On the Jamming Power Allocation and Signal Design in DF Relay Networks

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***Abstract*—This paper studies a simple decode-and-forward (DF) relay network in the presence of a jammer. The jammer is able to send noise-like signals to interfere with the signal reception at both the relay and the destination. From the attacker's point of view, we investigate the following two design problems: i) What is the optimal probability distribution of the random jamming signals? ii) Given a total jamming power budget, how to optimally allocate the power between attacking the relay and attacking the destination? We provide analytical solutions to these design problems for both quasi-static and ergodic fading channels.**

I. Introduction

Wireless networks are susceptible to numerous security threats due to the open nature of the wireless medium. In particular, the jamming attack is a long-standing security issue. A significant amount of effort has been devoted to analyzing the impact of jamming on both point-to-point communication systems [1–5] and relay-assisted cooperative networks [6–8]. For the communication between a source-destination pair assisted by a relay, the jammer has the opportunity to jam both the relay and the destination. However, prior studies on relay networks in [6–8] restricted the jammer's attack to the destination node only, which is generally not optimal from the jammer's viewpoint.

Intuitively, there are many occasions in which the jammer can cause detrimental effects to a relay network by jamming the relay instead of the destination. For an amplify-and-forward (AF) relay network, jamming the relay effectively amplifies the noise forwarded to the destination, resulting in a low signal-to-noise ratio (SNR) at the destination. Our recent work in [9] confirmed that the optimal jamming strategy against AF relay networks is to jam both the relay and destination with some optimal jamming power allocation.

In this work, we extend our study to decode-and-forward (DF) relay networks and allow the jammer to attack both the relay and destination. We investigate the optimal jamming signal design in both quasi-static and ergodic fading channels, where the performance of the communication system is measured by outage probability and ergodic capacity, respectively. The design parameters include the probability distribution of the random jamming signals as well as the jamming power allocation between attacking the relay and the destination with a total jamming power constraint. Our results show that Gaussian jamming signaling is optimal in quasi-static fading channels but not in ergodic fading channels. Regarding the optimal jamming power allocation, it is generally optimal to jam both the relay and destination in quasi-static fading channels, while it is always best to spend all power to jam either the relay or destination in ergodic fading channels depending on the relative strength of the source-relay and relay-destination links. The understanding on the optimal jammer design obtained in this work is useful for certain military applications and can assist future studies on anti-jamming design in DF relay networks.

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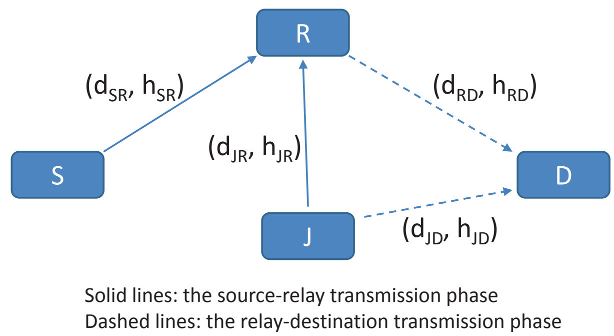

Fig. 1. The two-hop DF relay network in the presence of a jammer.

II. System Model

Consider a wireless network shown in Fig. 1, consisting of a source node (S), a half-duplex DF relay node (R), and a destination node (D), each equipped with a single antenna. In the absence of the direct link between the source and destination, the communication is assisted by the relay in a two-hop manner. Apart from the legitimate nodes, there exists a single-antenna jammer (J) trying to degrade the performance of the communication network by interfering with the signal reception at both the relay and destination.

The signal propagation is affected by both path loss with exponent α and small-scale Rayleigh fading. The distance between nodes X and Y is denoted by d_{XY} as shown in Fig. 1. The fading channel gain of the link from node X to node Y is denoted by h_{XY} as shown in Fig. 1. All channel gains follow zero-mean complex Gaussian distribution with

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unit variance, i.e., $\mathcal{CN}(0,1)$. For all links, we assume no instantaneous channel state information at the transmitter side. In addition, we assume that the instantaneous realizations of h_{JR} and h_{JD} are unknown to the relay and destination, since typically there is no mechanism to obtain such information.

