Observing what can’t be seen

J. Trumpf

June 2008
Outline

1. The idea behind observers
2. The problem I am trying to solve
3. An application I am interested in
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Time signals

time signals: \{\text{times}\} \rightarrow \{\text{values}\}

- Fuel consumption
- Fuel to electricity conversion rate
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Time signals

time signals: \{\text{times}\} \rightarrow \{\text{values}\} \quad s: \mathbb{R} \rightarrow M (M = \mathbb{R})

Fuel consumption
Fuel to electricity conversion rate

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Signals interact

“static” interaction:

\[ F(t) = m \cdot a(t) \]
Signals interact

“static” interaction:

\[ F(t) = m \cdot a(t) \]

dynamic interaction:

\[ C \frac{dV}{dt}(t) = I(t) \]
Signals interact

“static” interaction:

\[ F(t) = m \cdot a(t) \]

dynamic interaction:

\[ C \frac{dV}{dt}(t) = I(t) \]

multiple variables:

\[ \dot{x}(t) = f(x(t), u(t)), \quad y(t) = h(x(t), u(t)) \]
The problem with inversion

“static” interaction:

\[ F(t) = m \cdot a(t) \]
The problem with inversion

"static" interaction:

\[ F(t) = m \cdot a(t) \]

dynamic interaction:

\[ C \frac{dV}{dt}(t) = I(t) \]
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The problem with inversion

“static” interaction:

\[ F(t) = m \cdot a(t) \]

*dynamic interaction:*

\[ C \frac{dV}{dt}(t) = I(t) \]

Time matters! There are differences between past, present and future!

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The million dollar idea

Plant

\[ \dot{x} = f(x, u), \quad y = h(x) \]

Observer

\[ \hat{x} = f(\hat{x}, u) + g(\hat{y}, y), \quad \hat{y} = h(\hat{x}) \]

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The million dollar idea

Plant

\[
\dot{x} = f(x, u), \quad y = h(x)
\]

Observer

\[
\hat{x} = f(\hat{x}, u) + \frac{g(\hat{y}, y)}{\dot{y} - h(\hat{x})}, \quad \hat{y} = h(\hat{x})
\]

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The million dollar idea

\[ \dot{x} = f(x, u), \quad y = h(x) \]

\[ \dot{\hat{x}} = f(\hat{x}, u), \quad \hat{y} = h(\hat{x}) \]

Plant

Observer

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The million dollar idea

\[ \dot{x} = f(x, u), \quad y = h(x) \]

Plant

\[ \dot{\hat{x}} = f(\hat{x}, u) + g(\hat{y}, y), \quad \hat{y} = h(\hat{x}) \]

Observer

\[ e \]

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How unique is this construction?
In the context of linear stochastic difference or differential equations (with some assumptions on noise characteristics), the *Kalman filter* is the optimal solution in the sense of minimal error variance. *g* is computed by solving a Riccati equation.
Luenberger avoids an answer

Although often quoted, Luenberger does not answer this question at all in his work on observers for linear deterministic systems.

A solution was provided by the author and Jan C. Willems in 2007.
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Pose

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Sensors

Cheap MEMS Sensors

- Inertial Measurement Unit (IMU)
  - angular velocity, linear acceleration
  - high frequency, time varying bias

- GPS or video camera
  - full pose
  - low frequency, high frequency noise

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Collaborators

Uwe Helmke, Paul Fuhrmann, Jan C. Willems

Rob Mahony, Christian Lageman, Grant Baldwin