (8), (5) can be rewritten as

$$1 - \exp\left(-\Theta\left(\mu_1 \ell_{Sj}^{\alpha} + \mu_2 \ell_{jD}^{\alpha}\right)\right) \le \delta, \tag{9a}$$

and therefore, the region $\mathcal{A}_{\text{sr}}$ can be characterized as

$$\mu_1 \ell_{\text{S}j}^{\alpha} + \mu_2 \ell_{j\text{D}}^{\alpha} \leq \ln\left(\frac{1}{\sqrt[\Theta]{1-\delta}}\right). \tag{9b}$$

It can be seen that for the case of free space where $\alpha = 2$, condition (9b) corresponds to the interior of an ellipse.

Finally, the following proposition shows conditions under which $\mathcal{A}_{\text{sr}}$ provides a tight approximation to $\mathcal{A}_{\text{hs}}$ (the proof is given in Appendix).

Proposition 1: For $\delta \ll 1$ and $\Theta > 0$ dB, $\mathcal{A}_{\text{sr}}$ is a good approximation of $\mathcal{A}_{\text{hs}}$, in a way that a smaller δ leads to a more accurate approximation.

B. Opportunistic Relaying and Overall Outage Probability

Using the definition of $\mathcal{A}_{\text{sr}}$, the best relay R_J, is selected opportunistically according to the following criteria,

$$J \triangleq \arg \max_{j\,:\,\text{R}_j \in \mathcal{A}_{\text{sr}}} \Gamma_j. \tag{10}$$

Following opportunistic relaying, a network is in outage if both the direct link and the best relay link are faded. Thus, the overall outage probability of the system can be written as

$$\begin{aligned} P_{\text{out}}^{\text{Tot}}(\delta, P_{\text{S}}, \ell_{\text{SD}}) &\triangleq \Pr\{\max(\Gamma_{\text{SD}}, \Gamma_J) \leq \Theta\} \\ &= P_{\text{out}}^{\text{SD}}(P_{\text{S}}, \ell_{\text{SD}}) \cdot P_{\text{out}}^{\text{BR}}(\delta, P_{\text{S}}, \ell_{\text{SD}}) \end{aligned} \tag{11}$$

where

$$\begin{aligned} P_{\text{out}}^{\text{SD}}(P_{\text{S}}, \ell_{\text{SD}}) &\triangleq \Pr\{\Gamma_{\text{SD}} \leq \Theta\} \\ &= 1 - \exp\left(-\mu_1 \Theta {\ell_{\text{SD}}}^{\alpha}\right), \end{aligned} \tag{12}$$

and the outage probability of the best relaying path can be expressed as[1] [10, eq. 12]

$$\begin{aligned} P_{\text{out}}^{\text{BR}}(\delta, P_{\text{S}}, \ell_{\text{SD}}) &\triangleq \Pr\{\Gamma_J \leq \Theta\} \\ &= \exp\left(-\lambda \int_{\mathcal{A}_{\text{sr}}} \left[1 - \Psi(\Theta, \ell_{\text{S}j}, \ell_{j\text{D}})\right] \, d\mathcal{A}\right), \end{aligned} \tag{13}$$

where the integral is a surface integral, $\mathcal{A}_{\text{sr}}$ is the selection region corresponding to δ expressed in (9b), $d\mathcal{A}$ is the surface element, and [10, eq.30]

$$\begin{aligned} &\Psi(\Theta, \ell_{\text{S}j}, \ell_{j\text{D}}) = 1 - 2\sqrt{\mu_1 \mu_2 \ell_{\text{S}j}^{\alpha} \ell_{j\text{D}}^{\alpha} (\Theta^2 + \Theta)} \\ &\times \exp\left(-\Theta\left(\mu_1 \ell_{\text{S}j}^{\alpha} + \mu_2 \ell_{j\text{D}}^{\alpha}\right)\right) K_1\left(2\sqrt{\mu_1 \mu_2 \ell_{\text{S}j}^{\alpha} \ell_{j\text{D}}^{\alpha} (\Theta^2 + \Theta)}\right) \end{aligned} \tag{14}$$

