Kalman Filter-based Channel Estimation for Amplify and Forward Relay Communications

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Abstract—We propose an autoregressive model for the combined amplify and forward time-varying relay channel and derive a causal iterative channel estimation method using Kalman filter. This formulation enables us to study and compare two widely-used pilot transmission strategies in terms of the channel estimation errors. We provide a single-letter formula for the power allocation between the source and relay to achieve near optimal bit error rate (BER) performance in dual-hop communications. For cooperative communications, we show that the relay speed has a significant impact on the BER performance, and hence it is important to be considered in practical system design.

I. INTRODUCTION

The use of relayed transmission increases the communication range and reduces the need of having high power at the transmitter [1]. Studies on cooperative transmission also show that the use of relays provides spatial diversity gains in wireless communication systems [2]. A summary of relaying strategies was provided in [3], among which the amplify-and-forward (AF) and decode-and-forward (DF) schemes have been extensively studied in the past few years, especially in resource-constrained scenarios [4]. Most of the existing studies on relayed and cooperative communications assume that the channel state information at the receiver (CSIR) is perfect.

Recently, the design of channel estimation methods for AF relaying systems has drawn an increasing attention. Under a block fading assumption, two channel estimation schemes were studied in [5, 6]. When the communicating terminals are mobile, the source-relay-destination (dual-hop) channel can be a cascade of fixed-to-mobile or mobile-to-mobile channels, and hence, can change rapidly with time. Therefore, the block fading channel model becomes less appropriate in these circumstances. Using a time-varying channel model, a linear minimum mean square error (LMMSE) channel estimation method was studied in [7].

In this paper, we propose a first order autoregressive model to characterize the time-vary nature of the dual-hop channel. We prove that under some mild conditions, the principle of Kalman filter (KF) can be applied to estimate the dual-hop channel gain in both time division multiplexing (TDM) [8] and superimposed transmission (SIT) [8] schemes. We show that TDM scheme outperforms SIT scheme in various practical scenarios. Furthermore, we propose a simple scheme for power allocation between source and relay in dual-hop communications with channel estimation errors. Our numerical results show that this simple scheme achieves near optimal BER performance over a wide range of power budget in the presence of channel estimation errors. Furthermore, equal power allocation is found to give good performance when the relay is close to the destination. For cooperative communication systems, we investigate the effects of relay speed on the bit error rate (BER) performance, as well as the optimal power allocation. Our numerical studies show that the relay speed has significant impact on the BER performance and optimal power allocation. The results suggest that a speed-dependent relay selection scheme can dramatically improve the system performance while reducing the need of adaptive power allocation.

II. SYSTEM DESCRIPTION

We consider a wireless communication system consisting of a source node (SN), a relay node (RN) and a destination node (DN), each equipped with a single antenna. The RN is able to amplify-and-forward (AF) the signal sent from the SN to the DN. We denote the channel gain between the SN and the RN as \( h_\ell \) and the channel gain between the RN and the DN as \( g_\ell \), where \( \ell \) is the time index. We assume that \( h_\ell \) and \( g_\ell \) are independent zero-mean circularly symmetric complex Gaussian (ZMCSG) random variables with variance \( \sigma_h^2 \) and \( \sigma_g^2 \), respectively.

When the SN sends out a signal \( x_\ell \) with \( E\{x_\ell^2\} = 1 \), the received signal at the DN is given by

\[
y_{\ell,D} = A \sqrt{\rho_N g_\ell h_\ell x_\ell} + A g_\ell n_\ell,R + n_\ell,D
\]

where \( A = \sqrt{\rho_R / \sigma_h^2} \) is a fixed relay gain, \( \rho_N \) and \( \rho_R \) are the transmit powers at the SN and the RN, respectively, \( n_{\ell,R} \) and \( n_{\ell,D} \) are the ZMCSG noises at the RN and the DN with variance \( N_0 \) (or \( N_0 / 2 \) per dimension), \( f_\ell = g_\ell h_\ell \) denotes the combined dual-hop channel gain and \( z_\ell = A g_\ell n_\ell,R + n_\ell,D \) denotes the combined noise for the dual-hop link.

When the direct (SN-DN) link is considered (for cooperative communications), its signal model is given by

\[
y_{\ell,D}' = \sqrt{\rho_N} f_\ell x_\ell + n_{\ell,D},
\]

1For simplicity, we use a single time index symbol \( \ell \) for all the communications. However, it should be noted that the communications in different links, e.g. the SN-RN link and the RN-DN link, can take place in different time slots, and hence, \( \ell \) is not a global time index.
where \( f'_\ell \) denotes the channel gain between the SN and the DN.

