Imposing pattern nulls on broadband array responses

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This paper considers the problem of altering a quiescent design for an array of omni-directional sensors so that the altered design rejects a far-field broadband signal from a given direction. This problem occurs where microphone arrays are to be used to acquire speech signals for telecommunication and interferring signals must be rejected. Three main results are presented in this paper. First, conditions for imposing an exact broadband null upon any given quiescent array response are derived. Second, an analysis of the sensitivity of exact nulling array responses to sensor position error is presented. Third, frequency domain formulations for broadband nulls are obtained and it is shown that these are less restrictive than the exact null conditions which are imposed in the time domain. A Lagrange multiplier approach is used to impose the null conditions upon a quiescent array response to minimize the square error between the quiescent and nulled responses. A design example is given. © *1999 Acoustical Society of America.* [S0001-4966(99)02706-X]

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INTRODUCTION

In this paper we consider the problem of altering a far-field quiescent design for an array of omni-directional sensors so that the altered design rejects a far-field broadband signal from a given (known) direction. Without loss of generality, we will use quiescent designs based on the work presented by Ward *et al.*[1] While far-field designs are presented in this paper, similar results hold for the near-field problem.[2]

Microphone arrays for use in telecommunications applications[3] are the motivation for this work. The arrays must acquire a broadband signal, such as speech, while dealing with interference, such as other speakers or office noise.

No adaptive algorithms are examined here, although this is certainly an interesting extension of the current work. Rather, this paper examines what is achievable in the time-invariant setting. This can be considered as a bound on the performance of any adaptive extensions.

In the past, several methods of controlling the positions of the nulls have been employed, including modification of the element amplitude and phase,[4,5] modification of the element phase only,[4,6] and element position perturbation.[7–9]

In phased arrays, where the beamformer weights are fixed as a function of frequency, broadband pattern nulling is effected by imposing a null over a wide spatial region centered on the required broadband null direction. Methods of producing such a null trough include

- imposing multiple pattern nulls,[10–12]
- imposing derivative constraints at the desired direction,[10,13,14] and
- constraining the average power over an angular region,[15,16]

in the vicinity of the required broadband null direction.

The broadband nulling problem this paper addresses is more precisely defined in Sec. I. The problem decomposes into two sub-problems: choice of the cost functional (a squared error cost is chosen) and determination of appropriate constraints. A standard Lagrange multiplier approach is used to solve this optimization problem.

In Sec. II, the conditions for an exact broadband null are derived. These conditions are also presented in a form compatible with the solution approach used. An analysis of the sensitivity of the exact null to sensor position uncertainty is presented. This result allows greater insight into the structural enforcement of pattern nulls and improves on the performance of previous techniques.

Problems with the exact null approach, namely the necessity for oversampling and overly restrictive sensor position constraints, lead to an examination of an alternative approach, multiple frequency nulling, shown in Sec. III.

Both nulling approaches are applied to a frequency invariant quiescent design in Sec. IV. Conclusions are drawn in Sec. V.

I. PROBLEM STATEMENT

We will now define some notation and define the problem addressed in this paper.

Consider a linear array of N omni-directional sensors, as illustrated in Fig. 1. Each sensor signal is filtered using a

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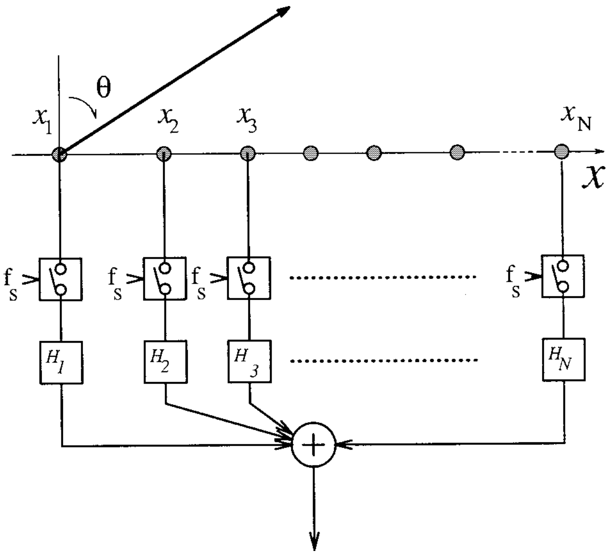

FIG. 1. The array geometry assumed.

causal finite impulse response (FIR) filter, $h_n[k]$ for $k=0,\ldots,L-1$, and the filtered signals are combined to give the following spatial response for a plane wave impinging on the array from a direction θ measured relative to broadside:

$$r(u,\omega)=\sum_{n=1}^{N} H_n(\omega)\exp(j\omega\tau_n(u)), \tag{1}$$

where

$$H_n(\omega)=\sum_{k=0}^{L-1} h_n[k]\exp(-j\omega k)$$

is the frequency response of the nth sensor filter, $\omega=2\pi f/f_s$ is the discrete-time frequency variable with a sampling rate of f_s, $\tau_n(u)=f_s c^{-1}x_n u$ is the relative propagation delay to the nth sensor with a propagation speed of c, x_n is the location of the nth sensor, and $u=\sin\theta$.

Let the NL-dimensional delay vector be

$$\mathbf{d}(u,\omega)=\mathbf{e}(\omega)\otimes\mathbf{a}(u,\omega),$$

where $\otimes$ indicates a Kronecker product, the L-dimensional discrete Fourier transform vector is

$$\mathbf{e}(\omega)=[1,e^{-j\omega},\cdots,e^{-j\omega(L-1)}]^T,$$

and the N-dimensional array response vector is

$$\mathbf{a}(u,\omega)=[e^{j\omega\tau_1(u)},e^{j\omega\tau_2(u)},\cdots,e^{j\omega\tau_N(u)}]^T.$$

The array response may then be written in vector form as

$$r(u,\omega)=\mathbf{h}^H\,\mathbf{d}(u,\omega),$$

where the NL vector of complex FIR filter coefficients is

$$\mathbf{h}=[h_1[0]\cdots h_N[0]\cdots h_1[L-1]\cdots h_N[L-1]]^H,$$

and x^H denotes the Hermitian transpose of x.

