ENGN2225 Systems Design

Simplifying Piano Tuning

Author:

Ross Pure (u5349749)

Abstract

This report documents the design process used to find a piano tuning system that allows the piano owner to be able to tune their instrument as easily as one tunes a guitar. The piano tuning scheme proposed is a Railsback stretch with an equally tempered temperament octave. The tools to be used are a universal electronic tuning device (ETD) for tuning the temperament octave and a device for turning the tuning pins that uses gear ratios for allow for fine, consistent pin adjustments. When compared with aural based and piano ETD based solutions performed by the piano user, the proposed solution was found to be superior. Compared to hiring a professional tuner, the proposed solution was found to be superior if cost was more important than the skill and time requirements of the user.

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1   Background

In general, it is recommended that a piano is to be tuned by a professional a minimum of four times a year in the first year of purchase, and a minimum of twice a year every year after that (PTG 1993). While hiring a professional certainly has its advantages, the disadvantages are the price (in Canberra you will probably pay $165 for the first tuning (Leslie 2015)) and the fact that it may be hard to find a time that is suitable for the tuner and piano owner.

The primary stakeholder (or client[1]) is a Canberra resident who has purchased a piano which has recently been moved to their home. Having been around guitar players that are able to easily tune their own instruments (often by ear), the client wishes to be able to do the same for their recently purchased piano.

1.1   The Musical Scale

Each key on the piano is identified with a 'note'. There are seven different letters that a note can be; A, B, C, D, E, F and G. These are the white keys as shown in Fig. 1. It can also be seen that there are black keys between some white keys. These are identified by their adjacent white keys; if a black key is above key X and below key Y then we may call it either X♯ (say: "x sharp") or Y♭ (say: "y flat"). For example, the leftmost black key could be called either A♯ or B♭. As can be seen the notes also correspond to a particular 'octave'. Within an octave there is exactly one of each note (A to G♯), and thus octaves are used to distinguish between notes. For example, the note A in the fourth octave would be identified as A4. it follows that the range of the piano is from A0 to C8, 88 keys in total.

Figure 1: The musical scale as it relates to the keys on a piano.

Each note has a fundamental frequency that the corresponding string(s) (there is not only one string for each key, some keys have 2 or 3 strings all tuned to the same note) vibrate at. For example, on a correctly tuned piano, the fundamental frequency at which the three strings corresponding to A4 will vibrate at is 440Hz. The 'distance' between two adjacent notes is called a 'semitone', where we note that two white keys are not adjacent if there is a black key between them; F is a semitone above E but G is not a semitone above F. From this comes the definition of a 'cent', which is defined by setting 100 cents = 1 semitone.

In an 'equally tempered' scale, every note is the same 'distance' apart and the a note one octave above another has double the frequency. For example, since A4 has a frequency of 440Hz, A5 has a frequency of 880Hz and A3 has a frequency of 220Hz in an equally tempered scale. Thus since all notes are equidistant, the ratio between two notes 1 semitone apart is $2^{\frac{1}{12}}$ (there are 12 notes in an octave including the black and white keys). If pianos were tuned to an equally tempered scale, tuning would be relatively easy, because one could simply tune each string according to a pre calculated frequency based upon A4 = 440Hz. However, pianos tuned with an equally tempered scale sound "inferior" and unnatural when compared to other standard tunings (Martin 1961). Pianos are not tuned to an equally tempered scale due to a property known as inharmonicity.

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[1]The client requirements will be based upon requirements of the author. However, the author doesn't own an acoustic piano (only an electric piano) and so the client is partially hypothetical.