A. Channel Model

In this work, we consider that the SN is stationary while the RN and the DN are mobile. An example of this scenario could be the communication between a fixed base station and a mobile terminal with another mobile terminal acting as a relay. Due to the node mobility, the channel gains of all the communications links change with time. We adopt a first order autoregressive model to characterize the time-varying nature of individual single-hop channels. For the SN-RN link, we have

\[
h_\ell = \alpha h_{\ell-1} + w_\ell,
\]

where \( w_\ell \) is a ZMCSG noise process. \( \alpha \) is the temporal correlation coefficient given by \( \alpha = J_0(2\pi F_D T_s) \), where \( J_0 \) is the zero-order Bessel function of the first kind, \( F_D \) is the Doppler frequency shift due to the mobility of RN, and \( T_s \) is the symbol duration. Similarly for the RN-DN link, we have

\[
g_\ell = \beta g_{\ell-1} + v_\ell,
\]

where \( v_\ell \) is a ZMCSG noise process. \( \beta \) is the temporal correlation coefficient given by \( \beta = J_0(2\pi F_D T_s)J_0(2\pi F_D T_s) \) [9], where \( F_D \) is the Doppler frequency shift due to the mobility of the DN. Therefore, the combined channel of the dual-hop link can be modeled as

\[
f_\ell = a f_{\ell-1} + u_\ell,
\]

where \( a = \alpha \beta \) and \( u_\ell = \alpha h_{\ell-1} v_\ell + \beta g_{\ell-1} w_\ell + w_\ell v_\ell \).

Similarly, the channel model for the direct link is given by

\[
f'_\ell = \gamma f'_{\ell-1} + u'_\ell,
\]

where \( u'_\ell \) a ZMCSG noise process. \( \gamma \) is the temporal correlation coefficient given by \( \gamma = J_0(2\pi F_D T_s) \).

III. Kalman Filter Channel Estimation

In this section, we derive a causal LMMSE channel estimation method using KF. The KF channel estimator for the direct link was derived in [8]. Here, we extend their results to the dual-hop relay link. In state-space model, the observation equation is given in (1) and the state update equation is given in (5).

With practical assumptions of independence between the channel gains and observation and process noises for different links, it can be shown that the signal and noise terms in (1) and (5) have the following important properties.

\[
E\{z_\ell f_{\ell-1}\} = 0, \quad \forall \ i \geq 0,
\]
\[
E\{u_\ell u_{\ell-i}\} = 0, \quad E\{z_\ell z_{\ell-i}\} = 0, \quad \forall \ i \neq 0,
\]
\[
E\{u_\ell f'_{\ell-1}\} = 0, \quad E\{u_\ell y'_{\ell-1,D}\} = 0, \quad E\{z_\ell y'_{\ell-1,D}\} = 0, \quad \forall \ i \geq 1.
\]

The above orthogonality properties enable the application of the standard KF [10]. Note that the KF is the optimal filter only in the linear sense due to the non-Gaussian distribution of the dual-hop channel gain. The channel estimation method depends on the pilot transmission scheme. Two commonly used transmission schemes are time division multiplexing (TDM) and superimposed transmission (SIT) [8]. We derive the KF channel estimator for both schemes in the following, and compare their performance in Section V-A.

A. TDM-Based Scheme

In TDM-based scheme, we consider a periodic pilot insertion of rate 1/L. Effectively, we have a block transmission scheme where each block consists of a pilot symbol transmission followed by \( L-1 \) data symbol transmissions.

During pilot transmission, the KF gain is given by

\[
K_\ell = \frac{(a^2 M_{\ell-1} + \sigma_u^2)\sqrt{\rho_s \sigma_g^2}}{(a^2 M_{\ell-1} + \sigma_u^2)\rho_s + \sigma_g^2},
\]

where \( M_\ell \) is the MSE of the channel estimate, \( \sigma_u^2 = E\{u_\ell u_\ell^*\} = (1-a^2)\sigma_R^2 \sigma_g^2 \), and \( \sigma_g^2 = E\{z_\ell z_\ell^*\} = A^2 \sigma_g^2 N_0 + N_0 \). The channel estimate update equation is given by

\[
\hat{f}_\ell = a \hat{f}_{\ell-1} + K_\ell (y_{\ell,D} - A\sqrt{\rho_s} a \hat{f}_{\ell-1} x_\ell),
\]

and the update equation for MSE of the channel estimate is

\[
M_\ell = (1 - K_\ell A\sqrt{\rho_s} x_\ell)(a^2 M_{\ell-1} + \sigma_u^2).
\]

