Spatial Aliasing for Nearfield Sensor Arrays*

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Abstract

This paper investigates the presence of spatial aliasing due to operating a linear array in the nearfield. It shows that the standard half wavelength sensor spacings rule, which guarantees no aliasing in the operation of farfield arrays, is not sufficient to prevent aliasing in the nearfield. This claim is justified by theoretical considerations and corroborated by simulation results.

1 Introduction

There has been a growing interest in the nearfield array processing due to the use of microphone arrays in teleconferencing and speech acquisition systems [1–3]. This paper considers the effect of spatially sampling a spherical wavefront received from a point source in the nearfield of a linear array, along the array axis.

2 Spatial Aliasing

Consider a linear array aligned to the x axis and a point source at a distance r from the array origin and angle θ measured relative to endfire. Then the signal received at a point x on the array is given by

$$s_{r,\theta}(x) = \frac{e^{jk\sqrt{r^2+x^2-2rx\cos\theta}}}{\sqrt{r^2+x^2-2rx\cos\theta}}, \tag{1}$$

where $k = 2\pi/\lambda$ is the wavenumber and λ is the wavelength of the received signal. If the source of interest is in the farfield of the array, the normalized signal received at a point x on the array is given by

$$s_{\infty,\theta}(x) = \lim_{r\to\infty} s_{r,\theta}(x)\, re^{-jkr} = e^{-jkx\cos\theta}. \tag{2}$$

By using an array, we effectively sample the signal $s_{r,\theta}(x)$ in spatial domain. To determine the sampling distance, i.e., array spacings, we need to examine the spectral content of the signal $s_{r,\theta}(x)$ with respect to x. Let the Fourier transform of $s(x)$ be

$$S(\xi) = \int_{-\infty}^{\infty} s(x)\, e^{j\xi x}\, dx \tag{3}$$

where ξ is the spatial frequency. Using (3), we can write the Fourier transform of (2) as

$$S_{\infty,\theta}(\xi) = 2\pi\delta(\xi - k\cos\theta) \tag{4}$$

where $\delta(\cdot)$ is the Dirac's delta function. By the usual Nyquist criterion, we need to sample $s_{\infty,\theta}(x)$ with a sampling distance of $d \leq \pi/(k\cos\theta) = \lambda/(2\cos\theta)$ to avoid spatial aliasing. Since we assume the possible range of $\theta \in [0,\pi]$, it suffices to take $d_{max} = \lambda/2$. This result, commonly known as the $\lambda/2$ rule, is standard in the array literature [4]. Until now, this rule has been used for designs in both farfield and nearfield (e.g., [5]). We show here that the $\lambda/2$ rule is generally not valid in the nearfield.

The Fourier Transform $S_{r,\theta}(\xi)$ of $s_{r,\theta}(x)$ can be obtained from the results in [6, page 31]:

$$S_{r,\theta}(\xi) = \begin{cases} j\pi\, e^{jr\xi\cos\theta}\, H_0^{(1)}\Big(r\sin\theta\sqrt{k^2-\xi^2}\Big), & |\xi| < k \\ 2\, e^{jr\xi\cos\theta}\, K_0\Big(r\sin\theta\sqrt{\xi^2-k^2}\Big), & |\xi| > k \end{cases}$$

where $H_0^{(1)}(\cdot)$ is the Hankel function of the first kind of order zero and $K_0(\cdot)$ is the modified Bessel function of order zero. Note that there is a singularity at $|\xi| = k$.

A graph of $|S_{r,\theta}(\xi)|$ vs normalized spatial frequency ξ/k for three different set of values (r,θ) is shown in Figure 1. From this result, it is evident that the function $s_{r,\theta}(x)$ is not bandlimited if the source is in the nearfield of the array at a smaller angle measured relative to the endfire, although it becomes more so as $r \to \infty$ or $\theta \to 90$ degrees. Thus, the use of the $\lambda/2$ rule is not strictly sufficient to ensure no aliasing error, and indeed no sampling distance will entirely eliminate such error.

3 Nearfield Rule of Thumb

To explain the above behaviour, we now examine $S_{r,\theta}(\xi)$ when $\xi > k$ for different values of r and θ. Since $K_0(z) \approx -\ln(z)$ for $z \to 0$ and $K_0(z) \approx \sqrt{\pi/(2z)}e^{-z}$ for large $z > 1$ [7, page 203], $|S_{r,\theta}(\xi)|$ decays rapidly as the argument of $K_0(\cdot)$ (i.e., $r \sin\theta\sqrt{\xi^2 - k^2}$) increases. Suppose there exists positive numbers M and z_0 such that $|S_{r,\theta}(\xi)| < M$ for $r \sin\theta\sqrt{\xi^2 - k^2} > z_0$ for a given r and θ. Then for a suitably small M we can assert that $S_{r,\theta}(\xi)$ is approximately bandlimited by

$$\xi_0 = \sqrt{k^2 + \frac{z_0^2}{r^2 \sin^2\theta}}, \tag{5}$$

and sampling distance of π/ξ_0 or less reduces the aliasing up to an acceptable level. It is difficult to find an analytic expression for z_0 in terms of M or quantify an acceptable level of aliasing. But a convenient rule of thumb is $z_0 \approx 1$.