1.2 Inhormonicity

When a string in a piano is struck, in addition to the fundamental frequency f (e.g. for A4 $f = 440$Hz), fainter higher frequencies are also produced; precisely the frequencies $2f$ (second harmonic), $3f$ (third harmonic), ..., nf (n^{th} harmonic) and so on (Wilson, Buffa, and Lou 2007). These frequencies are also called 'partials' where the nth partial is the $(n+1)^{th}$ harmonic. Notice that this means that in an equally tempered scale the first partial should be the same frequency as the fundamental frequency of the note one octave above. However, the assumption made for these partials is that the string is perfectly flexible, which the piano string is not. The piano strings are stiff to the extent that the partials are 'stretched' so that they are higher than predicted by the previous formula (Young 1952). Due to this, and the fact that for a tuning to be similar to one achieved by aural methods the more audible partials of a note should be in agreement with the other notes (Martin 1961), the notes above the reference A4 are slightly higher, and the notes lower are slightly lower than they would be in an equally tempered scale. Also, in the deviations from equal temperament measured by Martin there is a deviation as great as 50 cents (half a semitone) which is easily perceptible. A reproduction of Figure 1 in Martin's paper can be found in Fig. 2.
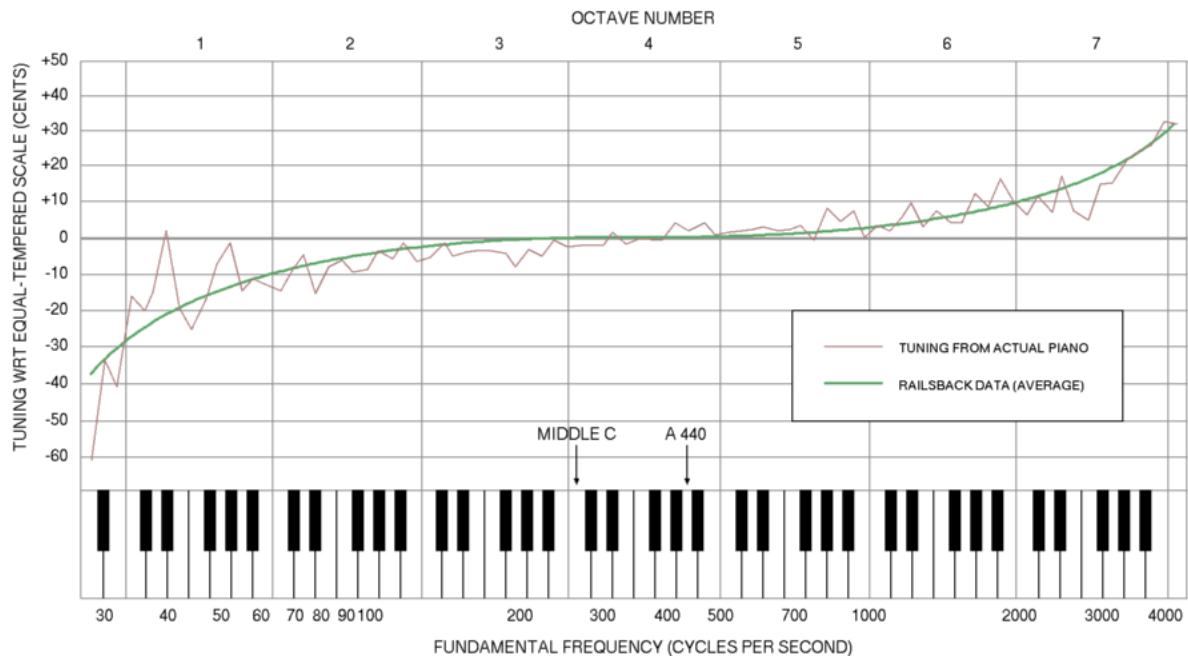
Figure 2: Graphs of the deviation from equal temperament of the Railsback Stretch and a fine tuning by a professional piano tuner. Image source is Wikipedia (Wikipedia 2015) but is a reproduction of Figure 1 in Martin's paper (Martin 1961).

Martin also identified that the 'Railsback Stretch' temperament was very close to being as good as a fine tuning performed by a professional piano tuner. This temperament accounts for inharmonicity in a simple way: after tuning one octave (the 'temperament octave'), tune the other notes so that their fundamental frequency is the same as the frequency of the first partial of the note one octave below it.

Even though the previous formulas for the the frequency of the partials don't apply to piano strings, the corrected formulas have been derived and found to predict the partials successfully (Lattard 1993). Lattard gives the formulas as shown in Eq. 1 and Eq. 2 where l is the speaking length (the length of the part of the string that vibrates), σ is the density, d is the diameter, E is Young's modulus and f_1 is the

fundamental frequency

$$B = \frac{\pi^2}{64} \frac{E}{\sigma} \frac{d^2}{(f_1 l^2)^2} \tag{1}$$

$$f_n = n f_1 \left(1 + \frac{B}{2}(n^2 - 1) \right) \tag{2}$$

Here B is called the coefficient of inharmonicity. Since the partials depend upon the length of the string among other factors, it follows that since string lengths vary from piano to piano every piano will need to be tuned differently.