During the source-relay transmission, the signal received at the relay is given by

$$y_R = \sqrt{P_S {d_{SR}}^{-\alpha}} h_{SR} x_S + \sqrt{P_{J1} {d_{JR}}^{-\alpha}} h_{JR} w_1 + n_R, \quad (1)$$

where P_S is the source transmit power, P_{J1} is the jamming power during the source-relay transmission, and n_R is the additive receiver noise at the relay following $\mathcal{CN}(0,1)$. We assume that the source uses Gaussian signaling, hence the normalized transmitted signal from the source, x_S, follows $\mathcal{CN}(0,1)$. On the other hand, the distribution of the normalized jamming signal, w_1, is treated as a design parameter in this work.

If the source message signal is successfully decoded, the relay re-encodes the message and starts the relay-destination transmission. The signal received at the destination is given by

$$y_D = \sqrt{P_R {d_{RD}}^{-\alpha}} h_{RD} x_R + \sqrt{P_{J2} {d_{JD}}^{-\alpha}} h_{JD} w_2 + n_D, \quad (2)$$

where P_R is the relay transmit power, P_{J2} is the jamming power during the relay-destination transmission, and n_D is the additive receiver noise at the destination following $\mathcal{CN}(0,1)$. Similarly, we assume that the relay uses Gaussian signaling, hence the normalized transmitted signal, x_R, follows $\mathcal{CN}(0,1)$. On the other hand, the distribution of the normalized jamming signal, w_2, is treated as a design parameter to be determined later.

In this work, we investigate the optimal design from the jammer's point of view. Specifically, we derive the optimal distribution of w_1 and w_2, as well as the optimal value of P_{J1} and P_{J2} with a given total power budget $P_J = P_{J1} + P_{J2}$. The design solution heavily depends on how much the jammer knows about the legitimate network. In this paper, we assume that the instantaneous fading channel gains are not known to the jammer. On the other hand, other system parameters, such as the transmit powers of source and relay, the location of each node, and the path loss exponent, are either known or can be accurately measured by the jammer. Furthermore, the jamming signaling design also depends on the type of fading channel experienced by the information-carrying signal. In the next two sections, we will study both quasi-static fading channel and ergodic fading channel and present the optimal jammer design solutions.

III. Jammer Design in Quasi-Static Fading Channels

In quasi-static fading channels, the channel gains are assumed to be constant over the transmission of a codeword. Hence, the outage probability is commonly used as the performance measure. For either the source-relay link or the relay-destination link, the transmission of a codeword is in outage when the achievable rate cannot support a prescribed rate denoted by $\mathcal{R}$. Denote the outage probabilities of the source-relay link and relay-destination link as P_{out}^R and P_{out}^D, respectively, which are mathematically defined as

$$P_{out}^R = \mathbb{P}\big(I(x_S; y_R) < \mathcal{R}\big), \quad (3)$$

$$P_{out}^D = \mathbb{P}\big(I(x_R; y_D) < \mathcal{R}\big), \quad (4)$$

where $I(x;y)$ denotes the mutual information between x and y. Since the dual-hop communication is in outage when at least one of the two links is in outage, the overall outage probability is given by

$$P_{out} = 1 - (1 - P_{out}^R)(1 - P_{out}^D) \quad (5)$$

The following theorem summarizes the optimal jammer design:

