Noise Modelling for Nearfield Array Optimization

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Abstract— **In this letter, an exact series representation for a nearfield spherically isotropic noise model is introduced. The proposed noise model can be utilized effectively to apply well established farfield array processing algorithms for nearfield applications of sensor arrays. A simple array gain optimization is used to demonstrate the use of the new noise model.**

Keywords— **Nearfield Beamforming, Reverberation, Noise Modeling, Optimization.**

I. Introduction

NEARFIELD sensor array design is important in teleconferencing and speech acquisition applications. Most array processing literature deals with situations where the desired source and the noise sources are assumed to be in the farfield of the array; this considerably simplifies the design problem. In most stochastic optimization techniques, the noise correlation matrix plays an integral part of the design. In fixed beamformer design, the noise field is assumed to be known, and usually modeled by either white gaussian noise or *farfield spherically isotropic noise* which results from a uniform distribution of noise sources over all directions in the farfield.

As an alternative, in this paper we model the noise field with uniformly distributed sources over all directions in the nearfield at a fixed distance from the array origin and call it *nearfield spherically isotropic noise*. This noise model can be utilized effectively to apply any signal processing criterion based on isotropic type noise correlation to nearfield applications. In our simulation example in section IV we will show that a design based on this nearfield noise model performs better than one based on a farfield noise model in a more realistic mixed farfield-nearfield noise field.

II. Gain optimization for an arbitrary array

As motivation for the theoretical development, we consider a simple array optimization technique as applied to a nearfield array.

The array gain is a key array performance indicator, defined by

$$G = 4\pi \, \frac{\text{power received from a desired location } (P_{source})}{\text{total noise power received } (P_{noise})}. \tag{1}$$

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Consider an array of $2N+1$ sensors, arbitrarily placed in a bounded region $\Omega \subset \mathcal{R}^3$. Then the response of this array to a source located outside the region Ω at $\boldsymbol{y}$, is

$$b(\boldsymbol{y}) = \sum_{n=-N}^{N} w_n \frac{e^{ik|\boldsymbol{y}-\boldsymbol{x}_n|}}{|\boldsymbol{y}-\boldsymbol{x}_n|} \, y e^{-iky}, \tag{2}$$

where $i=\sqrt{-1}$, w_n is the complex gain associated with the sensor positioned at $\boldsymbol{x}_n \in \Omega$, $y = |\boldsymbol{y}|$ and $k \triangleq 2\pi f/c = 2\pi/\lambda$ is the wavenumber which can be expressed in terms of the propagation speed c and the frequency f, or the wavelength λ. Thus, the power received from the desired location $\boldsymbol{y}_s$ is given by $P_{source} = b^*(\boldsymbol{y}_s)\, b(\boldsymbol{y}_s)$ where * denotes complex conjugate transpose. Arranging the weights in an $(2N+1)$-element column vector $\boldsymbol{W} = [w_{-N} \cdots w_N]^T$ and defining a square $((2N+1)\times(2N+1))$ Hermitian matrix $\boldsymbol{R}_{source} = \boldsymbol{a}\boldsymbol{a}^*$ in terms of the $(2N+1)$ column vector

$$\boldsymbol{a} = \begin{bmatrix} \dfrac{e^{ik|\boldsymbol{y}-\boldsymbol{x}_{-N}|}}{|\boldsymbol{y}-\boldsymbol{x}_{-N}|} \\ \vdots \\ \dfrac{e^{ik|\boldsymbol{y}-\boldsymbol{x}_{N}|}}{|\boldsymbol{y}-\boldsymbol{x}_{N}|} \end{bmatrix} y e^{-iky}, \tag{3}$$

leads to the matrix formulation $P_{source} = \boldsymbol{W}^* \boldsymbol{R}_{source} \boldsymbol{W}$.

By assuming the *nearfield spherically isotropic noise field*, i.e., having uniformly distributed noise sources on a sphere of radius y, we can write the total noise power received as, $P_{noise} = \int b^*(\boldsymbol{y})\, b(\boldsymbol{y})\, d\hat{\boldsymbol{y}}$, where $\hat{\boldsymbol{y}} = \boldsymbol{y}/y$ is a unit vector in the direction of $\boldsymbol{y}$ and the integration is over all directions. We define the $((2N+1)\times(2N+1))$ matrix $\boldsymbol{R}_{noise} = [r_{nm}]$ with elements

$$r_{nm} = \frac{y^2}{4\pi} \int \frac{e^{ik|\boldsymbol{y}-\boldsymbol{x}_n|}}{|\boldsymbol{y}-\boldsymbol{x}_n|} \, \frac{e^{-ik|\boldsymbol{y}-\boldsymbol{x}_m|}}{|\boldsymbol{y}-\boldsymbol{x}_m|} d\hat{\boldsymbol{y}}. \tag{4}$$