2 Project Scope and Requirements

2.1 Problem Scope

The first step taken in the design process was defining the system boundary. This identifies those variables that affect the process of tuning a piano which we control (exogenous), those which we don't control (endogenous), and any of the latter which were considered to be considered to be out of the scope of the project (excluded).

Table 1: System Boundary Chart corresponding to variables that affect the process of tuning a piano.

Endogenous	Exogenous	Excluded
●String tension	●Inharmonicity	●Desired temperament ●Existing damage ●Age of piano ●Degree to which piano is out of tune

Table 1 shows the general system boundary for the process of tuning a piano. The variable 'Desired temperament' encompasses any tuning other than the Railback Stretch. This allows there to be a precise state in which the piano is 'correctly' tuned, which will aid the design process. Additionally, alternate tunings are ruled out because piano string lengths, thicknesses etc. are usually designed to be optimal for the standard temperaments (Conklin 1996b). The other excluded variables attempt to encapsulate properties of the piano that would call for different tuning processes. For example, if there are damaged parts or the structural integrity of some parts is in question changing the tension in the strings could be catastrophic. Some strings are under more than 1.5kN of tension, meaning that if they snap they can cause significant damage to the piano or anyone in the vicinity (Conklin 1996a). With this narrowing of scope more information can be gathered from a system boundary chart which is much more specific. With this in mind the system boundary chart corresponding to the tuning of a single string was constructed in which the variable 'inharmonicity' is unpacked (Table 2).

Table 2: System Boundary Chart corresponding to variables that affect the desired tuning of a string.

Endogenous	Exogenous	Excluded
●Tension	●Speaking length ●Density ●Diameter ●Young's modulus ●Harmonics of tuned strings	

Recall that except for the first string all strings are tuned to the relationship between its harmonics and the harmonics of the previously tuned strings. Hence the endogenous and exogenous variables are precisely those variables which affect the harmonics of the string; the nth harmonic, f_n, is given by Eq. 1 and Eq. 2.

From Table 2 it is clear that the only variable that the system needs to control is the tension in each string, which is controlled by the corresponding tuning pin. This means that the system will need to be able to attach to tuning pins and apply enough force to turn them.

2.2 Requirements Analysis

The client wishes to be able to tune their own piano (i.e. not hiring a professional) and has specified the following requirements:

- The tuning method can't rely heavily upon aural techniques (tuning 'by ear').

- Any tuning tools must be relatively cheap and require little maintenance.

- The tuning process shouldn't be significantly longer than the time it takes for a professional to tune a piano.

- The tuning method should be simple enough for the client to perform relatively easily.

What is meant by the first point is that the process should require no greater an aural faculty than is required for tuning a guitar by ear. The general process for tuning a guitar by ear is to tune one string using a known reference (electronic tuning device (ETD), tuning fork etc.) and then to use previously tuned strings to produce frequencies (either fundamental or higher degree harmonics) that are the same as the desired fundamental for an untuned string. the way to know that the string is tuned is using 'beats': when two sinusoidal waves of frequencies f_1 and f_2 are added together (as occurs when two notes are played together), the sum can be represented as follows using a trigonometric identity:

$$\sin(2\pi f_1) + \sin(2\pi f_2) = \sin(\pi(f_1 + f_2))\cos(\pi(f_1 - f_2))$$

The frequency of the second term in the product is called the beat frequency. We can see that if f_1 is very close to f_2 then the beat frequency will be quite low. This means that when two frequencies that are quite close are played together, a perceptible 'beating' is heard. When tuning a guitar string that is out of tune, it generally be quite close to what it should be, and so a beat is heard when played with a reference frequency (from a previously tuned string or otherwise). The untuned string is then tuned so as to reduce the beat frequency; this will bring the two frequencies closer together. When at last no beat is heard, this means that the frequencies are equal and so the string is tuned. In short, the aural capacity required is to be able to perceive the beat frequency and tune accordingly.

The client requirements are translated into design requirements with associated metrics to quantify them. This quantification can be found in Table 3. The direction of improvement desired for each metric is represented by ↑ for increasing and ↓ for decreasing.

