ISOTROPIC NOISE MODELLING FOR NEARFIELD ARRAY PROCESSING

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ABSTRACT

In this paper, an exact series representation for a nearfield spherically isotropic noise model is introduced. The methodology uses the spherical harmonics expansion of the wavefield at a sensor to obtain the correlation between two sensors due to the nearfield isotropic noise field. The result is useful in nearfield application of sensor arrays. The proposed noise model can be utilized effectively to apply well established farfield array processing algorithms for nearfield applications. Specifically, any signal processing criterion based on farfield isotropic noise correlation can be reformulated with nearfield noise with this representation. A simple array gain optimization is used to demonstrate the new noise model.

1. INTRODUCTION

Nearfield sensor array design is of considerable importance in teleconferencing and speech acquisition applications. The majority of array processing literature deals with the situations where the desired source and the noise sources are assumed to be in the farfield of the array; this considerably simplifies the design problem. In most stochastic optimization techniques, the noise correlation matrix plays an integral part of the design. In fixed beamformer design, the noise field is assumed to be known, and usually modeled by either white gaussian noise or *farfield spherically isotropic noise* which results from a uniform distribution of point noise sources over all directions in the farfield.

For nearfield applications of sensor arrays such as teleconferencing, the noise field consists of undesirable nearfield sound sources as well as reverberation caused by the desired and noise sources. Using the *source-image* method [1], we can model reverberation with point sources. In an average size room, some or all first order reflected reverberant sources will be in the nearfield of the array while multiply reflected ones will be in the farfield. Due to absorption by walls, a multiply reflected reverberant noise source contributes less power compared to first order reflected ones. Thus, the overall noise field is due to nearfield as well as farfield noise sources, and an assumption of farfield spherically isotropic noise or white gaussian noise is a very crude approximation. In [2], the farfield spherically isotropic noise was used to model the effect of reverberation without considering the effect of nearfield noise sources.

As an alternative, in this paper we model the noise field with uniformly distributed sources over all directions in the nearfield at a fixed distance from the array origin. We call this *nearfield spherically isotropic noise*. This noise model can be utilized effectively to apply any signal processing criterion based on isotropic type noise correlation to nearfield applications. In our simulation example in section 4 we will show that a design based on this nearfield noise model performs better than one based on a farfield noise model in a more realistic mixed farfield-nearfield noise field. As motivation for the theoretical development, we consider a simple array optimization technique as applied to a nearfield array in the following section.

2. GAIN OPTIMIZATION FOR AN ARBITRARY ARRAY

The array gain is often used as an indicator of overall array performance. It is defined by

$$G = 4\pi \, \frac{\text{power received from a desired location}\,(P_{source})}{\text{total noise power received}\,(P_{noise})} \tag{1}$$

Consider an array of $2N+1$ sensors, arbitrarily placed in a bounded region $\Omega \subset \mathcal{R}^3$. Then the response of this array to a source located outside the region Ω at $\boldsymbol{y}$, is given by

$$b(\boldsymbol{y}) = \sum_{n=-N}^{N} w_n \, \frac{e^{ik|\boldsymbol{y}-\boldsymbol{x}_n|}}{|\boldsymbol{y}-\boldsymbol{x}_n|} \, y e^{-iky}, \tag{2}$$

where w_n is the complex gain associated with the sensor positioned at $\boldsymbol{x}_n \in \Omega$, $y = |\boldsymbol{y}|$ and $k \triangleq 2\pi f/c = 2\pi/\lambda$ is the wavenumber which can be expressed in terms of the propagation speed c and the frequency f, or the wavelength λ. Thus, the power received from the desired location $\boldsymbol{y}_s$ is given by

$$P_{source} = b^*(\boldsymbol{y}_s)\, b(\boldsymbol{y}_s),$$

where * denotes complex conjugate transpose. Arranging the weights in an $(2N+1)$-element column vector

$$\boldsymbol{W} = \begin{bmatrix} w_{-N} \\ \vdots \\ w_N \end{bmatrix}$$

and defining a square $((2N+1) \times (2N+1))$ Hermitian matrix $\boldsymbol{R}_{source} = \boldsymbol{a}\boldsymbol{a}^*$ in terms of the $(2N+1)$ column vector

$$\boldsymbol{a} = \begin{bmatrix} \dfrac{e^{ik|\boldsymbol{y}-\boldsymbol{x}_{-N}|}}{|\boldsymbol{y}-\boldsymbol{x}_{-N}|} \\ \vdots \\ \dfrac{e^{ik|\boldsymbol{y}-\boldsymbol{x}_{N}|}}{|\boldsymbol{y}-\boldsymbol{x}_{N}|} \end{bmatrix} y e^{-iky}, \tag{3}$$

leads to the matrix formulation

$$P_{source} = \boldsymbol{W}^* \boldsymbol{R}_{source} \boldsymbol{W}. \tag{4}$$