(Note that $\boldsymbol{R}_{noise}$ is Hermitian and positive definite). Then, $P_{noise} = 4\pi \boldsymbol{W}^* \boldsymbol{R}_{noise} \boldsymbol{W}$, and equation (1) becomes a ratio of quadratic forms

$$G = \frac{\boldsymbol{W}^* \boldsymbol{R}_{source} \boldsymbol{W}}{\boldsymbol{W}^* \boldsymbol{R}_{noise} \boldsymbol{W}}. \tag{5}$$

The usual goal is to find $\boldsymbol{W}$ which maximizes G. The form of (5) is familiar and $\boldsymbol{R}_{source}$ and $\boldsymbol{R}_{noise}$ are commonly known as the source correlation matrix and noise correlation matrix respectively. The optimum array weights are [1]

$$\boldsymbol{W}_{opt} = \boldsymbol{R}_{noise}^{-1} \boldsymbol{a}. \tag{6}$$

III. Nearfield isotropic noise

In this section, we find an exact series representation for the noise correlation r_{nm} (4) between two sensors due to nearfield isotropic noise field.

We write the wavefield at the sensor location $\boldsymbol{x}_n$ due to a source at $\boldsymbol{y}$ for $y > x_n$ using the spherical harmonic expansion [2, page 30] as

$$\frac{e^{ik|\boldsymbol{y}-\boldsymbol{x}_n|}}{|\boldsymbol{y}-\boldsymbol{x}_n|} = 4\pi ik \sum_{p=0}^{\infty} \sum_{q=-p}^{p} h_p^{(1)}(ky) Y_{pq}(\hat{\boldsymbol{y}}) j_p(kx_n) Y_{pq}^*(\hat{\boldsymbol{x}}_n) \tag{7}$$

where p and q are integers; $j_p(t) \triangleq \sqrt{\pi/2t}\, J_{p+1/2}(t)$ and $h_p^{(1)}(t) \triangleq \sqrt{\pi/2t}\,(J_{p+1/2}(t) + iN_{p+1/2}(t))$ are the *spherical Bessel and Hankel functions* of first kind [3, page 125], $J_{p+1/2}(\cdot)$ and $N_{p+1/2}(\cdot)$ are the half integer order Bessel functions of the first and second kind respectively; $x_n = |\boldsymbol{x}_n|$; $Y_{pq}(\hat{\boldsymbol{y}})$ are spherical harmonics [2, page 25],

$$Y_{pq}(\hat{\boldsymbol{y}}) = \sqrt{\frac{2p+1}{4\pi}\frac{(p-|q|)!}{(p+|q|)!}}\, P_p^{|q|}(\cos\theta)\, e^{iq\phi},$$

where (θ, ϕ) is the elevation and azimuth of the location given by $\hat{\boldsymbol{y}}$, and $P_p^{|q|}(\cdot)$ are the Associated Legendre functions. It is known that $\{Y_{pq}(\cdot) : p = 0, 1, 2, \ldots ; q = -p, \ldots , p\}$, form a complete orthonormal system in the unit sphere, where

$$\int Y_{pq}(\hat{\boldsymbol{y}})\, Y_{p'q'}^*(\hat{\boldsymbol{y}})\, d\hat{\boldsymbol{y}} = \begin{cases} 1 & \text{if } p = p' \text{ and } q = q', \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

where integration is over all directions.

We now determine an exact expression for the nearfield isotropic noise correlation matrix. We substitute (7) and its conjugate in to (4), interchange integration and summations, and evaluate the resulting integral using (8) to obtain

$$r_{nm} = 4\pi k^2 y^2 \sum_{p=0}^{\infty} \sum_{q=-p}^{p} |h_p^{(1)}(ky)|^2\, Y_{pq}^*(\hat{\boldsymbol{x}}_n)\, Y_{pq}(\hat{\boldsymbol{x}}_m) \times j_p(kx_n)\, j_p(kx_m). \tag{9}$$

Equation (9) is a novel result, which gives the noise correlation between a pair of sensors for a noise field generated by uniformly distributed noise sources on the surface of a sphere radius of y which encloses the pair of sensors. Another form of (9) can be derived using the relationship [2, page 27]

$$\sum_{q=-p}^{p} Y_{pq}^*(\hat{\boldsymbol{x}}_n) Y_{pq}(\hat{\boldsymbol{x}}_m) = \frac{2p+1}{2\pi} P_p(\cos\gamma_{nm}), \tag{10}$$

where $\cos\gamma_{nm} = \hat{\boldsymbol{x}}_n \cdot \hat{\boldsymbol{x}}_m$ and $P_n(\cdot)$ are the Legendre functions. Combining (9) and (10), we write the correlation between two sensors as

$$r_{nm} = 2k^2 y^2 \sum_{p=0}^{\infty} (2p+1) |h_p^{(1)}(ky)|^2\, j_p(kx_n)\, j_p(kx_m) \times P_p(\cos\gamma_{nm}). \tag{11}$$

An attractive feature of (11) is that for each term in the series, the dependence on the distance to the noise source y, the angle between two sensors γ_{nm}, and the distance to two sensors x_n and x_m appear as separate factors.