We wish to find the coefficients $\hat{\mathbf{h}}$ that produce a similar array response to $\mathbf{h}$, but with a broadband null in a given direction. We will assume that coefficients $\hat{\mathbf{h}}$ are formed by adding a set of perturbing coefficients to $\mathbf{h}$; i.e., $\hat{\mathbf{h}}=\mathbf{h}+\mathbf{b}$. The resulting response $\hat{r}(u,\omega)$ is equivalent to the quiescent response $r(u,\omega)$ plus the response of a nulling beamformer with coefficients $\mathbf{b}$:

$$\begin{aligned}\hat{r}(u,\omega)&=\hat{\mathbf{h}}^H\,\mathbf{d}(u,\omega)\\&=\mathbf{h}^H\,\mathbf{d}(u,\omega)+\mathbf{b}^H\,\mathbf{d}(u,\omega)\\&=r(u,\omega)+\mathbf{b}^H\,\mathbf{d}(u,\omega).\end{aligned}$$

Problem 1 (Imposing a Broadband Null). *The general broadband nulling problem can be formulated as: determine the coefficient vector* $\mathbf{b}$ *that minimizes*

$$\min_{\mathbf{b}} J(\mathbf{b}) \tag{2}$$

$$\text{subject to}\quad |\hat{r}(u_0,\omega)|\leqslant\epsilon,\quad \forall\omega\in\Omega, \tag{3}$$

where J is some suitably defined cost functional that measures the distance between $r(u,\omega)$ and $\hat{r}(u,\omega)$, ϵ is the desired null depth in the nulling direction u_0, and Ω is the set of frequencies of interest (typically the bandwidth of the source and interfering signals).

Several candidate cost functionals and null-imposing constraints will now be considered.

A. Cost functionals

The choice of J in Eq. (2) is very application dependent. In many situations use of a min–max (or L_∞) error criterion is of interest, however, computationally simple procedures to solve such problems occur only rarely[17] and closed-form solutions generally exist only for trivial cases.

As we wish to find closed-form, computationally simple functionals we will only consider square-error (L_2) techniques. Any one of several candidate functionals could be used, including:

- minimizing the weighted square distance between the quiescent response and the perturbed response,
- minimizing the output power of the nulling beamformer (which would be appropriate in an adaptive environment), and
- minimizing the gain of the nulling beamformer.

Although there is no one ''right'' choice of J, the last one listed above is by far the simplest to compute, and it will be considered exclusively in the remainder of this paper (for a more complete treatment of cost functionals see Ref. 18). Specifically, we wish to minimize the L_2 norm between the quiescent coefficients and the perturbed coefficients:

$$J_{NG}=\|\hat{\mathbf{h}}-\mathbf{h}\|^2=\|\mathbf{b}\|^2,$$

where $\|\mathbf{b}\|=(\mathbf{b}^H\mathbf{b})^{1/2}$ is the L_2 norm of $\mathbf{b}$.

B. Constraints

Having determined a cost functional, the practicality of the constraint in Eq. (3) must be assessed. The main results of this paper allow the following constraints to be imposed.

1. Exact nulling

Consider the following problem:

$$\min_{\mathbf{b}} \mathbf{b}^H \mathbf{b} \tag{4}$$

$$\text{subject to} \quad \hat{r}(u_0, \omega) = 0, \quad \forall \omega. \tag{5}$$

This corresponds to a constraint on the time domain impulse responses of the sensor filters, as shown in Sec. II.

2. Multiple frequency nulling

An alternative to an exact broadband null is to impose multiple zeros in the frequency domain response of the beamformer at the null angle. In this case, the problem is formulated as:

$$\min_{\mathbf{b}} \mathbf{b}^H \mathbf{b} \tag{6}$$

$$\text{subject to} \quad \hat{r}(u_0, \omega) = 0, \quad \omega = \omega_1, \cdots, \omega_M. \tag{7}$$

The challenge now is in choosing the set of M frequencies $\{\omega_m\}_{m=1}^{M}$ at which to impose the frequency domain zeros in order to obtain a broadband null of a given depth. This problem is considered in Sec. III.

C. Producing a broadband null in a quiescent response

The problem of producing a broadband null in a quiescent broadband response while perturbing that pattern the least with regard to J_{NG} may be tackled as follows. If $r(u,\omega) = \mathbf{h}^H \mathbf{d}(u,\omega)$ is the quiescent beamformer response, find coefficients $\mathbf{b}$ such that

$$\hat{r}(u,\omega) = (\mathbf{h}^H + \mathbf{b}^H)\mathbf{d}(u,\omega) \tag{8}$$

is close to $r(u,\omega)$ as measured by J_{NG} and a broadband null is imposed.

The optimum $\mathbf{b}$ is found as the solution to the constrained optimization problem:

$$\min_{\mathbf{b}} \mathbf{b}^H \mathbf{b} \tag{9}$$

$$\text{subject to} \quad \mathbf{C}^H(\mathbf{h} + \mathbf{b}) = \mathbf{0}_K, \tag{10}$$

where $\mathbf{C}$ is a $NL \times K$ constraint matrix, $\mathbf{0}_K$ is the K vector of zeros, and K represents the finite number of constraints to be imposed.

The solution to this constrained optimization problem may be found using Lagrange multipliers as stated in the following Proposition; see Frost[19] for a more complete treatment.