Without loss of generality, we initialize \( \hat{f}_0 = 0 \) and \( M_0 = \sigma_u^2 \sigma_g^2 \). During data transmission, the channel prediction is given by

\[
\hat{f}_\ell = a \hat{f}_{\ell-1},
\]

and the update equation for MSE of the channel estimate is

\[
M_\ell = a^2 M_{\ell-1} + \sigma_u^2.
\]

It can be shown that this channel estimation process always converges to a periodic steady state for \( a < 1 \) [10]. At the steady state, we have \( M_\ell = M_{\ell,T} \), \( \forall \ \ell \). Therefore, the steady-state MSE of the channel estimate at the pilot transmission is given by (derivation is similar to that in [8])

\[
M_{1,ss} = \frac{\sigma_g^2 \sigma_u^2}{\frac{a^2}{2}(1 + \epsilon) + \sqrt{\left(\frac{a^2}{2}(1 + \epsilon)\right)^2 + \frac{\sigma_u^2 \sigma_g^2}{\frac{a^2}{2}(1 + \epsilon)}}}\epsilon,
\]

where \( \epsilon = \frac{a^2 \rho_s \sigma_g^2}{\sigma_u^2} = \frac{A^2 \rho_s \sigma_g^2}{\sigma_u^2 \sigma_R^2 + N_0} \), which is the average signal-to-noise ratio (SNR) at the DN. The steady-state MSE of the channel estimate at the data transmission can be found using (11) as

\[
M_{\ell,ss} = \sigma_R^2 \sigma_u^2 - \sigma_u^2 (\ell^2 - 1) \left(\frac{a^2}{2} \sigma_g^2 - M_{1,ss}\right), \quad \ell = 2, 3, ..., L.
\]

B. SIT-Based Scheme

In SIT-based scheme, the transmitted signal is given by \( x_\ell = d_\ell + p_\ell \), where \( d_\ell \) is the data part and \( p_\ell \) is the pilot part. For channel estimation, the observation equation in (1) needs to be rewritten as

\[
y_{\ell,D} = A\sqrt{\rho_s} f_{\ell-1} + A\sqrt{\rho_d} f_{\ell-1} + z_\ell,
\]

\[
\ell = 2, 3, ..., L.
\]

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where \( \rho_p \) and \( \rho_d \) are the transmit power for pilot and data, respectively, with \( \rho_p + \rho_d = \rho S \). Similar to the TDM case, the KF equations are derived as

\[
K_\ell = \frac{(a^2 M_{\ell-1} + \sigma^2_h)A\sqrt{\rho_p}}{(a^2 M_{\ell-1} + \sigma^2_h)A^2 + A^2\rho_p \sigma^2_h + \sigma^2_z},
\]

\[
\hat{\ell}_t = a \hat{\ell}_{t-1} + K_\ell (y_{t,D} - A \sqrt{\rho_p} \hat{\ell}_{t-1} + \rho_t),
\]

\[
M_\ell = (1 - K_\ell A \sqrt{\rho_p})p_t (a^2 M_{\ell-1} + \sigma^2_h).
\]

At the steady state, we have \( M_\ell = M_{\ell-1}, \forall \ell \). Therefore, the MSE of the channel estimate is given by

\[
M_{ss} = \frac{\sigma^2_h \sigma^2_g}{\frac{1}{2} \left( 1 + \epsilon \right) + \sqrt{\frac{1}{2} \left( 1 + \epsilon \right)^2 + \frac{a^2}{1-\epsilon}}},
\]

where \( \epsilon = \frac{a^2 \rho_p \sigma^2_g}{A^2 \rho_p \sigma^2_h + A^2 N_0 + N_0} \). Note that \( M_{ss} \) is independent of time.

### IV. Power Allocation for Dual-Hop Link

In power constrained scenarios, an important design problem is the optimal power allocation between the SN and the RN. The objective function for power optimization can be short-term system performance such as outage probability or long-term system performance such as BER. Optimizing short-term performance requires the power allocation to be adaptive to the instantaneous channel gains. However, this requirement becomes overwhelming when the channel is time-varying. Therefore, we focus on the long-term system performance and only require knowledge of the second order statistics of the channel gains and noises to design the power allocation scheme between the SN and the RN. Furthermore, finding the optimal power allocation is very difficult and mathematically intractable when channel estimation error is considered. One practical strategy is to find a low-complexity solution that achieves close to optimal system performance.