By assuming the *nearfield spherically isotropic noise field*, i.e., having uniformly distributed noise sources on a sphere of radius y, we can write the total noise power received as,

$$P_{noise} = \int b^*(\boldsymbol{y})\, b(\boldsymbol{y})\, d\hat{\boldsymbol{y}} \tag{5}$$

where $\hat{\boldsymbol{y}} = \boldsymbol{y}/y$ is a unit vector in the direction of $\boldsymbol{y}$ and the integration is over the unit sphere. We define the $((2N+1) \times (2N+1))$ matrix $\boldsymbol{R}_{noise} = [r_{nm}]$ with elements

$$r_{nm} = \frac{y^2}{4\pi} \int \frac{e^{ik|\boldsymbol{y}-\boldsymbol{x}_n|}}{|\boldsymbol{y}-\boldsymbol{x}_n|}\, \frac{e^{-ik|\boldsymbol{y}-\boldsymbol{x}_m|}}{|\boldsymbol{y}-\boldsymbol{x}_m|} d\hat{\boldsymbol{y}}. \tag{6}$$

(Note that $\boldsymbol{R}_{noise}$ is Hermitian and positive definite). Then,

$$P_{noise} = 4\pi \boldsymbol{W}^* \boldsymbol{R}_{noise} \boldsymbol{W},$$

and equation (1) becomes a ratio of quadratic forms

$$G = \frac{\boldsymbol{W}^* \boldsymbol{R}_{source} \boldsymbol{W}}{\boldsymbol{W}^* \boldsymbol{R}_{noise} \boldsymbol{W}}. \tag{7}$$

The usual goal is to find the weights which maximize G. Equation (7) is a well known result for array gain [3, page 141] and [4, page 164] and $\boldsymbol{R}_{source}$ and $\boldsymbol{R}_{noise}$ are commonly known as the source correlation matrix and noise correlation matrix respectively. The optimum array gain and weights are given by [5]

$$G_{opt} = \boldsymbol{a}^* \boldsymbol{R}_{noise}^{-1} \boldsymbol{a} \tag{8}$$

and

$$\boldsymbol{W}_{opt} = \boldsymbol{R}_{noise}^{-1} \boldsymbol{a} \tag{9}$$

respectively.

3. NEARFIELD ISOTROPIC NOISE

In this section, we find an exact series representation for the noise correlation r_{nm} (6) between two sensors due to nearfield isotropic noise field.

We write the wavefield at the sensor location $\boldsymbol{x}_n$ due to a source at $\boldsymbol{y}$ for $y > x_n$ using the spherical harmonic expansion [6, page 30] as

$$\frac{e^{ik|\boldsymbol{y}-\boldsymbol{x}_n|}}{|\boldsymbol{y}-\boldsymbol{x}_n|} = 4\pi i k \sum_{p=0}^{\infty} \sum_{q=-p}^{p} h_p^{(1)}(ky) Y_{pq}(\hat{\boldsymbol{y}}) j_p(kx_n) Y_{pq}^*(\hat{\boldsymbol{x}}_n) \tag{10}$$

where $x_n = |\boldsymbol{x}_n|$; $j_n(\cdot)$ and $h_n^{(1)}(\cdot)$ are the so called *spherical Bessel and Hankel functions* of first kind which are defined as [7, page 125]

$$j_p(t) = \sqrt{\frac{\pi}{2t}}\, J_{p+\frac{1}{2}}(t),$$
$$h_n^{(1)}(t) = \sqrt{\frac{\pi}{2t}}\, (J_{p+\frac{1}{2}}(t) + iN_{p+\frac{1}{2}}(t))$$

where p and q are integers; $J_{p+\frac{1}{2}}(\cdot)$ and $N_{p+\frac{1}{2}}(\cdot)$ are the half integer order Bessel functions of the first and second kind respectively; $Y_{pq}(\hat{\boldsymbol{y}})$ are known as spherical harmonics and are given by [6, page 25]

$$Y_{pq}(\hat{\boldsymbol{y}}) = \sqrt{\frac{2p+1}{4\pi} \frac{(p-|q|)!}{(p+|q|)!}}\, P_p^{|q|}(\cos\theta)\, e^{iq\phi},$$

where (θ, ϕ) is the elevation and azimuth of the location given by $\hat{\boldsymbol{y}}$, and $P_p^{|q|}(\cdot)$ are the Associated Legendre functions. It is known that $\{Y_{pq}(\cdot) : p = 0, 1, 2, \ldots; q = -p, \ldots, p\}$, form a complete orthonormal system in the unit sphere, where

$$\int Y_{pq}(\hat{\boldsymbol{y}})\, Y_{p'q'}^*(\hat{\boldsymbol{y}})\, d\hat{\boldsymbol{y}} = \begin{cases} 1 & \text{if } p = p' \text{ and } q = q', \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

where integration is over the unit sphere as before.