where $K_1(\cdot)$ denotes the first order modified Bessel function of the second kind. Using the approximation given in (31), is possible to re-write (13) as

$$P_{\text{out-ap}}^{\text{BR}}(\delta, P_{\text{S}}, \ell_{\text{SD}}) = \exp\left(-\lambda[|\mathcal{A}_{\text{sr}}| - \Theta(\mu_1 \Upsilon_{\text{SR}} + \mu_2 \Upsilon_{\text{RD}})]\right) \tag{15a}$$

with

$$\Upsilon_{\text{SR}} = \int_{\mathcal{A}_{\text{sr}}} \ell_{\text{S}j}^{\alpha} \; d\mathcal{A}, \tag{15b}$$

$$\Upsilon_{\text{RD}} = \int_{\mathcal{A}_{\text{sr}}} \ell_{j\text{D}}^{\alpha} \; d\mathcal{A}. \tag{15c}$$

C. Fair Relay Selection Strategy

Here, we first explain how the total outage probability relates to δ as an implementation parameter. Assume that the corresponding selection region for two different values of $\delta = \delta_1$ and $\delta = \delta_2$ with $\delta_1 \leq \delta_2$, are denoted by $\mathcal{A}_{\text{sr}}^{(1)}$ and $\mathcal{A}_{\text{sr}}^{(2)}$. Also assume that an arbitrary point $\mathcal{P}_0 = (\ell_{\text{S}0}, \ell_{0\text{D}})$ is located in $\mathcal{A}_{\text{sr}}^{(1)}$ (i.e. $\mathcal{P}_0 \in \mathcal{A}_{\text{sr}}^{(1)}$), where $\ell_{\text{S}0}$ and $\ell_{0\text{D}}$ are the source to $\mathcal{P}_0$ and $\mathcal{P}_0$ to the destination distance, respectively. Then from (9b) we have

$$\mu_1 \ell_{\text{S}0}^{\alpha} + \mu_2 \ell_{0\text{D}}^{\alpha} \leq \ln\left(\frac{1}{\sqrt[\Theta]{1-\delta_1}}\right). \tag{16}$$

Now, since $\delta_1 \leq \delta_2$, one can also find that

$$\ln\left(\frac{1}{\sqrt[\Theta]{1-\delta_1}}\right) \leq \ln\left(\frac{1}{\sqrt[\Theta]{1-\delta_2}}\right). \tag{17}$$

From (16) and (17) we obtain

$$\mu_1 \ell_{\text{S}0}^{\alpha} + \mu_2 \ell_{0\text{D}}^{\alpha} \leq \ln\left(\frac{1}{\sqrt[\Theta]{1-\delta_2}}\right). \tag{18}$$

Thus $\mathcal{P}_0 \in \mathcal{A}_{\text{sr}}^{(2)}$ and as a result $\mathcal{A}_{\text{sr}}^{(1)} \subseteq \mathcal{A}_{\text{sr}}^{(2)}$. Therefore,

$$P_{\text{out}}^{\text{BR}}(\delta_1, P_{\text{S}}, \ell_{\text{SD}}) \geq P_{\text{out}}^{\text{BR}}(\delta_2, P_{\text{S}}, \ell_{\text{SD}}), \tag{19}$$

or equivalently

$$P_{\text{out}}^{\text{Tot}}(\delta_1, P_{\text{S}}, \ell_{\text{SD}}) \geq P_{\text{out}}^{\text{Tot}}(\delta_2, P_{\text{S}}, \ell_{\text{SD}}). \tag{20}$$

Consequently by varying δ the area of $\mathcal{A}_{\text{sr}}$ and the total outage probability can be adjusted to meet the reference target outage performance.