We focus on the TDM-based scheme since it usually outperforms SIT based scheme in practical scenarios (which will be shown in the next section). To obtain a simple power allocation solution, we choose the average SNR at the DN during pilot transmission as the objective function given as

\[
\epsilon = \frac{A^2 \rho_S \sigma^2_h \sigma^2_g}{A^2 \sigma^2_h N_0 + N_0}, \quad \text{with} \quad A^2 = \frac{\rho_R}{\rho_S \sigma^2_h + N_0}.
\]

When a total transmit power constraint for the SN and the RN is given by \( \rho = \rho_S + \rho_R \), we denote the ratio of the total power allocated to the SN as \( \phi \), that is to say, \( \rho_S = \phi \rho \) and \( \rho_R = (1 - \phi) \rho \). Therefore, it is easy to show that the optimal \( \phi \) that maximizes \( \epsilon \) is given by

\[
\phi = \begin{cases} \rho^2 \sigma^2_h N_0 - \sqrt{\rho^2 \sigma^2_h N_0 + \rho^2 \sigma^2_h \sigma^2_g + \rho^2 \sigma^2_h N_0} \rho^2 \sigma^2_h \sigma^2_g \rho^2 \sigma^2_h \sigma^2_g \rho^2 \sigma^2_h N_0 \rho^2 \sigma^2_h \sigma^2_g & \sigma^2_h \neq \sigma^2_g \\ \frac{\rho^2}{2} & \sigma^2_h = \sigma^2_g \end{cases}
\]

(18)

When the total transmit power is sufficiently high, we have

\[
\phi \approx \frac{1}{1 + \sqrt{\frac{\sigma^2_h}{\sigma^2_g}}},
\]

(19)

which shows that more power should be allocated to the SN if the SN-RN link is weaker than the RN-DN link, and vice versa. In the next section, we will numerically show that the closed-form solution given in (18) achieves near optimal BER performance.

### V. Numerical Results

In this section, we numerically study the performance of the KF channel estimator and power allocation scheme. We also investigate the effect of relay mobility on the BER performance of cooperative communication systems. In all numerical results, we consider that the SN, RN and DN are on the same line. We denote the distance between SN and RN, RN and SN, SN and RN as \( d_{sr} \), \( d_{sd} \) and \( d_{rd} \), respectively. The distance between the SN and the DN is normalized to unity, i.e., \( d_{sd} = d_{sr} + d_{rd} = 1 \). Assuming a path loss exponent of 3, the variances of the channel gains are given by \( \sigma^2_h = 1/d^3_{sr} \) and \( \sigma^2_g = 1/d^3_{sd} \). We also set \( N_0 = 1 \). We consider a carrier frequency of 2.5 GHz and a bandwidth of 10 kHz, which is suitable for Mobile WiMAX, i.e., IEEE 802.16e [11]. In our numerical examples, we will use normalized Doppler frequency (i.e., \( F_D T_s \) or \( F_D T_d \)) of 0.001, 0.005 and 0.02, corresponding to mobile speeds of 4.3 km/h, 21.6 km/h and 86.4 km/h, to represent slow fading, moderate fading and fast fading scenarios, respectively.

#### A. Dual-hop Communications

Fig. 1 and Fig. 2 show the steady-state MSE of the channel estimates for the dual-hop link (normalized by \( \sigma^2_h \sigma^2_g \)) in slow fading and fast fading scenarios, respectively. For comparison, we include KF channel estimation in both TDM and SIT-based transmission with MSE expressions given in Section III-A and Section III-B, as well as a non-causal channel estimation method in TDM-based transmission discussed in [7]. The non-causal estimator uses 4 pilot observations (2 from the past...
and 2 from the future) closest to the data transmission slot of interest. To obtain a fair comparison between TDM and SIT schemes, we keep the ratio of power allocated to pilots and data in each scheme the same by setting $p_p = \rho / L$ in SIT scheme [8]. For TDM based transmission scheme, we include the MSE of the channel estimates at both the first and the last data transmission within a block.\footnote{Due to symmetry, the MSE of the non-causal channel estimates at the first and the last data slots is identical, and does not differ too much from the MSE at other data slots.}

From Fig. 1 and Fig. 2, it is clear that the KF channel estimator works better in TDM scheme than in SIT scheme. For the slow fading channel, we see from Fig. 1 that the KF estimator outperforms the non-causal estimator at low to moderate transmit power. On the other hand, we see from Fig. 2 that the non-causal estimator outperforms KF estimator for the fast fading channel. Since the implementation of KF estimator requires minimum memory storage and does not incur any delays in the detection process, it gives a good trade-off between the system performance and complexity.