Table 3: Customer requirements to design requirements and TPMs

| Customer Requirements	ID	Design Requirements	Technical Performance Measures	Metric
Non aural	DR01-01	Beats requiring aural detection have low frequency	↓Beat frequency	Hz
	DR01-02	Harmonics requiring aural detection have high relative volume	↑Volume ratio	Ratio (no units)
Cost	DR02-01	Low initial upfront cost	↓Material cost	$AUD
	DR02-02	Low ongoing cost	↓Maintenance cost	$AUD/year
Time	DR03-01	Low time spent tuning	↓Tuning time	Hrs
Simple to use	DR04-01	Low time to learn to use tools	↓Time spent learning	mins
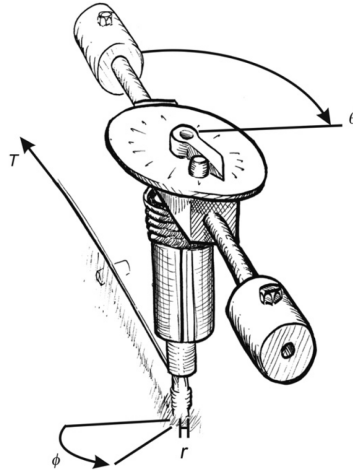
Figure 3: Repeatable impact tuning hammer designed by Millard and Tizhoosh

3   Logic and Subsystems

3.1   Logic and Function

The process of tuning a piano is described in the Functional Flow Block Diagram (FFBD) in Fig. 5. From the FFBD, functions that the system must be able to perform are identified:

- Determining whether a note being played has a frequency of 440Hz, and if it isn't, providing information that allows this to be corrected (e.g. a standard tuning device that displays whether the note is above or below 440Hz).

- Muting one or many strings at a time.

- Turning the tuning pin.

- Determining whether an untuned string in the temperament octave is correct relative to a tuned string.

- Determining whether an untuned string in the other octaves is correct relative to a tuned note one octave below.

The first two identified functions can be easily achieved by using existing tools; any standard tuner (like on that would be used for tuning a guitar) or 440Hz tuning fork can perform the first task, and many tools for muting strings exist and are used by professional piano tuners. The third function can certainly be performed using an existing tool: the tuning hammer. However, further consideration needs to be taken because efficient use of the hammer is one of the difficulties of tuning a piano, since one needs "to make the most minute of tuning-pin adjustment" (Martin 1961). If the solution only used a tuning hammer, with which the client has no experience, the tuning process could take significantly longer than it would take a professional tuner to do it, and thus using a tuning hammer might not satisfy all of the client requirements. What is needed is a tool that allows one to easily make small and consistent changes to the tuning pin rotation. One proposed solution is using a "repeatable impact tuning hammer" which aims to cause consistent pin rotations by storing a known energy in a torsion string by turning the handles (shown in Fig. 3). Upon release, the energy stored will be used to turn the tuning pin (Millard and Tizhoosh 2007). One downfall of this solution noted by Millard and Tizhoosh is that for small impact energies, the force required for movement due to the static coefficient of friction was not overcome and so the pin didn't move at all, and for small impact energies that did move the pin there was significant
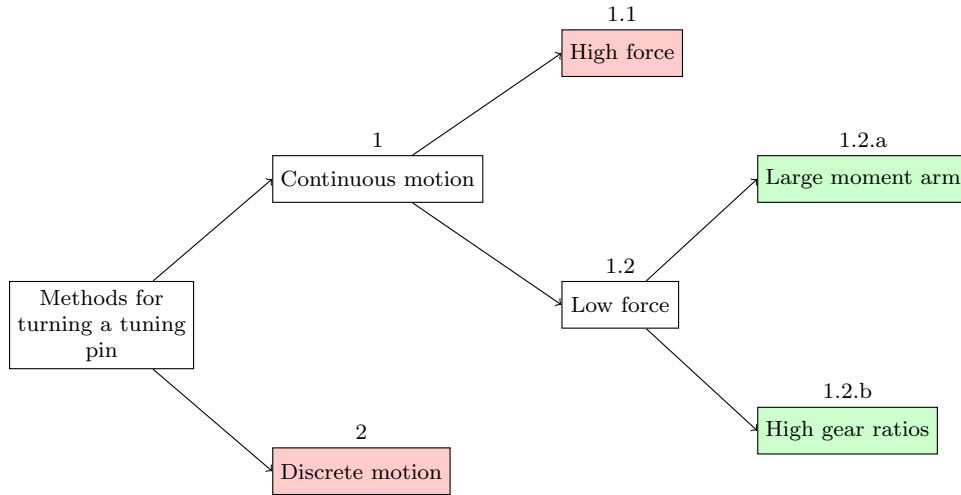
Figure 4: Concept generation tree for methods to turn a tuning pin.

variance in how much the pin rotated in response. This identifies the problem of turning the tuning pin as one that requires attention.