Proposition 1 (Coefficients for a Broadband Null). *The optimum perturbations* $\mathbf{b}_{\text{opt}}$ *from the quiescent weights* $\mathbf{h}$ *to ensure a broadband null in the required direction are given by*

$$\mathbf{b}_{\text{opt}} = -\mathbf{C}[\mathbf{C}^H \mathbf{C}]^{-1} \mathbf{C}^H \mathbf{h}. \tag{11}$$

Having defined a suitable cost functional and examined the solution technique to be used, the next two sections show how the two broadband null constraints may be placed in the solution framework. That is, we identify suitable choices for $\mathbf{C}$ to impose broadband nulls.

II. EXACT NULLING

We now show how it is possible through proper array design to produce an exact pattern null (a pattern zero that is present over all frequencies). However, we also show that stringent requirements are imposed on the array geometry and sampling rate to achieve this. In Sec. II D we evaluate the degradation of an exact null that would occur in a practical setting with sensor location errors.

A. Exact broadband nulling

The requirements for an exact null are presented in Proposition 2 and its Corollary.

Proposition 2 (Condition for a Broadband Null). *A broadband null at* u_0 *will be available if and only if either*

$$r(u_0, \omega) = \sum_{n=1}^{N} H_n(\omega) e^{j\omega\tau_n(u_0)} = 0, \quad \forall \omega \tag{12}$$

or, equivalently,

$$\sum_{n=1}^{N} (h_n[k] * \text{sinc}(k + \tau_n(u_0))) = 0, \quad \forall k \tag{13}$$

where $*$ *denotes convolution in the* k *index and* $\text{sinc}(x) = \sin(\pi x)/(\pi x)$.

Proof: From Eq. (1), the response in direction u_0 is

$$\begin{aligned} r(u_0, \omega) &= \sum_{n=1}^{N} H_n(\omega) e^{j\omega\tau_n(u_0)} \\ &= \sum_{n=1}^{N} \sum_{k=0}^{L-1} h_n[k] e^{-j\omega k} e^{j\omega\tau_n(u_0)} \qquad (14) \\ &= \sum_{k=0}^{L-1} \left(\sum_{n=1}^{N} h_n[k] * \text{sinc}[k + \tau_n(u_0)] \right) e^{-j\omega k}. \qquad (15) \end{aligned}$$

Equation (14) yields Eq. (12) and the inverse discrete-time Fourier transform of Eq. (15) yields Eq. (13).

It is not immediately clear how Eqs. (12) and (13) may be easily enforced. The following result shows this.

Corollary 1 (Integer Delay Property). *If* $\tau_n(u_0)$ *is an integer for all* n*, and*

$$\sum_{n=1}^{N} h_n[k + \tau_n(u_0)] = 0, \quad \forall k, \tag{16}$$

then Eq. (13) is satisfied.

Proof: If $\tau_n(u_0)$ is an integer for all n, then Eq. (13) becomes

$$\sum_{n=1}^{N} h_n[k] * \delta[k + \tau_n(u_0)] = 0, \quad \forall k,$$

where

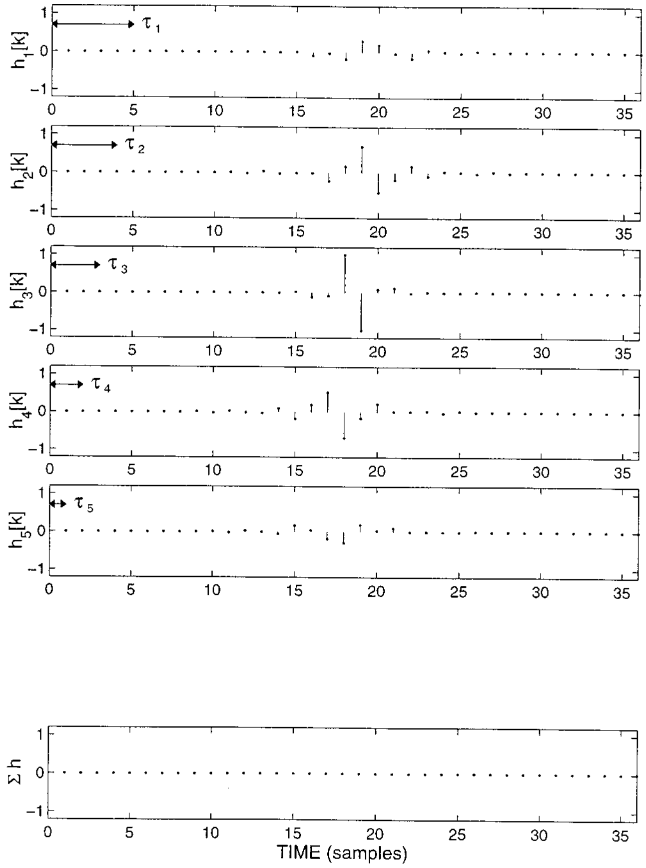

FIG. 2. Time domain nulling constraint.

$$\delta[k]=\begin{cases}1, & k=0\\ 0, & k\neq 0.\end{cases}$$

The null constraint Eq. (16) is illustrated in Fig. 2. Specifically, when $\tau_n(u_0)$ is an integer for all n, the sum of the delayed filter impulse responses can be made exactly zero.

Frost[19] noted that, for a signal arriving from broadside, the broadband beamformer with N sensors and L taps per sensor was equivalent to an L tap filter whose coefficients were the sum over N sensors. The integer delay property of the Corollary is equivalent, but for delays other than $\tau_n(u_0)=0$, $\forall n$.

B. Constraints for exact nulling

Note that it is always possible to place a null at $u_0=0$ because in this case

$$\tau_n(u_0)=\frac{f_s}{c}x_n u_0=0, \quad \forall n$$

and Eq. (16) reduces to requiring

$$\sum_{n=1}^{N} h_n[k]=0, \quad \forall k.$$

Using this idea, the condition for an exact null at broadside may be written as

$$\mathbf{C}_0^H\mathbf{h}=\mathbf{0}_L,$$

where

$$\mathbf{C}_0=\mathbf{I}_L\otimes\mathbf{1}_N,$$

where $\mathbf{I}_L$ is the $L\times L$ identity matrix and $\mathbf{1}_N$ is the N-vector of ones. The resulting $\mathbf{C}_0$ is an $NL\times L$ matrix.

