Optimising Bounds in Simple Temporal Networks with Uncertainty under Dynamic Controllability Constraints

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Overview

• Background
  • Simple Temporal Network with Uncertainty (STNU)
  • Dynamic Controllability (DC)

• Motivation
  • Optimising STNU under DC

• Problem Formulation

• Applications and Result
  • Maximum Delay under DC
  • Chance-constraint DC

• Conclusion
Scenario Example – Evacuation Plan
Scenario Example – Evacuation Plan
STN for the Example

Region A starts to evacuate

Flow A enter the main route

Region B starts to evacuate

Flow B reach the main route

Flow A pass the entrance of the main route

[60, 70]

[30, 35]

[20, 25]

[0, 10]

Simple Temporal Network (STN)

• General temporal reasoning for temporal plan and schedules
  • Nodes:
  • time points
  • Links:
  • time constraints
STNU of the Scenario Example

Region A starts to evacuate

[60, 70]

Flow A enter the main route

Flow A pass the entrance of the main route

[30, 35]

Flow B reach the main route

Region B starts to evacuate

[20, 25]

[0, 10]

STN with Uncertainty (STNU)

• Vidal and Fargier, 1999
• Extension from STN
• Contingent (CTG)
  • Uncontrollable
  • Uncertainty
  • Unknown
Dynamic Controllability of STNU

Region A starts to evacuate

Flow A enter the main route

Observe: [60, 70]

Flow A pass the entrance of the main route

Decide: +15

Flow B reach the main route

Region B starts to evacuate

[0, 10] [20, 25] [30, 35]

• Dynamic Controllability (DC) (Vidal and Fargier, 1999)
  • Observe the Past
  uncontrollable nodes
  • Decide the Future
  controllable nodes
  • A dynamic strategy satisfying every constraint
**Dynamic Strategy for the Example**

Flow A enter the main route

Flow A pass the entrance of the main route

Region B start to evacuate

Flow B reach the main route

Timeline

[0, 10] [30, 35] [60, 70] [20, 25]
Dynamic Strategy for the Example

Flow A enter the main route

Flow A pass the entrance of the main route

Region B start to evacuate

Flow B reach the main route

Timeline

Observe

[0, 10]

[30, 35]

[60, 70]

[20, 25]
Dynamic Strategy for the Example

- **Flow A enter the main route**
  - [60, 70]

- **Flow A pass the entrance of the main route**
  - [30, 35]

- **Region B start to evacuate**
  - [20, 25]

- **Flow B reach the main route**
  - [0, 10]

**Timeline**

- Observe
- Decide +15
- T
- T + 15
Dynamic Strategy for the Example

Flow A enter the main route

[60, 70]

Flow A pass the entrance of the main route

[30, 35]

Region B start to evacuate

[20, 25]

Flow B reach the main route

[0, 10]

Observe

Decide +15

Timeline

T

T + 15

T + [30, 35]
Dynamic Strategy for the Example

Flow A enter the main route

Flow A pass the entrance of the main route

Region B start to evacuate

Flow B reach the main route

[0, 10] [30, 35] [60, 70]

Timeline

Observe

Decide +15

T

T + 15

T + [30, 35]

T + 15 + [20, 25]
Dynamic Strategy for the Example

Flow A enter the main route
- [60, 70]

Flow A pass the entrance of the main route
- [30, 35]

Region B start to evacuate
- [20, 25]

Flow B reach the main route
- [0, 10]

Timeline
- T
- T + 15
- T + 15 + [20, 25]

Observe
- T
- T + 15
- T + [30, 35]

Decide +15
- Diff=[0, 10]
DC Checking Algorithms

- Answer: Yes/No
- Time Complexity: Polynomial

- Morris, Muscettola and Vidal’s (2001)
  - Reduction Rules
  - Local and Global DC
  - Cubic Algorithm (2014)

- Other Algorithms
  - Fast IDC (Stedl and Williams, 2005)
  - Efficient IDC, cubic algorithm (Nilsson, Kvarnstrom and Doherty, 2014)
Motivation

By Checking DC, we get:
- Yes/NO
- Polynomial
- Dynamic Strategy
- Large Scale

We cannot get:
- How far?
  - DC->Not DC
  - Not DC -> DC
- Other questions

Region A starts to evacuate

Flow A enter the main route

Decide +15

Flow A pass the entrance of the main route

[60, 70]

Observe

[30, 35]

Flow B reach the main route

[20, 25]