Now, in order to ensure fairness among all users in the network, appropriate values of δ are chosen for the source nodes with different distances from the destination. In this way, all users at different locations should experience the same overall outage probability. Assume that an initial value of P_{S}, i.e., $P_{\text{S}} = P_{\text{S}}^0$, is given and a target outage probability, ϵ, is to be reached. Solving

$$P_{\text{out}}^{\text{Tot}}(\delta, P_{\text{S}}^0, \ell_{\text{SD}}) = \epsilon, \tag{21}$$

one can obtain a unique $\delta = \delta(\ell_{\text{SD}}, \epsilon)$ and its corresponding $\mathcal{A}_{\text{sr}} = \mathcal{A}_{\text{sr}}(\ell_{\text{SD}}, \epsilon)$. Thus, if the source node transmits its data with power P_{S}^0 and by the help of the best relay from $\mathcal{A}_{\text{sr}} = \mathcal{A}_{\text{sr}}(\ell_{\text{SD}}, \epsilon)$, the target outage probability is achieved. Obviously, as ℓ_{SD} increases its corresponding $\delta(\ell_{\text{SD}}, \epsilon)$ and $\mathcal{A}_{\text{sr}}(\ell_{\text{SD}}, \epsilon)$ from (21) becomes larger due to the degradation of the direct link quality.

For users in vicinity of D, the quality of direct link is less likely to be worse than the target ϵ, and therefore, they are able to reach a lower outage probability without employing a relay. For these users, the corresponding direct link outage

[1] The relaying path is in outage if no relay is within the area (i.e. $\Gamma_J = 0$) or if the best relay suffers from the undesirable end-to-end channel condition.

Algorithm 1 Fair relay assignment to the source nodes

Given ϵ, λ, μ_1, μ_2, α, Θ.
Calculate $\mathcal{R}$ using (23).
Make a lookup table by computing $\mathcal{A}_{\text{sr}}(\ell_{\text{SD}}, \epsilon)$ and $\tilde{P}_{\text{S}}(\ell_{\text{SD}})$ using (24) and (21) for various values of ℓ_{SD}, respectively.
Refresh the lookup table in case that the system parameters are changed.
Determine ℓ_{SD}.
if $\ell_{\text{SD}} \leq \mathcal{R}$ **then**
  Extract the corresponding $\tilde{P}_{\text{S}}(\ell_{\text{SD}})$ from the lookup table.
  $P_{\text{S}} \leftarrow \tilde{P}_{\text{S}}(\ell_{\text{SD}})$
  Do not assign any relay to the source.
else $\{\ell_{\text{SD}} > \mathcal{R}\}$
  $P_{\text{S}} \leftarrow P_{\text{S}}^0$.
  Extract $\mathcal{A}_{\text{sr}}(\ell_{\text{SD}}, \epsilon)$ from the lookup table.
  $\mathcal{A}_{\text{sr}} \leftarrow \mathcal{A}_{\text{sr}}(\ell_{\text{SD}}, \epsilon)$
  Assign the best relay to the source from $\mathcal{A}_{\text{sr}}$.
end if

probability must satisfy

$$P_{\text{out}}^{\text{SD}}(P_{\text{S}}^0, \ell_{\text{SD}}) = 1 - \exp\left(-\mu_1 \Theta {\ell_{\text{SD}}}^{\alpha}\right) \leq \epsilon, \quad (22)$$

which provides the following bound for the S–D distance:

$$\ell_{\text{SD}} \leq \mathcal{R} = \left[\frac{1}{\mu_1} \ln\left(\frac{1}{\sqrt[\Theta]{1-\epsilon}}\right)\right]^{\frac{1}{\alpha}}. \quad (23)$$

The transmission power of a source node with the distance ℓ_{SD} from the destination satisfied in (23) can be reduced from $P_{\text{S}} = P_{\text{S}}^0$ to $P_{\text{S}} = \tilde{P}_{\text{S}}$ to meet the target outage performance ϵ without any help of relays. In other words, $P_{\text{S}} = \tilde{P}_{\text{S}}$ is obtained by solving $P_{\text{out}}^{\text{SD}}(P_{\text{S}}, \ell_{\text{SD}}) = \epsilon$. Using (12), $\tilde{P}_{\text{S}}$ can be expressed as

$$\tilde{P}_{\text{S}} = \frac{N_0 \ell_{\text{SD}}^{\alpha}}{\ln\left(\frac{1}{\sqrt[\Theta]{1-\epsilon}}\right)} \triangleq \tilde{P}_{\text{S}}(\ell_{\text{SD}}). \quad (24)$$

Now based on the above discussions we summarize our fair relay selection approach in Algorithm 1.