To demonstrate how a null may be placed at directions other than broadside, we rewrite the integer delay property Eq. (16) as

$$\sum_{n=1}^{N}\tilde{h}_n[k]=0,$$

where $\tilde{h}_n[k]=h_n[k+\tau_n(u_0)]$. The $\tilde{h}_n[k]$ are effectively the filter coefficients that steer the null direction to broadside. Thus if the coefficients $\tilde{h}_n[k]$ were used as the beamformer filter coefficients, the resulting response would be steered to $-u_0$ and the null would appear at broadside. This null steering is only exact for $\tau_n(u_0)\in\mathbb{Z}$, $\forall n$.

The null constraint can now be written as

$$\mathbf{C}_0^H\tilde{\mathbf{h}}=\mathbf{0}.$$

Define a linear transformation matrix $\mathbf{T}_u$ that satisfies $\tilde{\mathbf{h}}=\mathbf{T}_u\mathbf{h}$. The null constraint now becomes

$$\mathbf{C}^H\mathbf{h}=\mathbf{0}, \tag{17}$$

where $\mathbf{C}=\mathbf{T}_u^H\mathbf{C}_0$ is the transformed constraint matrix.

C. Implications for practical array design

It is instructive to consider what the integer delay property implies for a practical design. The requirement for a broadband null is

$$\tau_n(u_0)=f_s c^{-1}x_n u_0\in\mathbb{Z}, \quad n=1,2,...,N.$$

Let $x_1=0$ and $x_n>0$, $n>1$, i.e., we are considering a single-sided linear array. The first constraint on the array geometry is

$$\frac{x_n}{d_0}\in\mathbb{Z}, \quad n>1, \tag{18}$$

regardless of the desired null angle, where we will refer to d_0 as the *fundamental spacing*. Second, we require

$$d_0=\frac{mc}{f_s u_0}, \quad m\in\mathbb{Z}. \tag{19}$$

Assume that the sensor spacings are logarithmically increasing,[20] and the part of the array closest to the origin is used only at the highest frequency, with more and more elements becoming active at lower frequencies. Assuming we want to use the minimum number of sensors to avoid spatial aliasing, then

$$d_0=\frac{c}{2f_U},$$

where f_U is the upper frequency of interest. The directions in which we may form a broadband null are then

$$u_0=\frac{m2f_U}{f_s}, \quad m\in\mathbb{Z}.$$

Clearly, for a minimum sampling rate of $f_s=2f_U$ it is only possible to produce a broadband null at $u_0\in\{-1,0,1\}$, or equivalently $\theta_0 \in \{-\pi/2, 0, \pi/2\}$. Hence, the minimum sampling rate required for a null at u_0 is

$$f_s = \frac{2 f_U}{u_0} \geqslant 2 f_U. \tag{20}$$

This demonstrates the major disadvantages of time domain nulling: Oversampling is required to produce a null that is away from broadside or endfire, and the sensor locations must be quantized to multiples of the fundamental spacing. These problems can be overcome by using interpolation beamforming[21] or fractional sample delay FIR filters.[22,23] However, since FIR filters must be quite long to obtain accurate fractional sample delays, these techniques will result in significantly longer sensor filters.

D. Effect of sensor position errors on a broadband null

Given that the optimum nulling coefficients have been determined to produce an exact null in a given direction, we now consider the effect of sensor position errors on the expected null depth. We follow the standard method in the antenna literature.[24]

Assume the actual position of the nth sensor is $\hat{x}_n = x_n + \chi_n$, where x_n is the ideal sensor location and the χ_n are independent zero-mean Gaussian random variables with variance σ_x^2. We assume that the location errors are uncorrelated from sensor to sensor.

The actual broadband response is

$$\hat{r}_a(u,\omega) = \sum_{n=1}^{N} \hat{H}_n(\omega) e^{j\omega \hat{\tau}_n(u)}, \tag{21}$$

where

$$\hat{H}_n(\omega) = \sum_{k=0}^{L-1} (h_n[k] + b_n[k]) e^{-j\omega k},$$

with the $b_n[k]$ calculated from Eq. (11) assuming ideal sensor locations. The actual propagation delay to the nth sensor is

$$\hat{\tau}_n(u) = f_s c^{-1}(x_n + \chi_n) u = \tau_n(u) + \Delta_n(u). \tag{22}$$

We now have

$$|\hat{r}_a(u,\omega)|^2 = \sum_{\substack{n=1 \\ n\neq m}}^{N} \sum_{m=1}^{N} \hat{H}_n(\omega)\hat{H}_m^*(\omega) e^{j\omega[\tau_n(u)-\tau_m(u)]} \times e^{j\omega[\Delta_n(u)-\Delta_m(u)]} + \sum_{n=1}^{N} |\hat{H}_n(\omega)|^2.$$

The expected value of the beampattern is

$$E[|\hat{r}_a(u,\omega)|^2] = \sum_{\substack{n=1 \\ n\neq m}}^{N} \sum_{m=1}^{N} \hat{H}_n(\omega)\hat{H}_m^*(\omega) e^{j\omega[\tau_n(u)-\tau_m(u)]} \times |E_\Delta(u,\omega)|^2 + \sum_{n=1}^{N} |\hat{H}_n(\omega)|^2, \tag{23}$$