To investigate the performance of the simple power allocation between the SN and the RN proposed in Section IV, we carry out Monte-Carlo simulation on BER performance of BPSK modulated transmission shown in Fig. 3. The solid lines are BER achieved by using the closed-form power allocation solution given in (18), and the markers are BER achieved by using the optimal power allocation found numerically. We see that the closed-form solution always achieves near optimal performance over a wide range of power budget for different fading rates and locations of RN. Furthermore, we include the BER performance using equal power allocation between the SN and the RN in Fig. 3 as the dashed lines. We see a general trend that equal power allocation gives good performance when the RN is close to the DN (e.g., $d_{sr} = 0.75$), while it results in a noticeable performance degradation when the RN is close to the SN (e.g., $d_{sr} = 0.25$).

B. Cooperative Communications

We now study the performance of cooperative communication, that is when both the dual-hop link and the direct link are used. Once the DN obtains the channel estimates for both links, it treats the estimates as the true channel gains and combines the received signals using maximum ratio combining. In particular, we investigate the effect of the speed of RN on the BER performance and optimal power allocation in cooperative communication systems.

We consider that the direct link is in moderate fading with the normalized Doppler frequency at the DN given by $F_D T_s = 0.005$. The block length is $L = 10$. We consider either a fast moving RN with $F_R T_s = 0.02$ or a slow moving RN with $F_R T_s = 0.001$, located either close to the SN with $d_{sr} = 0.25$ or close to the DN with $d_{sr} = 0.75$. Fig. 4 shows the BER performance of BPSK modulated transmission. Both direct link communication and cooperative communication are considered. The power allocation between the SN and the RN is optimized numerically in all cases. We see that the use of cooperation always results in a significant BER reduction. Moreover, the speed of RN makes a considerable difference in BER performance. For example, for a target BER of $10^{-3}$ the power saving by selecting a slow moving RN rather than a fast moving RN is around 5.4 dB when $d_{sr} = 0.75$ or around 3.3 dB when $d_{sr} = 0.25$. This result suggests that a speed-dependent relay selection strategy is another way to significantly improve the system performance. In addition, we see that it is generally better to select an RN that is close to
the DN rather than close to the SN, which agrees with the trend observed in the dual-hop scenarios in Section V-A.

Fig. 5 shows the optimal power allocation between the SN and the RN in terms of $\phi$ for the system parameters used in Fig. 4. For comparison, we also include the optimal power allocation when perfect CSIR is available. We see that the optimal values of $\phi$ for slow RN (dashed lines) is very close to those for perfect CSIR (markers) due to the small channel estimation errors in the case of slow RN. When the channel estimation of the dual-hop link becomes poor which happens for fast RN, the optimal $\phi$ changes significantly towards (but not reaching) unity, especially at high transmit power. Therefore, the optimal power allocation is significantly affected by the speed of RN. Since the power optimization needs to be done numerically, the computational complexity is relatively high. An alternative solution is again a speed-dependent relay selection strategy which ensures the near-optimality of the power allocation scheme obtained from the perfect CSIR case.

VI. CONCLUSION

In this paper, we have studied the dual-hop and cooperative communication systems with AF relaying. To characterize the time-varying nature of the dual-hop channel gain, we proposed a first-order autoregressive channel model, based on which we derived a KF channel estimator. Our numerical results have shown that the KF channel estimator works better in TDM scheme than in SIT scheme. For dual-hop communication systems, we have provided a closed-form solution to the power allocation between the SN and the RN, which achieves near optimal BER performance. We have also seen that equal power allocation between the SN and the RN can also achieve near optimal BER performance when the RN is close to the DN. For cooperative communication systems, our numerical results have shown that the speed of RN has a significant impact on the BER performance through channel estimation errors. Furthermore, optimizing power allocation between the SN and the RN is crucial when the RN is fast moving. Therefore, a speed-dependent relay selection strategy can result in good system performance while reducing the need of adaptive power allocation.

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