IV. Numerical Results

In this section we provide numerical results to demonstrate how the area of $\mathcal{A}_{\text{sr}}(\ell_{\text{SD}}, \epsilon)$ depends on the user location using the analytical expressions derived in Section III. Moreover, we study the outage behaviour of the relaying path (both the exact and approximated expressions) in the proposed algorithm for a user at various places. We set $\mu_1 = \mu_2 = -40$dB, $\Theta = 3$dB, $\lambda = 0.25$ and $r = 12$.

Fig. 2 depicts the proportion of the system coverage area, occupied by the $\mathcal{A}_{\text{sr}}(\ell_{\text{SD}}, \epsilon)$. It was found that the bound $\mathcal{R}$, derived in (23), is always smaller than 3 for all cases. Thus, as illustrated, for $\ell_{\text{SD}} \geq 3$ we have $\mathcal{A}_{\text{sr}}(\ell_{\text{SD}}, \epsilon) > 0$. Our results show that the area of $\mathcal{A}_{\text{sr}}(\ell_{\text{SD}}, \epsilon)$ in the proposed algorithm is considerably smaller than the area of the entire network. It means that only a small percentage of relays are considered as candidate for cooperation. In addition, this area becomes larger as the source node moves farther from D, to contain larger number of candidate relays. In this way, the worse direct channel quality is compensated by the higher quality of relaying path channel. This figure also shows that a smaller α results in smaller selection area due to the fact that the direct source to destination channel is less affected by the path loss large-scale fading, so for the same target outage performance the smaller number of candidate relays are required to participate in the cooperation.

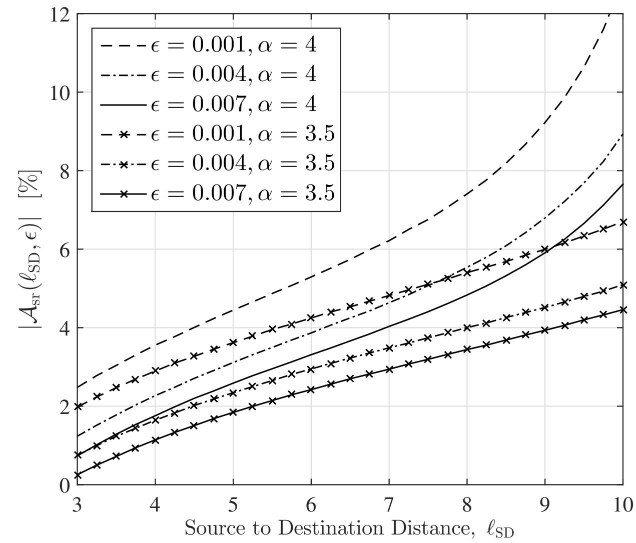

Fig. 2. The percentage of network area taken up by $\mathcal{A}_{\text{sr}}(\ell_{\text{SD}}, \epsilon)$ in Algorithm 1.

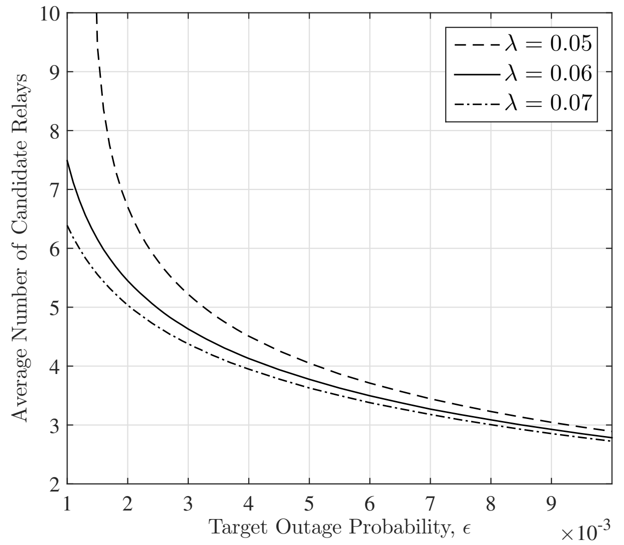

Fig. 3. The average number of candidate relays versus ϵ for a user located at $\ell_{\text{SD}} = 5$.