Theorem 1: With the goal of maximizing the outage probability of the DF relay network in (5), the optimal jamming signals, w_1 in (1) and w_2 in (2), follow independent $\mathcal{CN}(0,1)$. The optimal jamming power allocation is given by

$$P_{J1}^* = \begin{cases} \frac{P_J}{2} + \kappa, & \text{if } \frac{P_J}{2} \ge \max\{\kappa, -\kappa\}, \\ P_J, & \text{if } \frac{P_J}{2} < \kappa, \\ 0, & \text{if } \frac{P_J}{2} < -\kappa, \end{cases} \quad (6)$$

$$P_{J2}^* = P_J - P_{J1}^*, \quad (7)$$

where $\kappa = \frac{1}{2\gamma}\left(\left(\frac{d_{JD}}{d_{RD}}\right)^\alpha P_R - \left(\frac{d_{JR}}{d_{SR}}\right)^\alpha P_S\right)$ *and* $\gamma = 2^{\mathcal{R}} - 1$.

Proof: See Appendix A.

The optimality of Gaussian jamming signal follows from a well-known result: Gaussian noise minimizes the mutual information [10]. The optimal jamming power allocation can be explained as follows: When the power budget is small, the jammer tends to spend all power to jam the weaker link. This is because either the second or third condition in (6) holds when P_J becomes sufficiently small and the sign of κ depends on the relative strength of the source-relay and relay-destination links. When the power budget is sufficiently large, the jammer should jam both links. As the power budget goes to infinity, equal power jamming is asymptotically optimal.

A. Numerical Results

The analytical results are further illustrated numerically in Fig. 2. In the scenarios considered in this figure, the relative strength of the source-relay and relay-destination links is determined by the transmit powers P_S and P_R: when $P_S > P_R$, the source-relay link is stronger than relay-destination link, and vice versa. From Fig. 2, it is clear that equal power allocation is asymptotically optimal when the jamming power budget is large, while it is always best to jam only the weaker link when the jamming power budget is sufficiently small.

Furthermore, Fig. 3 shows the outage probability performance of the same DF relay network under either optimized or non-optimized jamming power allocation. The non-optimized case is represented by equal jamming power during both source-relay and relay-destination transmission phases (i.e.,

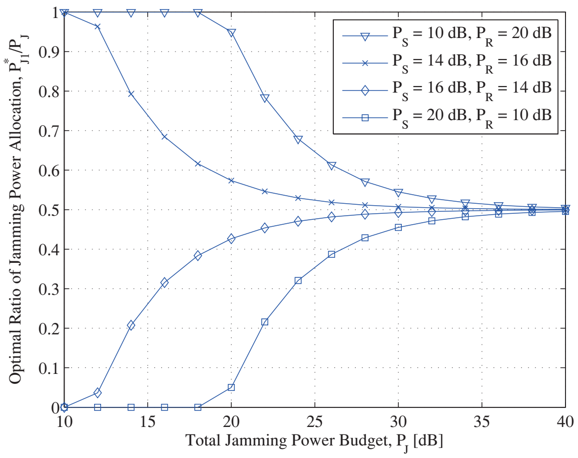

Fig. 2. The optimal ratio of jamming power allocation expressed in terms of P_{J1}^*/P_J versus the total jamming power budget P_J. Other system parameters are set as: $d_{SR} = d_{JR} = d_{RD} = d_{JD} = 1$, $\gamma = 1$, $\alpha = 3$. Different sets of values of P_S and P_R are used to represent the change in the relative signal strengths of the two links.

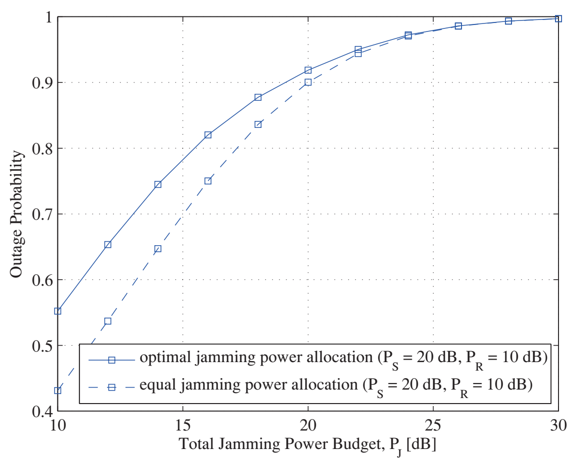

Fig. 3. The outage probability of the DF relay-assisted communication versus the total jamming power budget P_J. Other system parameters are set as: $d_{SR} = d_{JR} = d_{RD} = d_{JD} = 1$, $\gamma = 1$, $\alpha = 3$, $P_S = 20$ dB, $P_R = 10$ dB.