To help find potential solutions to the tuning pin problem, a concept generation tree (Fig. 4) was used to identify possible solutions. The first distinction is made between methods that turn the pin continuously (e.g. tuning hammer) and methods that turn the pin discretely (e.g. repeatable impact tuning hammer). The latter option is ruled out because anything in this category is likely to have the same problem of consistency at low movements that the repeatable impact tuning hammer exhibited. The continuous motion methods are separated into high force and low force methods. The latter category is for those methods that don't attempt to reduce the force required to turn the pin. This category was ruled out because tuning hammers, which fall into the low force category, already require upwards of 90N of force (Millard and Tizhoosh 2007). This is a significant force, and any high force methods are likely to require an even greater force. Finally, the last two categories distinguish between the main two methods of reducing the force required to turn the pin: increasing the moment arm to reduce the force required to produce the required torque, and using gear ratios to reduce the force required to produce the required torque. Standard tuning hammers fall into the former category. The latter category gives an idea for a unique solution: a device with a handle that through gears applies a torque to the tuning pin such that one revolution of the handle is much less that one revolution of the tuning pin.

It is important to make sure that this new idea would not have the same problem as the repeatable impact tuning hammer. It is possible for the gear ratio idea to be inconsistent because the members through which the force is transferred from the handle to the tuning pin may not be resistant to twisting from torque. This would mean that turning the handle would initially not turn the pin, because the members would twist due to the initial frictional force resisting the motion. Then when the members have twisted enough such that the torsional force is grater than the frictional force, the pin would jump inconsistently. This means that the members transferring the force would need to be resistant to twisting, which can be achieved by increased shear modulus of elasticity and polar moment of inertia, and reduced length (Hibbeler 2011). A cost efficient way to increase the polar moment of inertia is to make the members wide hollow cylinders. This poses a potential problem because the width of the cylinders would be constrained by the diameters of the gears, as they would be connecting gears. This means that to achieve the desired gear ratio, more larger gears would be needed possibly making the device bulkier and more expensive. Additionally, if the device gets too heavy there is a potential problem. It can be seen from Fig. 5 that while tuning a string there may need to be multiple attempts at turning the pin the
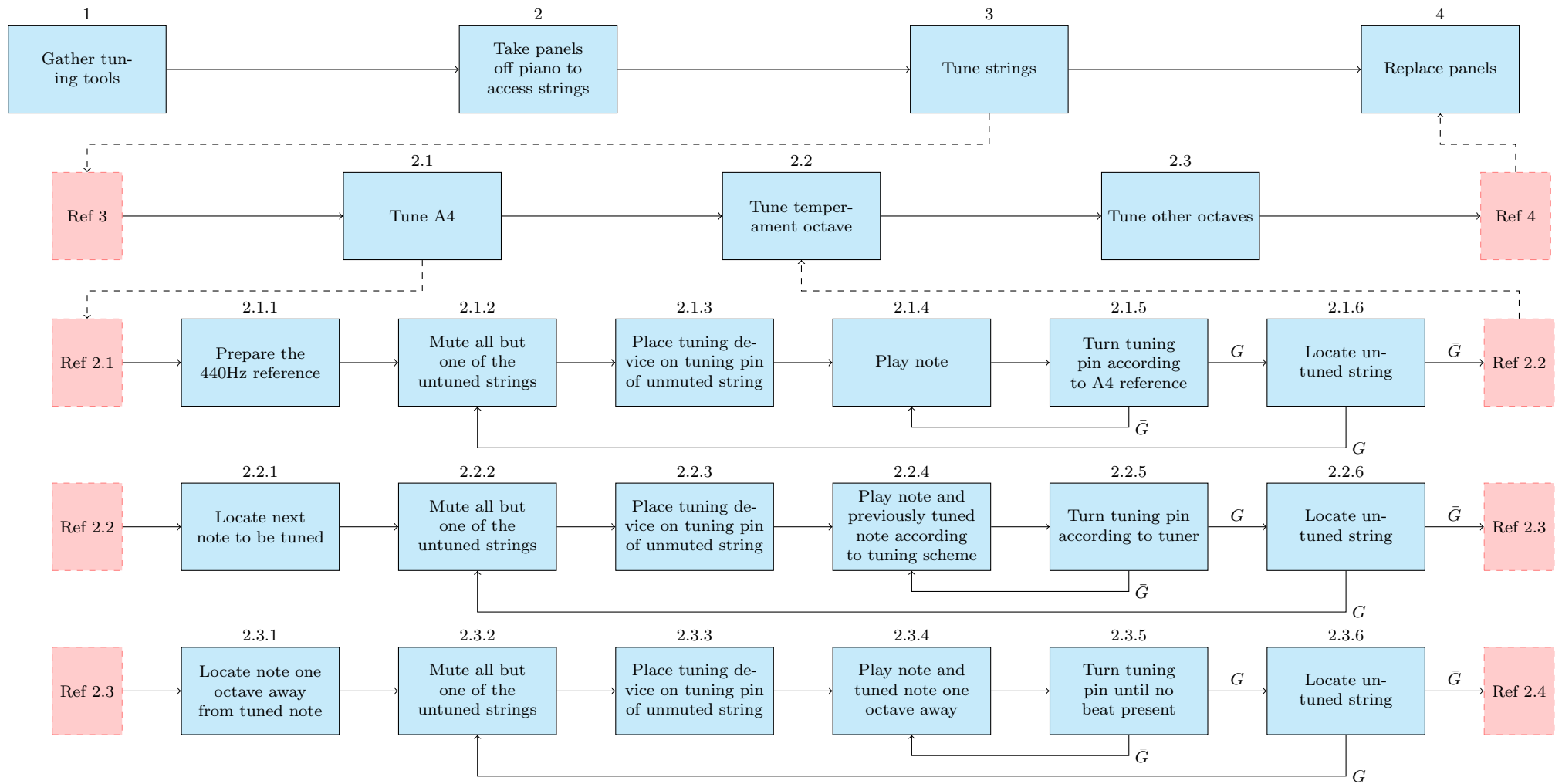
Figure 5: Functional flow block diagram (FFBD) depicting the process of tuning a piano.