The impact of two fundamental system parameters ϵ and λ on the average number of candidate relays for cooperation

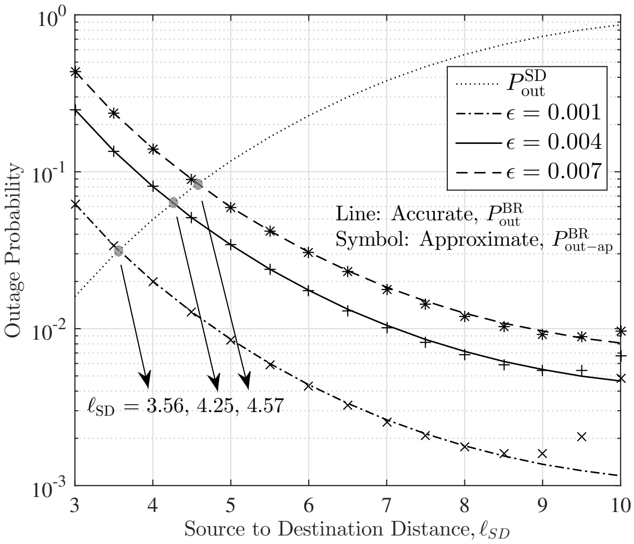

Fig. 4. The outage probability of both direct and relaying path in Algorithm 1.

are depicted in Fig. 3. As expected, the number of candidate relays falls by relaxation of reference outage performance of the network in which in the case of smaller λ the rate of reduction is considerably faster. Indeed, λ has two inverse effects on the number of candidate relays; first of all the average number of relays in a region is linearly dependent on the density of relays, and secondly the region $\mathcal{A}_{\text{sr}}(\ell_{\text{SD}}, \epsilon)$ obtained from (21) enlarges as λ diminishes. However, as seen in the figure, the final effect is a growth in the average number of candidate relays with reduction in λ. As ϵ grows, the lines for different λ becomes closer. This means that for a large ϵ the average number of candidate relays are almost independent of λ and thus, according to our discussion, one finds that $|\mathcal{A}_{\text{sr}}(\ell_{\text{SD}}, \epsilon)| \propto \lambda^{-1}$.

The system outage performance for different values of ℓ_{SD} is depicted in Fig. 4. As opposed to the conventional relay selection method, in our proposed algorithm the relaying path outage probability, $P_{\text{out}}^{\text{BR}}$, decreases as ℓ_{SD} increases. This is due to the fact that the average number of candidate relays that participate in the cooperation grows with the source to destination distance. The reduction of $P_{\text{out}}^{\text{BR}}$ is such that its production by $P_{\text{out}}^{\text{SD}}$ is fixed and equal to the target outage probability, ϵ. Thus, since $P_{\text{out}}^{\text{SD}}$ is not dependent on ϵ, the $P_{\text{out}}^{\text{BR}}$ decreases as ϵ is decreased. In the figure, the intersection of $P_{\text{out}}^{\text{SD}}$ and $P_{\text{out}}^{\text{BR}}$ has been marked, as it shows source to destination distance at which the communication starts depending more strongly on the relaying path than on the direct link. This is in accordance with the fact that at those distances relaying is much less likely to be in outage than the direct transmission. Moreover, as it can be seen from the figure, the approximated expression for the outage probability of relaying, $P_{\text{out-ap}}^{\text{BR}}$, matches closely to the exact outage probability, $P_{\text{out}}^{\text{BR}}$, for a wide range of source to destination distances. Indeed, as discussed in Section III, a smaller relay selection area is associated to a better approximation.

V. Conclusion

A fair relay selection algorithm in cooperation networks has been proposed where the relays are randomly distributed. To this end, a selection region comprising qualified candidate relays has been introduced. This method avoids too many number of channel estimations and considerably reduces the complexity and delay of the best relay selection, while a target outage performance is guaranteed. The numerical results confirm the effectiveness of the proposed algorithm.