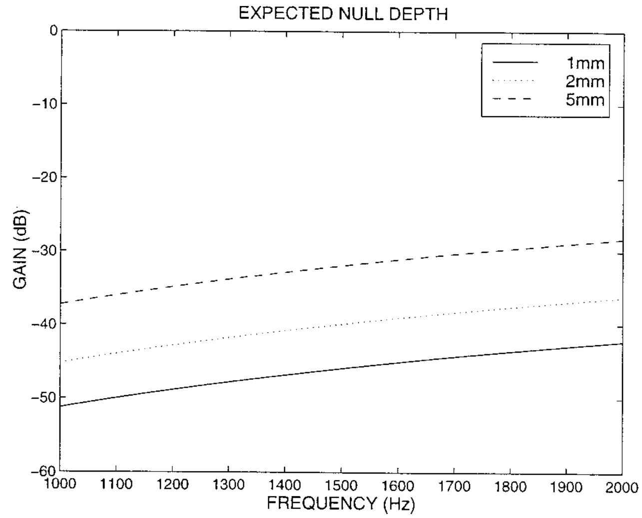

FIG. 3. Expected null depth for $\sigma_x = 1$ mm (——), $\sigma_x = 2$ mm (···), and $\sigma_x = 5$ mm (--).

where $E_\Delta(u,\omega) = E[e^{j\omega f_s c^{-1} \chi u}]$. Noting that $E_\Delta(u,\omega)$ is the characteristic function of the Gaussian random variable χ, gives[25]

$$E_\Delta(u,\omega) = \exp(-\sigma_x^2(\omega f_s c^{-1} u)^2/2). \tag{24}$$

Recall that the ideal beampattern is

$$|\hat{r}(u,\omega)|^2 = \sum_{\substack{n=1 \\ n\neq m}}^{N} \sum_{m=1}^{N} \hat{H}_n(\omega)\hat{H}_m^*(\omega) e^{j\omega[\tau_n(u)-\tau_m(u)]} + \sum_{n=1}^{N} |\hat{H}_n(\omega)|^2.$$

Hence, the actual beampattern can be expressed as

$$E[|\hat{r}_a(u,\omega)|^2] = |\hat{r}(u,\omega)|^2 |E_\Delta(u,\omega)|^2 + \sum_{n=1}^{N} |\hat{H}_n(\omega)|^2 (1 - |E_\Delta(u,\omega)|^2).$$

Assuming that the ideal pattern has an exact null at u_0, then $\hat{r}(u_0,\omega) = 0, \forall \omega$, and the expected actual null direction response is

$$E[|\hat{r}_a(u_0,\omega)|^2] = \sum_{n=1}^{N} |\hat{H}_n(\omega)|^2 \times \left(1 - \exp\left(-\sigma_x^2 \frac{(\omega f_s c^{-1} u_0)^2}{2}\right)^2\right) \tag{25}$$

$$\approx \sigma_x^2 (\omega f_s c^{-1} u_0)^2 \sum_{n=1}^{N} |\hat{H}_n(\omega)|^2, \quad \text{for small } \sigma_x. \tag{26}$$

The expected null depth for several different positioning errors, for the array design detailed in Sec. IV, is shown in Fig. 3. Unsurprisingly, the depth of the null decreases as the sensor position errors increase.

III. MULTIPLE FREQUENCY NULLING

In this section we consider an alternative to exact broadband nulling that avoids the oversampling problem identified in exact nulling. Specifically, we consider the problem of placing multiple zeros in the null direction frequency response such that a given null depth is achieved while minimizing the disturbance to the quiescent broadband response.

A. Constraints for multiple zeros in null direction frequency response

Let $r(u,\omega)=\mathbf{h}^H\mathbf{d}(u,\omega)$ be the quiescent broadband response. The constraint we wish to impose here is

$$\mathbf{d}(u_0,\omega_m)^H(\mathbf{h}+\mathbf{b})=0, \tag{27}$$

for $m=1,...,M$. The cost functional is still the nulling gain J_{NG}.

This problem is identical to Eqs. (9) and (10) with $\mathbf{C}=[\mathbf{d}(u_0,\omega_1),\ldots,\mathbf{d}(u_0,\omega_M)]$. The resulting $\mathbf{C}$ is an $NL\times M$ matrix. The solution is again given by Eq. (11).

Clearly, we are not limited to only placing multiple frequency nulls at the direction u_0. A wider null may be specified by placing further nulls at a direction $u_0+\Delta u_0$ (this is demonstrated by an example in Sec. IV).

It is now necessary to determine the zero locations ω_m, $m=1,...,M$ in order to achieve the required null depth. As a simplification we will assume the M zeros are equally spaced within the design frequency band, so the problem now is only to choose the number of zeros.

B. Determining the required number of frequency zeros

Consider the problem of determining the number M of equally spaced zeros to impose in the null direction frequency response in order to achieve a given null depth within the design frequency band $\Omega=[\omega_L,\omega_U]$. An analogous problem was considered by Steyskal[11] in which he determined the number of constraints required to achieve a required null depth over a spatial sector for a phased array. Steyskal's method can be modified as follows.

The constraint Eq. (27) may be rewritten as

$$\hat{r}(u_0,\omega_m)=0, \tag{28}$$

for $m=1,\ldots,M$.