the jamming power is fixed at all times). The gap between the two outage curves shows the advantage of optimizing the jamming power allocation. One can see that, when the jamming power budget is small, optimizing the jamming power allocation significantly increases the outage probability.

IV. Jammer Design in Ergodic Fading Channels

In ergodic fading channels, the transmission of a codeword experiences a sufficiently large number of channel realizations. Hence, the ergodic capacity is commonly used as the performance measure, given by

$$C = \frac{\min\{C^R, C^D\}}{2}, \tag{8}$$

where C^R and C^D denote the ergodic capacities of the source-relay and relay-destination links, respectively, which are mathematically defined as[1]

$$C^R = I(x_S; y_R \,|\, h_{SR}), \tag{9}$$
$$C^D = I(x_R; y_D \,|\, h_{RD}), \tag{10}$$

where the conditioning is due to the knowledge of h_{SR} and h_{RD} at the relay and destination, respectively.

The following theorem summarizes the optimal jammer design:

Theorem 2: With the goal of minimizing the ergodic capacity of the DF relay network in (8), the optimal jamming signals, w_1 in (1) and w_2 in (2), both have a (constant) unit magnitude and random phases uniformly distributed over $[0, 2\pi)$. The optimal jamming power allocation is given by

$$P_{J1}^* = \begin{cases} P_J, & \text{if } C(P_{J1} = P_J) \le C(P_{J1} = 0), \\ 0, & \text{if } C(P_{J1} = P_J) > C(P_{J1} = 0), \end{cases} \tag{11}$$
$$P_{J2}^* = P_J - P_{J1}^*, \tag{12}$$

where $C(P_{J1} = a)$ denotes the ergodic capacity with the jamming power $P_{J1} = a$ and $P_{J2} = P_J - a$, and

$$C^R = \int_0^\infty \log_2\left(1 + \frac{{d_{SR}}^{-\alpha} P_S\, z}{{d_{JR}}^{-\alpha} P_{J1} + 1}\right) \exp(-z) \mathrm{d}z, \tag{13}$$
$$C^D = \int_0^\infty \log_2\left(1 + \frac{{d_{RD}}^{-\alpha} P_R\, z}{{d_{JD}}^{-\alpha} P_{J2} + 1}\right) \exp(-z) \mathrm{d}z. \tag{14}$$

Proof: See Appendix B.

Note that the optimal transmit jamming signals do not follow Gaussian distribution. This is due to the fact that, in a fast fading scenario, the fading channel gain from the jammer to the legitimate receiver (either relay or destination) already creates the Gaussian randomness. In order to preserve the Gaussian nature of the received jamming signal, the jammer should send out signals with constant magnitude. In addition, the random phase with uniform distribution not only preserves the Gaussian nature of the received jamming signal, but also makes it impossible for the legitimate receiver to detect and cancel the jamming signal.

A. Numerical Results

Fig. 4 compares the ergodic capacity achieved under optimal jamming power allocation and that under equal power jamming. From Theorem 2, the optimal jamming power allocation is to spend all power to jam the weaker link (or any link when the two links have equal strength). From this figure, we see a large capacity gap between the optimized and non-optimized jamming strategies, even when the source-relay link and the relay-destination link have the same strength. This clearly shows the detrimental effect that the jammer is able to make by optimizing its power allocation.