We can now obtain an exact expression for the nearfield isotropic noise correlation matrix as follows: we substitute (10) and its conjugate in to (6), interchange integration and summations, and evaluate the resulting integral using (11) to obtain

$$r_{nm} = 4\pi k^2 y^2 \sum_{p=0}^{\infty} \sum_{q=-p}^{p} |h_p^{(1)}(ky)|^2 \, Y_{pq}^*(\hat{\boldsymbol{x}}_n) \, Y_{pq}(\hat{\boldsymbol{x}}_m) \times j_p(kx_n) \, j_p(kx_m). \tag{12}$$

Equation (12) is a novel result, which gives the noise correlation between a pair of sensors for a noise field generated by uniformly distributed point noise sources on the surface of a sphere radius of y which encircles the pair of sensors. Another form of (12) can be derived using the relationship [6, page 27]

$$\sum_{q=-p}^{p} Y_{pq}^*(\hat{\boldsymbol{x}}_n) Y_{pq}(\hat{\boldsymbol{x}}_m) = \frac{2p+1}{2\pi} P_p(\cos\gamma_{nm}), \tag{13}$$

where $\cos\gamma_{nm} = \hat{\boldsymbol{x}}_n \cdot \hat{\boldsymbol{x}}_m$ is the cosine of the angle between $\hat{\boldsymbol{x}}_n$ and $\hat{\boldsymbol{x}}_m$ and $P_n(\cdot)$ are the Legendre functions. Combining (12) and (13), we write the correlation between two sensors as

$$r_{nm} = 2k^2 y^2 \sum_{p=0}^{\infty} (2p+1) |h_p^{(1)}(ky)|^2 \, j_p(kx_n) \, j_p(kx_m) \times P_p(\cos\gamma_{nm}). \tag{14}$$

An attractive feature of (14) is that for each term in the series, the dependence on the distance to the noise source y, the angle between two sensors γ_{nm}, and the distance to two sensors x_n and x_m appear as separate factors.

3.1. Linear array

For the simple case of a line array through the origin, γ_{nm} would be equal to either 0 or π depending on the location of the origin, for all pairs of sensors. That is

$$\cos\gamma_{nm} = \mathrm{sgn}(\hat{\boldsymbol{x}}_n \cdot \hat{\boldsymbol{x}}_m)$$

where $\mathrm{sgn}(\cdot)$ is the Signum function. Since

$$\begin{aligned} P_p(-\cos\gamma) &= (-1)^p P_p(\cos\gamma), \\ P_p(1) &= 1 \end{aligned} \tag{15}$$

for all integers p [8, page 208], the correlation between two sensors for a line array is

$$r_{nm} = 2k^2 y^2 \sum_{p=0}^{\infty} (2p+1) \left\{\mathrm{sgn}(\hat{\boldsymbol{x}}_n \cdot \hat{\boldsymbol{x}}_m)\right\}^p |h_p^{(1)}(ky)|^2 \times j_p(kx_n) \, j_p(kx_m). \tag{16}$$

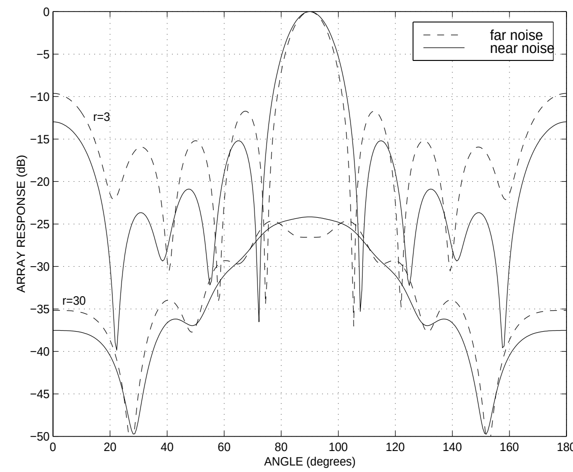

Figure 1: Response of the optimum array based on nearfield noise model (solid line) to sources at 3 and 30 wavelengths from the array origin. Also shown is the response of the farfield noise model based array response (dashed line).