Steyskal[10] showed that the weights which solve the constrained optimization are given by

$$\hat{\mathbf{h}}=\mathbf{h}-\sum_{m=1}^{M}\alpha_m\mathbf{d}_m,$$

where $\mathbf{d}_m=\mathbf{d}(u_0,\omega_m)$. Equivalently, the optimum response is given by

$$\hat{r}(u,\omega)=r(u,\omega)-\sum_{m=1}^{M}\alpha_m q^{(m)}(u,\omega), \tag{29}$$

where

$$q^{(m)}(u,\omega)=\mathbf{d}_m^H\mathbf{d}(u,\omega). \tag{30}$$

The parameters α_m are obtained by solving the set of M simultaneous equations

$$\begin{bmatrix} r(u_0,\omega_1)\\ \vdots \\ r(u_0,\omega_M)\end{bmatrix}=\begin{bmatrix} q^{(1)}(u_0,\omega_1) & \cdots & q^{(M)}(u_0,\omega_1)\\ \vdots & & \vdots\\ q^{(1)}(u_0,\omega_M) & \cdots & q^{(M)}(u_0,\omega_M)\end{bmatrix}\times\begin{bmatrix}\alpha_1\\ \vdots\\ \alpha_M\end{bmatrix}. \tag{31}$$

Define the null depth as

$$\epsilon=\max_{\omega\in\Omega}|\hat{r}(u_0,\omega)|^2, \tag{32}$$

where $\hat{r}(u,\omega)$ is given by Eq. (29), and $q^{(m)}(u,\omega)$ is given by Eq. (30).

1. Simplification for a frequency invariant beamformer

It is extremely difficult to gain any insight into the variation of ϵ with M using the general equations above. Hence, in this section our aim is to determine M for the specific case where the quiescent beamformer is a frequency invariant beamformer (FIB). We will see that in this case it is relatively simple to determine *a priori* the required value of M for a desired null depth.

Consider the following assumptions.

(1) The M nulls are equally spaced over the region Ω.
(2) For an FIB, the quiescent response in the nulling direction is approximately constant over the nulling frequency band, that is, $r(u_0,\omega)\approx A$, $\omega\in\Omega$, where $A=|r(u_0,\omega_L)|$.
(3) $\hat{r}(u_0,\omega)$ is oscillatory and crosses the ω axis at $\omega=\omega_1,\ldots,\omega_M$.
(4) $\hat{r}(u_0,\omega)$ attains its maximum value at $\omega_{\max}=(\omega_1+\omega_2)/2$.

Of these assumptions, only the last one requires further comment. When M equally spaced frequency nulls are imposed within a region Ω, the null-direction response will exhibit $(M-1)$ lobes within Ω. The peak of each lobe will occur approximately midway between the adjacent pair of null frequencies. From many simulations, we have observed that it is generally the first or last lobe that has maximum height. For simplicity, we will assume that the first lobe has maximum height.

With these assumptions, the null depth is given by

$$\epsilon=\left|\hat{r}\left(u_0,\frac{\omega_1+\omega_2}{2}\right)\right|^2, \tag{33}$$

where

$$\hat{r}(u_0,\omega)=A-\sum_{m=1}^{M}\alpha_m q^{(m)}(u_0,\omega), \tag{34}$$

with $q^{(m)}(u_0,\omega)$ given by Eq. (30), and α_m given by the solution of Eq. (31) with $r(u_0,\omega_m)=A$, $m=1,\ldots,M$.

2. Incremental null depth

At this point it is useful to define the *incremental null depth*, ϵ_{inc} as the null depth imposed in the direction u_0 relative to the quiescent FIB response, i.e.,

$$\epsilon_{\rm inc} \triangleq |\hat{r}(u_0,\omega_{\max}) - r(u_0,\omega_{\max})|^2, \tag{35}$$

which for a FIB reduces to

$$\epsilon_{\rm inc} = |\hat{r}(u_0,\omega_{\max}) - A|^2. \tag{36}$$

Hence, from Eq. (34) we have

$$\epsilon_{\rm inc} = \left| \sum_{m=1}^{M} \alpha_m q^{(m)}(u_0,\omega_{\max}) \right|^2, \tag{37}$$

with α_m given by the solution of Eq. (31) with $r(u_0,\omega_m) = 1$, $m = 1,...,M$.

Although Eq. (37) falls short of providing an explicit expression for M in terms of $\epsilon_{\rm inc}$, it does, however, provide a simple means of calculating $\epsilon_{\rm inc}$ for a given M. This calculation depends on the array geometry, number of filter coefficients, sampling rate, design bandwidth, and null direction. For a given beamformer all of these variables will be fixed. Importantly, however, the calculation does not depend on the filter coefficients themselves (only on the number of filter coefficients used). Thus the incremental null depth achieved for a given M is independent of the quiescent beampattern; even if the FIB filter coefficients are determined from an adaptive algorithm, the incremental null depth in the null direction will be as determined by Eq. (37).

C. Effect of sensor position errors

In Sec. II we derived an expression, Eq. (26), for the expected null depth in a practical situation where sensor position errors are present. In the case of multiple frequency nulls this equation becomes

$$E[|\hat{r}_a(u_0,\omega)|^2] = |\hat{r}(u_0,\omega)|^2 |E_\Delta(u_0,\omega)|^2 + \sum_{n=1}^{N} |\hat{H}_n(\omega)|^2 (1 - |E_\Delta(u_0,\omega)|^2),$$

where $\hat{r}(u_0,\omega)$ is the ideal null direction response (assuming ideal sensor locations).

IV. DESIGN EXAMPLES

In order to illustrate the methods developed above, consider the following example.

A broadband array was designed to operate in the band 1–2 kHz, with an aperture size of six half-wavelengths (for acoustic waves in air) and a sampling rate of $f_s = 8$ kHz. The null was to be located at $\theta = 30°$. For exact nulling, this required a fundamental spacing of $d_0 = 0.085$ m. The resulting sensor positions are tabulated in Table I.

A. Quiescent response

The quiescent response was found following the frequency invariant beamformer (FIB) ideas presented by Ward *et al.*[1,26] The FIR filters each had $L = 32$ coefficients. The quiescent array response is shown in Fig. 4.

TABLE I. Sensor locations and sensor-to-sensor delays relative to center sensor for a source at 30°.