Note that when $r \to \infty$, $\xi_0 \to k$, hence $S_{r,\theta}(\xi)$ is bandlimited by k for this case. For the case of $\theta = 90$ degrees, $\xi_0 = \sqrt{k^2 + 1/r^2} \approx k$ for all practical values of r in the nearfield. For example if $r = 3\lambda = 6\pi/k$ then $\xi_0 = k\sqrt{1 + 1/36\pi^2} \approx k$. Hence, for angles close to 90 degrees, $S_{r,\theta}(\xi)$ is bandlimited by k even for nearfield signals. However, nearfield signals from small angles are not spatially bandlimited which can be gleaned from (5).

4 Simulations and Conclusion

To conclude we show the effect of spatial aliasing due to sampling a signal from a nearfield source at 3.5λ from an array origin, where λ is the wavelength of the signal. Figure 2 shows the magnitude response of three arrays with different sensor spacings of $\lambda/2$, $\lambda/4$ and $\lambda/6$, to the above source as a function of θ. For comparison purposes, we make all three arrays to have equal aperture length, thus they have 7, 13 and 19 elements, respectively. The effect of aliasing is clearly evident from the response of the $\lambda/2$ spaced array, however there is little or no effect of aliasing present in the response of the $\lambda/6$ spaced array. This result is in agreement with (5) which gives sensor spacing of $\lambda/5.6$ to avoid aliasing for the case of $r = 3.5\lambda$ and $\theta = 1$ degrees.

Thus we can conclude that the received signal from a point source in the nearfield is not bandlimited in spatial frequency and hence use of standard half wavelengths spaced arrays introduce undesirable aliasing effects to the array output. The use of finer sensor spacings can overcome limitations imposed by aliasing.

References

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List of captions

- **Figure 1.** Magnitude of the Fourier transform $S_{r,\theta}(\xi)$ of the signal $s_{r,\theta}(x)$ for $r = 3.5\lambda$, $\theta = 1$ degrees (solid line); $r = 3.5\lambda$, $\theta = 5$ degrees (dashed dot line); $r = 3.5\lambda$, $\theta = 90$ degrees (dashed line) and $r = 100\lambda$, $\theta = 1$ degrees (dotted line) plotted against the normalized spatial frequency ξ/k.

- **Figure 2.** Magnitude of the array response of a $\lambda/2$ spaced 7 sensor array (dotted line), $\lambda/4$ spaced 13 sensor array (dashed line) and $\lambda/6$ spaced 37 sensor array (solid line line) to a nearfield source at 3.5λ from the array origin. Aperture length of each of three arrays are equal to 3λ.

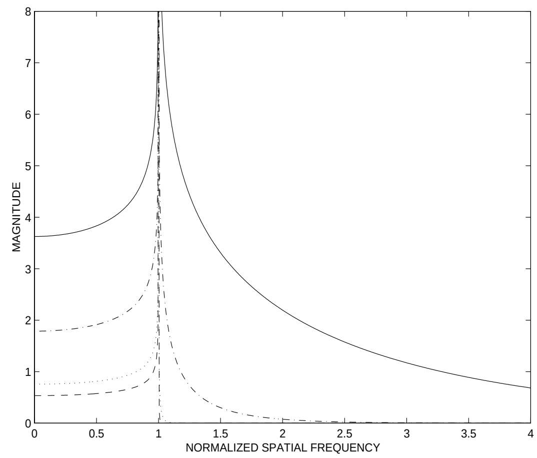
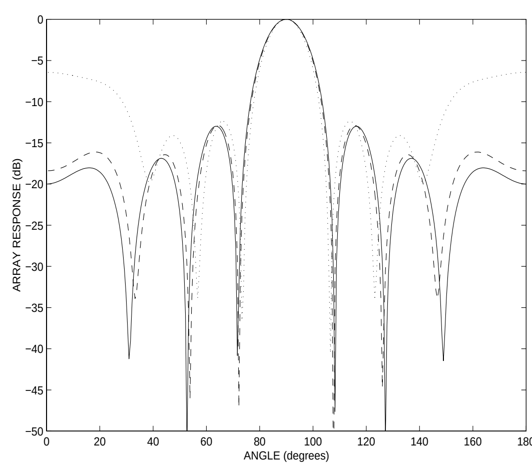


Figure 1: Magnitude of the Fourier transform $S_{r,\theta}(\xi)$ of the signal $s_{r,\theta}(x)$ for $r = 3.5\lambda$, $\theta = 1$ degrees (solid line); $r = 3.5\lambda$, $\theta = 5$ degrees (dashed dot line); $r = 3.5\lambda$, $\theta = 90$ degrees (dashed line) and $r = 100\lambda$, $\theta = 1$ degrees (dotted line) plotted against the normalized spatial frequency ξ/k.



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