Flow B starts to evacuate

[0, 10]
Motivation

Region A starts to evacuate

$[60, 60 + D_1]$

Flow A enter the main route

$[30, 30 + D_2]$

Flow A pass the entrance of the main route

$[0, 10]$

Region B starts to evacuate

$[20, 20 + D_3]$

Flow B reach the main route

$D = \min(D_1, D_2, D_3)$

Q  How far?
DC -> Not DC

Maximum value of D under DC

robustness measure
Problem Formulation

Optimisation model of STNU under Dynamic Controllability

\[ \text{opt } f_{\text{obj}} (l_{ij}, u_{ij} \mid e_{ij} \in E) \]

s.t. \[ L_{ij} \leq l_{ij} \leq u_{ij} \leq U_{ij} \]

\[ N(l_{ij}, u_{ij} \mid e_{ij} \in E) \text{ is dynamically controllable} \]

application - specific side constraints

- Bounds of links are \textbf{variables} (lower case)
- \textit{How to formulate DC constraints?}
Problem Formulation

How to formulate DC constraints?

\[ N(l_{ij},u_{ij} \mid e_{ij} \in E) \text{ is dynamically controllable} \]

- Non-linear Programming (NLP) model
  - Wah and Xin, 2004
- Disjunctive Linear Model
  - Mixed Integer Programming (MIP): **Binary Variables**
  - Follow the **reduction rules** (Morris, Muscettola and Vidal, 2001)
Generate Constraints by Reduction Rules

A

\[ [L_{AB}, U_{AB}] \]

\[ [U_{AC} - U_{BC}, L_{AC} - L_{BC}] \]

\[ [L_{AC}, U_{AC}] \]

\[ [L_{BC}, U_{BC}] \]

C

Reduction rule
when \( L_{BC} \geq 0 \)

if \( L_{BC} \geq 0 \), then \( L_{AB} \geq U_{AC} - U_{BC} \)
and \( U_{AB} \leq L_{AC} - L_{BC} \).

Constraints for precede case

\[ l_{BC} < 0 \lor \begin{cases} 
    l_{AB} \geq u_{AC} - u_{BC} \\
    u_{AB} \leq l_{AC} - l_{BC}
\end{cases} \]
Applications I
Robustness with Non-Probabilistic Uncertainty

Maximum Delay (MD) under Dynamic Controllability

$$\max D = \min (D_1, D_2, D_3)$$

$$N(l_{ij}, u_{ij} \mid e_{ij} \in E)$$ is DC

Q What is the Maximum value of D for which the STNU is still Dynamically Controllable?
Applications I
Robustness with Non-Probabilistic Uncertainty

Formulation

• Maximum delay of STNU

\[
\begin{align*}
\max \quad & \min_{e_{ij} \in ctg} (d_{ij}) \\
\text{s.t.} \quad & l_{ij} = L_{ij}, \quad u_{ij} = l_{ij} + d_{ij}, \quad d_{ij} \geq 0 \quad e_{ij} \in ctg \\
& L_{ij} \leq l_{ij} \leq u_{ij} \leq U_{ij}, \quad e_{ij} \in rqm \\
& N(l_{ij}, u_{ij} | e_{ij} \in E) \text{ is DC}
\end{align*}
\]
## Data

<table>
<thead>
<tr>
<th>Description</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>3400 Partial Order Schedules (POS) for RCPSP/max</td>
<td>Generated by POSL (Banerjee and Haslum, 2011)</td>
</tr>
<tr>
<td>Number of CTG per POS</td>
<td>10-18</td>
</tr>
<tr>
<td>Number of RQM per POS</td>
<td>50-300</td>
</tr>
</tbody>
</table>

## Solver

<table>
<thead>
<tr>
<th>Description</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disjunctive Linear Model (CDRU)</td>
<td></td>
</tr>
<tr>
<td>Mixed Integer Programming Model (Gurobi)</td>
<td></td>
</tr>
<tr>
<td>Non-Linear Programming Model (SNOPT)</td>
<td></td>
</tr>
<tr>
<td>Conflict-directed Search (Yu, Fang, and Williams, 2014)</td>
<td></td>
</tr>
</tbody>
</table>
Result I Runtime

Convergence of Non-linear programming:
- Single run 70%
- repeatedly run 93%
Result II

Robustness (MD) vs. Fluidity and Flexibility

- Fluidity (average time slack)
- Flexibility (average temporal tolerance)
  - Wilson et al. (2014)
- How can fluidity/flexibility predict the robustness (MD)?

<table>
<thead>
<tr>
<th></th>
<th>Robustness (Maximum Delay under DC)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Increase</td>
</tr>
<tr>
<td>Fluidity Increase</td>
<td></td>
</tr>
<tr>
<td>38.90%</td>
<td>44.39%</td>
</tr>
<tr>
<td>Flexibility Increase</td>
<td>40.30%</td>
</tr>
</tbody>
</table>
Other Applications of DC Constraint Model

Application List

- Relaxing Over-Constrained Problems
- Minimizing Flexibility
- Probabilistic Robustness Measure on STNU
- Dynamic Controllability with Chance-constraints
Conclusion

DC Constraint Model

• A more useful representation than DC checking process
• Measure robustness
• Other optimisation problem (DC vs. SC)

Comparison of solvers

• CDRU
• MIP
• NLP

Faster
More General Problems