Sensor position (m)	Time delay (samples)
−0.5100	−6
−0.4250	−5
−0.3400	−4
−0.2550	−3
−0.1700	−2
−0.0850	−1
0	0
0.0850	1
0.1700	2
0.2550	3
0.3400	4
0.4250	5
0.5100	6

B. Exact nulling

Placing a null at $\theta = 30°$ corresponds to one of the quiescent array sidelobes. Using the techniques of Sec. II, the array response of the exact nulling design is shown in Fig. 5.

Note, from Table I, that the sensor-to-sensor delays (for a signal arriving at $\theta = 30°$) in this array are integer numbers of samples.

C. Multiple frequency nulling

In order to avoid the problem of integer delays (or the associated problem of large increases in filter order required by fractional delay FIR filters) the multiple frequency nulling technique can be used. Applied in this example, with $M = 10$ and nulling direction again $\theta = 30°$, the resulting array response is shown in Fig. 6.

Figure 7 shows the multiple frequency technique applied at bearings of 27.5° and at 32.5°. Ten nulls were placed at each bearing. The purpose of this is to produce a null over a wider range of bearings.

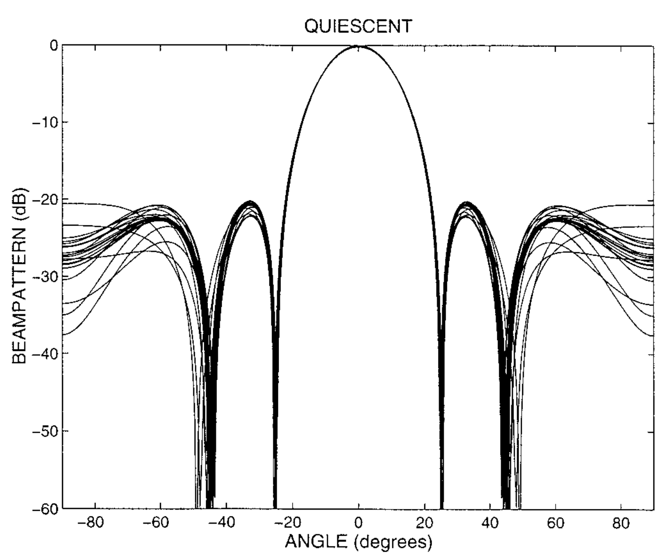

FIG. 4. Quiescent array response at 21 equispaced frequencies within the design band.

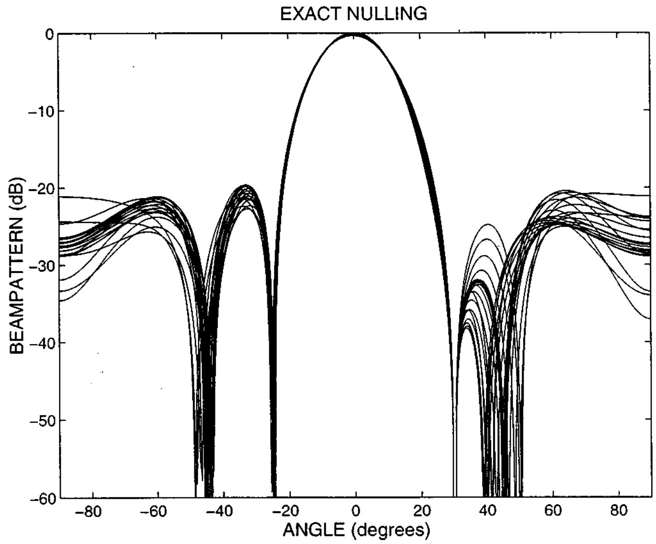

FIG. 5. Array response with exact null at 30° at 21 equispaced frequencies within the design band.

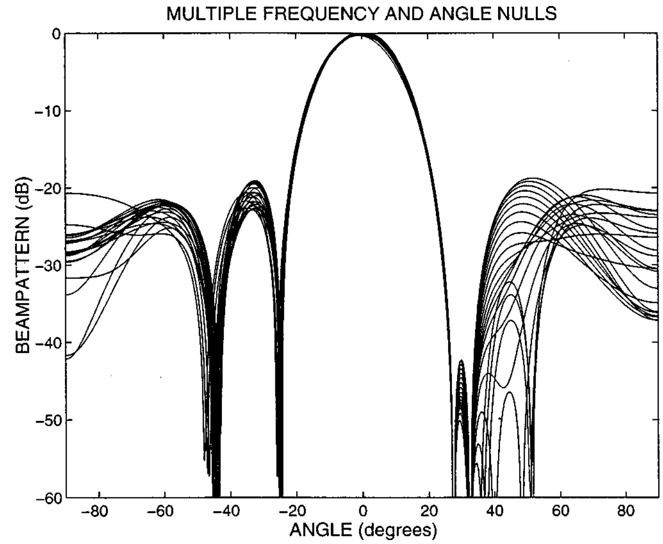

FIG. 7. Array response with multiple frequency nulls ($M=10$) at 27.5° and multiple frequency nulls ($M=10$) at 32.5° at 21 equispaced frequencies within the design band.

D. Null depths

The array responses of the quiescent and multiple frequency nulls at the required null direction are displayed in Fig. 8, normalized with respect to the quiescent null direction response at 1 kHz, i.e., these plots show the incremental null depth that will be obtained relative to the quiescent response. Observe that the predicted null depth (-46 dB) is very close to the actual null depth (-49 dB).

The exact null depth is not shown as it was zero for all intents and purposes (lower than 200 dB over the frequency range of interest).

The predicted incremental null depth for the multiple frequency nulling approach Eq. (37) is plotted versus number of nulls in Fig. 9. Also shown is the actual incremental null depth, which demonstrates that the predicted null depth is in good agreement with the actual depth over a large range of M. This demonstrates the validity of the prediction technique derived in Sec. III.