correct amount, and so naturally the device that turns the pin will remain on the pin unsupported for brief periods of time. If the device is too heavy this may apply unsafe pressure to the pin and cause damage.

A similar trade off exists for large moment arm solutions. An inconsistent jump can occur because before the frictional force is overcome the long member will bend, and then when enough force is applied the pin will jump. The trade off here is that reducing the length of the long member will reduce bending, but this will increase the force required for use since the moment arm would be shorter. This trade off is more difficult to optimise than the gear ratio option, so the gear ratio option will be chosen for consideration.

For the fourth function, tuning the temperament octave, the precise tuning needs to be chosen. Tuning this octave by ear is much more difficult compared to the other octaves because there are no tuned notes that are one octave away from notes in the temperament octave at this stage (initially only A4 will be tuned). This means that to use the method of picking a beat frequency that should zero would require considering much higher partials, because there will be no overlap of lower degree partials for notes that are within an octave of eachother. This means that a non zero beat frequency would need to be achieved for some partials of the two notes. Even if suitable partials were found to compare for which the beat frequency when tuned was low (less than one second, say), they would very hard to perceive as they would be higher partials which are very quiet and last a very short time (Schuck 1943). For these reasons it was decided that the temperament octave should be tuned to equal temperament, because this can be done with any standard ETD. It should be noted that this is not invalidated by the inharmonicity of the strings; if we choose a suitable octave containing A4 then equal temperament will be a good approximation to the Railsback stretch. For example, we can see that octave 4 (C4 to B4) matches the Railsback stretch to within about 1 cent from Fig. 2. Additionally, the temperament octave is often tuned to equal temperament with good results (Chang 2009). The inharmonicity will be accounted for when tuning the other octaves.