[1]Strictly speaking, it is more accurate to name the defined quantities as "ergodic rates" instead of "ergodic capacities" as we have assumed a specific distribution of input signals, i.e., Gaussian inputs, which may or may not maximize the mutual information under general jamming signaling.

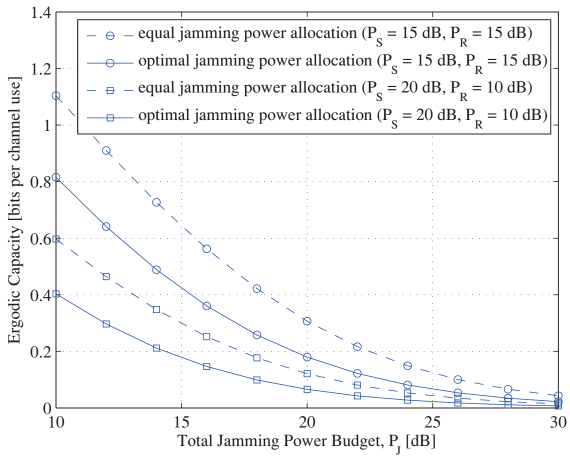

Fig. 4. The ergodic capacity of the DF relay-assisted communication versus the total jamming power budget P_J. Other system parameters are set as: $d_{SR} = d_{JR} = d_{RD} = d_{JD} = 1$, $\gamma = 1$, $\alpha = 3$. Different sets of values of P_S and P_R are used.

V. Conclusion

In this paper, we studied the optimal jamming signaling design against a simple two-hop DF relay network. We derived the optimal jammer design for both quasi-static and ergodic fading channels. Both the optimal distribution of the jamming signals and the optimal jamming power allocation are different for different types of fading channels. Specifically, Gaussian jamming is optimal in quasi-static but not ergodic fading channels. In addition, it is always optimal to jam only one link in ergodic fading channels but this is often not optimal in quasi-static fading channels.

Appendix A
Proof of Theorem 1

We first determine the optimal probability distribution of the jamming signals and then derive the optimal jamming power allocation.

Consider the source-relay transmission, referring to (1), the optimal probability distribution of the jamming signal w_1 should minimize the mutual information $I(x_S; y_R)$. Note that the channel gains, h_{SR} and h_{JR}, are constants but not random variables when computing $I(x_S; y_R)$. In order to minimize $I(x_S; y_R)$, the overall noise term must be a Gaussian random variable [10], which happens when w_1 is Gaussian. Therefore, the optimal distribution of w_1 is zero-mean complex Gaussian. Applying the same argument on the relay-destination transmission, one can show that the optimal probability distribution of the jamming signal w_2 is also zero-mean complex Gaussian.

After determining the Gaussian distribution of the jamming signals, the outage probabilities can be explicitly derived by averaging over the fading channel gains, as

$$\begin{aligned} P_{out}^{R} &= \mathbb{P}\big(I(x_S; y_R) < \mathcal{R}\big), \\ &= \mathbb{P}\left(\log_2\left(1 + \frac{{d_{SR}}^{-\alpha} P_S |h_{SR}|^2}{{d_{JR}}^{-\alpha} P_{J1} |h_{JR}|^2 + 1}\right) < \mathcal{R}\right), \\ &= 1 - \frac{P_S {d_{JR}}^{\alpha}}{{d_{SR}}^{\alpha}\gamma P_{J1} + P_S {d_{JR}}^{\alpha}} \exp\left(-\frac{{d_{SR}}^{\alpha}\gamma}{P_S}\right), \quad (15) \end{aligned}$$

and similarly,

$$P_{out}^{D} = 1 - \frac{P_R {d_{JD}}^{\alpha}}{{d_{RD}}^{\alpha}\gamma P_{J2} + P_R {d_{JD}}^{\alpha}} \exp\left(-\frac{{d_{RD}}^{\alpha}\gamma}{P_R}\right), \quad (16)$$

where $\gamma = 2^{\mathcal{R}} - 1$ representing a threshold SNR. Note that all the fading channel power terms, in the form of $|h|^2$, follow exponential distribution with unit variance.