3.2. Farfield isotropic noise

The simplest special case of (14) is for farfield isotropic noise, in which case $y \to \infty$. Making use of

$$\lim_{y\to\infty} y^2 |h_p^{(1)}(ky)|^2 = \frac{1}{k^2},$$

[6, page 30] we find (14) reduces to

$$r_{nm} = 2 \sum_{p=0}^{\infty} (2p+1) j_p(kx_n) \, j_p(kx_m) \, P_n(\cos\gamma_{nm}). \tag{17}$$

Using [9, page 366], (17) is reduced to

$$\begin{aligned} r_{nm} &= \frac{2\sin(k\sqrt{x_n^2 + x_m^2 - 2x_n x_m \cos\gamma_{nm}})}{k\sqrt{x_n^2 + x_m^2 - 2x_n x_m \cos\gamma_{nm}}} \\ &= \frac{2\sin(k|\boldsymbol{x}_n - \boldsymbol{x}_m|)}{k|\boldsymbol{x}_n - \boldsymbol{x}_m|}, \end{aligned} \tag{18}$$

which is a well-known result for farfield spherically isotropic noise fields [3, page 49]. For the simple case of a linear array with half wavelength spacings, the observed noise are uncorrelated between sensors; this fact is readily evident from (18).

4. SIMULATION EXAMPLE

We now present a design example to demonstrate the use of nearfield isotropic noise modelling for nearfield beamforming. Our demonstration is based on the simple array gain optimization technique outlined in section 2, however

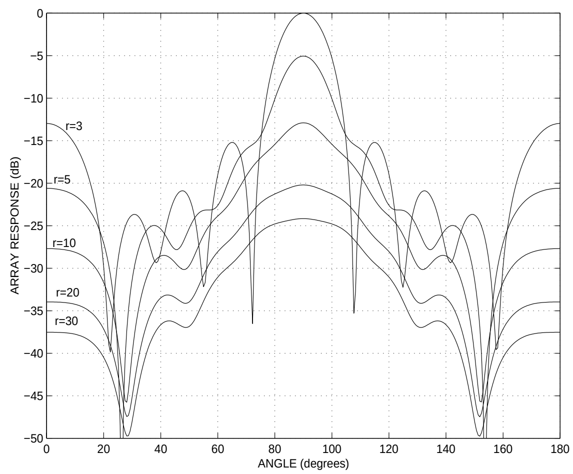

Figure 2: Response of the optimum array (nearfield noise model) at 3, 5, 10, 20 and 30 wavelengths from the array origin

this noise model can be applied to wide class of optimization methods such as Minimum Variance (MV), Maximum Likelihood (ML) and Mean Square Error (MSE) as applied to beamforming.

The design is for a double-sided linear array of 9 sensors with an inter-sensor spacing of $\lambda/2$, where λ is the wavelength. Suppose the desired source is in the nearfield at 3λ from the array origin, on the broadside of the array. We calculate the optimum weight vector (9), with the noise correlation matrix $\boldsymbol{R}_{noise}$ for nearfield isotropic noise (16) at a sphere of radius 2λ. Even though the series (16) has infinite number of terms, we approximate it by first 21 terms for this example. Generally these series expansions are convergent and could be approximated by finite number of terms depending on the array configuration and the desired operating distance.

The responses of the resulting array (solid line) to a nearfield source at 3 wavelengths from the array origin and to a farfield source at 30 wavelengths are given in Figure 1. Also shown is the response of a optimum array designed using farfield isotropic noise model (18) (dashed). Observe that the nearfield noise model based design provides a better directional array gain in the nearfield and simultaneously provides similar farfield noise rejection when compared with the farfield noise model based design. For both design methods, the power received from a source at 30λ at the look direction is about 25dB less than that of the desired source at 3λ. The trade-off for using the nearfield noise model is the better directional gain at the expense of slightly wider main lobe width. Figure 2 shows the response of the optimum array (designed to operate at 3λ using nearfield noise model) at different radial distances from the array origin. From this figure, we can note that the nearfield noise other than in look direction and farfield noise in all directions are attenuated with respect to the signal from the desired source. Thus we can conclude that our design has acceptable performance in a mixed farfield-nearfield noise environment.

5. CONCLUSION

In this paper, we have introduced an exact series representation for nearfield/farfield isotropic noise field, which may be useful in sensor array applications in the nearfield. For each term in these series representation, the dependence on the position coordinates of the sensors is factored in to components, each of which depends on a single coordinate. This property of the series expansion facilitates the calculation of the correlation matrix for various sensor orientations. While the model has only been demonstrated here for a small line array, it generally applicable to more complex arrays (2D and 3D). More importantly, this result can be utilized to apply well established farfield array processing algorithms for the nearfield applications. A study of a more general noise model using an arbitrary directional distribution of spatial harmonics is currently underway.

6. REFERENCES

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