Finally, it is proposed that the other octaves should be tuned by comparing each note with a tuned note one octave away. According to the Railsback stretch, the string will be in tune when the first partial of the lower note and the fundamental of the higher note are the same frequency; no beat will be heard. Since the beat will have zero frequency, this is the optimum value for the design requirement DR01-01. The volume ratio of the frequencies to be listened for will also be relative high. The fundamental frequency is the loudest of all the harmonics, and the first partial is the second loudest, being of about equal volume to the fundamental (Schuck 1943). For lower octaves, Schuck's graph also shows that the first partial decays only about as slow as the fundamental, making the beat easier to identify. This process is about as aurally demanding as tuning a guitar, being only slightly harder due to the presence of one high volume 'noise' frequency (the fundamental of the lower note). It follows that the proposed method for tuning the other octaves also satisfies design requirement DR01-02, so that the entire first customer requirement is met.

3.2 Susbsystems

The proposed subsystem decomposition of the piano tuning system is shown in Fig. 6. Note that the tools for muting strings do not appear on the diagram, because their interactions with the rest of the system are trivial. The fact that these muting tools will be used is one of the reasons that the system was separated as much as possible into separate tools. The separation makes the system highly modular which is desirable, and since there are already going to be a number of tools due to the mutes a few more tools isn't going to make the system particularly more cumbersome. Modularity is desirable because it means separate parts of the system can be easily replaced when damaged or when an upgrade is needed

without affecting the performance of the other parts of the system. The two major subsystems are the Electronic Tuning Device and the Pin Turning Mechanism. The former has a microphone to detect the string vibrations, a signal processor to identify the fundamental frequencies and associate them to their relative placement among the notes in the equally tempered scale, a display to indicate this placement, and a power system to power the electronics. The latter has an appropriate socket to apply force to the tuning pin, a handle for the client to apply force to turn the pin, and a gear system connecting the two such that a rotation of the handle causes a much smaller rotation of the tuning pin.

The subsystems were chosen so that every function identified in the FFBD (Fig. 5) can be performed by one of the subsystems. The note A4 is tuned using the pin turning mechanism and the A4 reference, which is built in to the functionality ETD which can identify correct frequencies for notes in the equally tempered scale. The temperament octave is tuned using the ETD and the pin turning mechanism. The other notes are tuned using the pin turning mechanism.
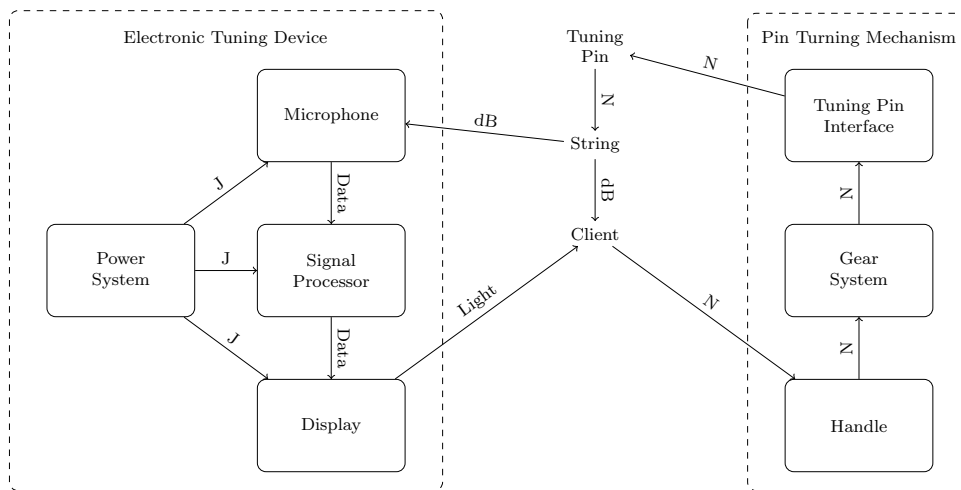

Figure 6: Subsystem interface diagram for the proposed piano tuning system.

How the proposed subsystems link back to the customer requirements is shown in Table 4. The primary attributes are the design requirements and the secondary attributes are a decomposition of the primary attributes. The subsystems in Fig. 6 are identified by the abbreviations Microphone ↦ MIC, Power System ↦ POW, Signal Processor ↦ SIG, Display ↦ DISP, Tuning Pin Interface ↦ INT, Gear System ↦ GEAR and Handle ↦ HAN. Note that MIC and SIG link to the non aural requirement because in their absence the temperament octave would need to be tuned by ear, increasing the aural demand. With these identifications it can be seen that the two most commonly appearing subsystems are the Microphone and the Signal Processor, and the least common is the Display. The former fact simply reinforces the importance of the existence of those subsystems. The latter fact indicates that the form of the display can be varied without significantly affecting how the system satisfies the customer requirements.