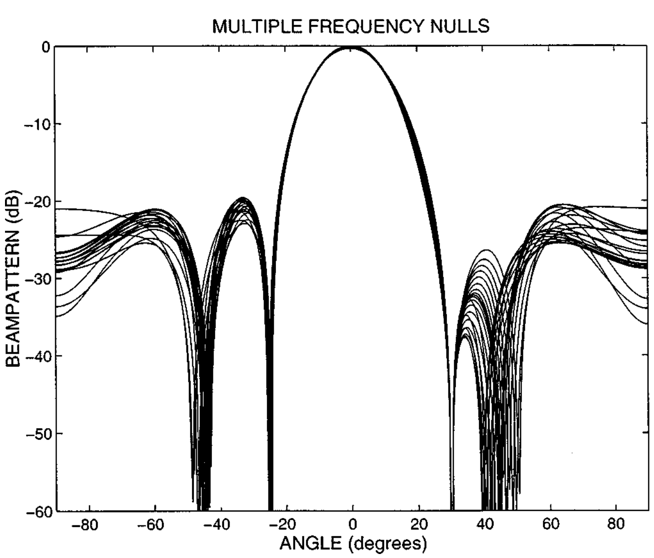

FIG. 6. Array response with multiple frequency nulls ($M=10$) at 30° at 21 equispaced frequencies within the design band.

V. CONCLUSION

The broadband nulling problem considered in this paper was formulated as follows. Given an NL vector of filter coefficients that produces some desired broadband response $r(u,\omega)$ for a beamformer with N sensors and L filter taps per sensor, find the coefficients that produce a broadband response $\hat{r}(u,\omega)$ which has a broadband null in a specified direction and was close in some respect to the original response $r(u,\omega)$. The problem was formulated in terms of the following constrained minimization problem: minimize the distance between $r(u,\omega)$ and $\hat{r}(u,\omega)$ subject to the constraint that $\hat{r}(u,\omega)$ exhibits a broadband null in the direction $u=u_0$.

Two null constraints were considered: one which placed an exact null in the null direction over all frequencies and one which placed multiple single-frequency nulls in the required null direction.

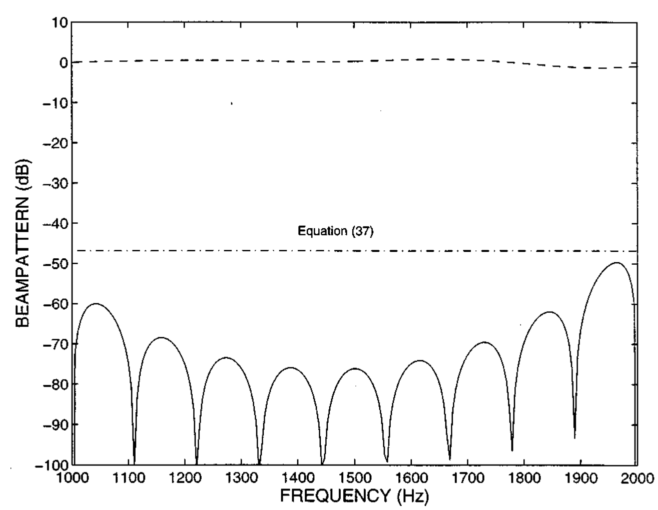

FIG. 8. Array responses of quiescent design (--) and multiple frequency null design (——) at the null direction ($\theta=30°$) over the array design frequency band. The predicted null depth from Eq. (37) for the multiple frequency nulling approach with $M=10$ is also shown (-.-).

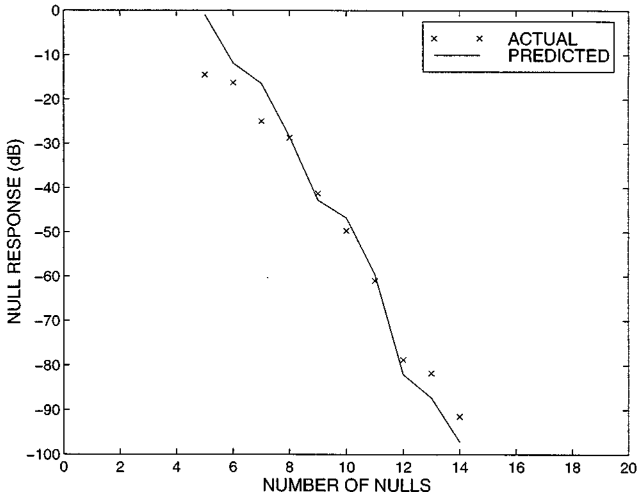

FIG. 9. The predicted null depth given by Eq. (37), and the actual obtained depth, versus number of nulls.

For the exact null constraint, it is possible to produce a pattern zero over all frequencies by formulating the constraint as a time domain constraint. This requires oversampling to produce a null at directions other than broadside or endfire, and places stringent constraints on the sensor locations. The degradation of the exact null that occurs in a practical setting with sensor positioning errors was considered. A reasonably deep null is still achieved for small sensor position perturbations. Sensor calibration errors [i.e., differences between the assumed array vector $\mathbf{a}(u,\omega)$ and the actual array vector] would also affect the null depth. A similar technique to that presented for sensor position errors could also be used to analyze the effects of sensor calibration errors on the expected null depth.

For the multiple frequency nulling constraint, a relationship was derived between the number of zeros to impose in the quiescent beampattern, and the broadband null depth over the design bandwidth. In the specific case of a frequency invariant beamformer, a simple expression was provided to *a priori* determine the incremental null depth that will be achieved for a given number of frequency zeros.

Examples show the strengths and weaknesses of each technique.

Finally, we conclude by noting that there is no single ''right'' answer to the broadband nulling problem considered in this paper. We have attempted to give a brief review of some possible formulations of the problem, and have obtained some new results concerning both approximate and exact broadband pattern nulls. These new results are specifically aimed at the frequency invariant beamformer, although they have wider application to more general broadband beamformers.

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