For the simple case of a line array through the origin, $\cos\gamma_{nm} = \pm 1$ for all pairs of sensors, depending on whether the origin is on one side of a sensor pair or in between them. Since $P_p(\pm 1) = (\pm 1)^p$ for all integers p [4, page 208], the correlation between two sensors for a line array is

$$r_{nm} = 2k^2 y^2 \sum_{p=0}^{\infty} (2p+1)(\pm)^p\, |h_p^{(1)}(ky)|^2\, j_p(kx_n) j_p(kx_m). \tag{12}$$

The simplest special case of (11) is for farfield isotropic noise, in which case $y \to \infty$. Exploiting the relation $\lim_{y\to\infty} y^2 |h_p^{(1)}(ky)|^2 = 1/k^2$ [2, page 30], we find (11) reduces to

$$r_{nm} = 2 \sum_{p=0}^{\infty} (2p+1) j_p(kx_n)\, j_p(kx_m)\, P_n(\cos\gamma_{nm}). \tag{13}$$

Using [5, page 366], (13) simplifies to

$$\begin{aligned} r_{nm} &= \frac{2\sin(k\sqrt{x_n^2 + x_m^2 - 2x_n x_m \cos\gamma_{nm}})}{k\sqrt{x_n^2 + x_m^2 - 2x_n x_m \cos\gamma_{nm}}} \\ &= \frac{2\sin(k|\boldsymbol{x}_n - \boldsymbol{x}_m|)}{k|\boldsymbol{x}_n - \boldsymbol{x}_m|}, \end{aligned} \tag{14}$$

which is a well-known result for farfield spherically isotropic noise fields [6, page 49]. For the simple case of a linear array with half wavelength spacings, the observed noise is uncorrelated between sensors; this fact is readily evident from (14).

IV. Simulation example

We now present a design example to demonstrate the use of nearfield isotropic noise modelling for nearfield beamforming. Our demonstration is based on the simple array gain optimization technique outlined in section II, however this noise model can be applied to wide class of optimization methods.

The design is for a double-sided linear array of 9 sensors with an inter-sensor spacing of $\lambda/2$, where λ is the wavelength. Suppose the desired source is in the nearfield at 3λ from the array origin, on the broadside of the array. We calculate the optimum weight vector (6), with the noise correlation matrix $\boldsymbol{R}_{noise}$ for nearfield isotropic noise (12) at a sphere of radius 2λ. We approximate the infinite series in (12) by the first 21 terms. Generally these series expansions are convergent and are readily approximated by finite number of terms depending on the array configuration and the desired operating distance.

Figure 1 shows the responses of the resulting array (solid line) to a nearfield source at 3λ from the array origin and to a farfield source at 30λ. Also shown is the response of an optimum array designed using farfield isotropic noise

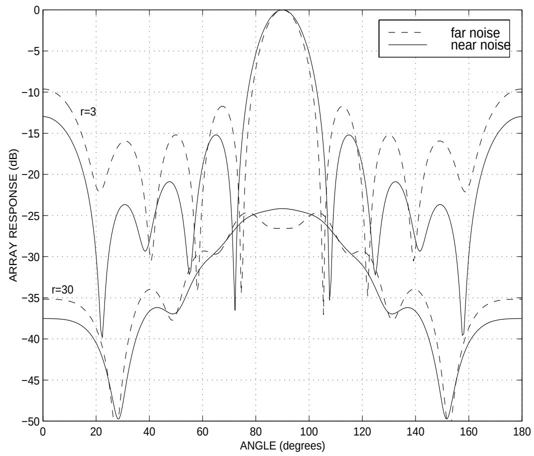

Fig. 1. Response of the optimum array based on nearfield noise model (solid line) to sources at 3 and 30 wavelengths from the array origin. Also shown is the response of the farfield noise model based array response (dashed line).

model (14) (dashed). The nearfield noise model based design provides a better directional array gain in the nearfield and simultaneously provides similar farfield noise rejection when compared with the farfield noise model based design. For both design methods, the power received from a source at 30λ at the look direction is about 25dB less than that of the desired source at 3λ. The trade-off for using the nearfield noise model is the better directional gain at the expense of slightly wider main lobe width.

V. Conclusion

In this letter, we have introduced an exact series representation for nearfield/farfield isotropic noise field, which may be useful in sensor array applications in the nearfield. While the model has only been demonstrated here for a small line array, it is generally applicable to more complex sensor geometries. More importantly, this result allows the application of well established farfield array processing algorithms for the nearfield applications.

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