Appendix
Proof of the Proposition 1

According to AM–GM inequality, for any relay node such as R_j in $\mathcal{A}_{\text{sr}}$, we can write

$$2\sqrt{\mu_1 \ell_{\text{S}j}^{\alpha} \mu_2 \ell_{j\text{D}}^{\alpha}} \leq \mu_1 \ell_{\text{S}j}^{\alpha} + \mu_2 \ell_{j\text{D}}^{\alpha}. \tag{25}$$

Thus, using (9b) we have

$$2\sqrt{\mu_1 \ell_{\text{S}j}^{\alpha} \mu_2 \ell_{j\text{D}}^{\alpha}} \leq \ln\left(\frac{1}{\sqrt[\Theta]{1-\delta}}\right). \tag{26}$$

Therefore, for $\Theta > 0$ dB, one can write

$$\begin{aligned} 2\sqrt{\mu_1 \mu_2 \ell_{\text{S}j}^{\alpha} \ell_{j\text{D}}^{\alpha} (\Theta^2 + \Theta)} &\leq \sqrt{2}\Theta \ \ln\left(\frac{1}{\sqrt[\Theta]{1-\delta}}\right) \\ &= -\sqrt{2}\ln(1-\delta) \end{aligned} \tag{27}$$

From above, using the fact that $-\ln(1-\delta) \ll 1$ for $\delta \ll 1$, we conclude that the expression on the left side is small enough to utilize the approximation of

$$\begin{aligned} &K_1\left(2\sqrt{\mu_1 \mu_2 \ell_{\text{S}j}^{\alpha} \ell_{j\text{D}}^{\alpha} (\Theta^2 + \Theta)}\right) \\ &\quad\cong \left(2\sqrt{\mu_1 \mu_2 \ell_{\text{S}j}^{\alpha} \ell_{j\text{D}}^{\alpha} (\Theta^2 + \Theta)}\right)^{-1}. \end{aligned} \tag{28}$$

Therefore, $\Psi(\Theta, \ell_{\text{S}j}, \ell_{j\text{D}})$ in (14) can be simplified to

$$\begin{aligned} &\Psi(\Theta, \ell_{\text{S}j}, \ell_{j\text{D}}) \cong 1 - 2\sqrt{\mu_1 \mu_2 \ell_{\text{S}j}^{\alpha} \ell_{j\text{D}}^{\alpha} (\Theta^2 + \Theta)} \\ &\times \exp\left(-\Theta\left(\mu_1 \ell_{\text{S}j}^{\alpha} + \mu_2 \ell_{j\text{D}}^{\alpha}\right)\right)\left(2\sqrt{\mu_1 \mu_2 \ell_{\text{S}j}^{\alpha} \ell_{j\text{D}}^{\alpha} (\Theta^2 + \Theta)}\right)^{-1} \\ &= 1 - \exp\left(-\Theta\left(\mu_1 \ell_{\text{S}j}^{\alpha} + \mu_2 \ell_{j\text{D}}^{\alpha}\right)\right). \end{aligned} \tag{29}$$

From (9b) we can write

$$\Theta\left(\mu_1 \ell_{\text{S}j}^{\alpha} + \mu_2 \ell_{j\text{D}}^{\alpha}\right) \leq -\ln(1-\delta) \ll 1. \tag{30}$$

Therefore, using the fact that $\exp(-x) \cong 1 - x$ for $x \ll 1$, (29) can be further simplified to

$$\Psi(\Theta, \ell_{\text{S}j}, \ell_{j\text{D}}) \cong \Theta\left(\mu_1 \ell_{\text{S}j}^{\alpha} + \mu_2 \ell_{j\text{D}}^{\alpha}\right). \tag{31}$$

From [10], we have

$$\Pr\{\Gamma_j \leq \Theta\} = \Psi(\Theta, \ell_{\text{S}j}, \ell_{j\text{D}}). \tag{32}$$

Therefore, combining (30)-(32) yields

$$\Pr\{\Gamma_j \leq \Theta\} \leq -\ln(1-\delta). \tag{33}$$

In conclusion, for $\delta \ll 1$ the region $\mathcal{A}_{\text{sr}}$ is approximately defined by (33). This area converges to the area of $\mathcal{A}_{\text{hs}}$ defined in (4) owing to the fact that $-\ln(1-\delta) \to \delta$ for $\delta \ll 1$.

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