The problem of finding the optimal jamming power allocation for maximizing the overall outage probability P_{out} can be written as

$$\begin{aligned} \arg \max_{P_{J1}, P_{J2}} \quad & 1 - (1 - P_{out}^{R})(1 - P_{out}^{D}), \\ \text{subject to} \quad & P_{J1} + P_{J2} = P_J. \end{aligned}$$

Using (15) and (16), the optimization problem reduces to

$$\begin{aligned} \arg \max_{P_{J1}, P_{J2}} \quad & ({d_{SR}}^{\alpha}\gamma P_{J1} + P_S {d_{JR}}^{\alpha})({d_{RD}}^{\alpha}\gamma P_{J2} + P_R {d_{JD}}^{\alpha}), \\ \text{subject to} \quad & P_{J1} + P_{J2} = P_J, \end{aligned}$$

which can be solved in a straightforward manner.

Appendix B
Proof of Theorem 2

Again, we first determine the optimal probability distribution of the jamming signals and then derive the optimal jamming power allocation.

Consider the source-relay transmission, referring to (1), the optimal probability distribution of the jamming signal w_1 should minimize the conditional mutual information $I(x_S; y_R | h_{SR})$. Note that the jammer's channel gain, h_{JR}, is unknown to the relay, and hence, is treated as a random variable. In order to minimize $I(x_S; y_R | h_{SR})$, the overall noise term must be a Gaussian random variable [10], which happens when $h_{JR} w_1$ is Gaussian. Since h_{JR} is already Gaussian, one choice of the probability distribution for w_1 is to have a constant amplitude with a uniformly distributed phase. With such a choice, the term $h_{JR} w_1$ remains Gaussian. Therefore, the optimal distribution of w_1 is a random variable with a unit magnitude and a random phase following a uniform distribution over $[0, 2\pi)$. Applying the same argument on the relay-destination transmission, one can show that the optimal probability distribution of the jamming signal w_2 is the same as w_1.

After determining the optimal distribution of the jamming signals, the ergodic capacities can be explicitly derived as in (13) and (14). The problem of finding the optimal jamming power allocation for minimizing the overall ergodic capacity C can be written as

$$\begin{aligned} \arg \min_{P_{J1},P_{J2}} \quad & \min\{C^R, C^D\}, \\ \text{subject to} \quad & P_{J1} + P_{J2} = P_J. \end{aligned}$$

Lemma 1: The solution to the above optimization problem is obtained at one of the extreme points, i.e., either $P^*_{J1} = P_J$ *or* $P^*_{J1} = 0$

Proof: This can be proved by contradiction. Assume that the solution is not obtained at the extreme points, i.e., $P^*_{J1} = \rho \in (0, P_J)$. At this optimal point, the ergodic capacity is given by either $C^R/2$ or $C^D/2$:

Case 1: $C = C^R/2$ at $P^*_{J1} = \rho \in (0, P_J)$. From (13) we know that C^R decreases as P_{J1} increases. Therefore, it is possible to find a value of $P_{J1} > \rho$ that further reduces the ergodic capacity. Hence, the optimal point cannot be at $P^*_{J1} = \rho$.

Case 2: $C = C^D/2$ at $P^*_{J1} = \rho \in (0, P_J)$. From (14) we know that C^D decreases as P_{J1} decreases (i.e., as P_{J2} increases). Therefore, it is possible to find a value of $P_{J1} < \rho$ that further reduces the ergodic capacity. Hence, the optimal point cannot be at $P^*_{J1} = \rho$.

In conclusion, the optimal value of P_{J1} must be at one of the extreme points. ■

With the result in Lemma 1, one can easily find the optimal value of P_{J1} by comparing the ergodic capacity with $P_{J1} = P_J$ and that with $P_{J1} = 0$.

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