Table 4: Attributes cascade linking the customer requirements to the subsystems.

CR	Primary Attribute	Secondary Attribute	Related Subsystems
Non aural	Low beat frequency		MIC/SIG
	High relative harmonic volume	Volume ratio for lower note	MIC/SIG
		Volume ratio for higher note	MIC/SIG
Cost	Low upfront cost		ALL
	Low ongoing cost		ALL
Time	Low time spent tuning	Time tuning temperament octave	HAN/GEAR/INT/MIC/SIG/ DISP
		Time tuning other octaves	HAN/GEAR/INT
Simple to use	Low learning time	ETD learning time	DISP
		Pin turning mechanism learning time	HAN/GEAR/INT

4 Testing and Life Cycle

4.1 Life Cycle

The main components of the proposed design at this stage are muting tools (wedge mutes, felt strips etc.), a mechanism for turning the tuning pins and a universal ETD. The majority of these components lend well to end of life retirement. The muting tools can come in very simple forms, including a single piece of rubber for wedge mutes, and a strip of felt for the strip mute and hence these may be easily recycled. These tools are also highly durable and should last a long time, and if no longer need for use by the owner could be recycled by selling them to someone else who needs them. Similarly for the universal ETD, if it is no longer needed it can be sold. Universal ETDs are also used for tuning guitars and many other instruments and so there should be a high probability that it can be sold easily. Naturally these tools also require little to no maintenance throughout their lifetime.

For the gear ratio mechanism, some thought should be given to designing the parts so that existing production techniques can be used. For example, using common shapes and cross sections, such as standard gears and hollow cylinders as was proposed, will help to achieve this. Considering later stages of the life cycle, it becomes apparent that care should be taken to allow the device to be easily maintained and recycled. This can be done by making it easy to disassemble and by using recyclable materials.

4.2 Testing and Verification

The proposed solution of a standard ETD combined with the gear ratio based mechanism for turning the tuning pins will be compared against the most common alternatives for tuning a piano. Initially, the benchmarks for the design requirements that any solution must meet are established, as shown in Table 5. The beat frequency threshold of 10Hz was chosen as the highest beat frequency that could be detected with greater than 90% probability (Perrott and Nelson 1969). The relative volume benchmark represents the normalised amplitude of the frequency. This means that the value of 0.25 implies that if the amplitudes of all other frequencies are 1, then the amplitude needs to be at least 0.25 to be perceived. The specific value was chosen as the threshold of what could be detected aurally (Duifhuis 1970). The cost benchmarks were calculated on the assumption that the client's time is worth as much as a Canberra piano tuner's time is, which is $66 an hour (Leslie 2015). From this the upfront cost threshold is calculated to be such that the cost of buying the tools and the client tuning the piano is the same as the price of the first tuning from a professional, which is $165. Using the benchmark of 2 hours to tune the piano, this gives the price of the system to be $165 - 2 \times 66 = \$33$. If the ongoing cost was calculated in the same way, since subsequent tunings from the professional cost $132 the calculated benchmark would be $0, so an arbitrary small number was chosen, $10. The tuning time of 2 hours is chosen to be half an hour longer than the standard time it takes for a professional piano tuner (Coxon 2004). This is a reasonable threshold given that the tuning scheme employed by professional piano tuners is more complicated than

the Railsback stretch with the further simplification that the temperament octave is tuned to equal temperament. The learning time was chosen to be the time taken to complete the first tuning. This is because the tuning process is composed of small steps repeated many times (due to there being up to three strings per key, pianos have 236 strings in total that would need to be tuned (Smit 2004)), so one tuning would provide enough practice to get comfortable with the process.

Table 5: Benchmarks for the design requirements.

Data	TPM	Benchmark
Low beat Frequency	Hz	<10
High relative volume	ratio	>0.25
Low upfront cost	$AUD	<$33
Low ongoing cost	$AUD/year	<$10
Low tuning time	hrs	<2
Low learning time	hrs	<2

With benchmarks established, data for several tuning options will be gathered for each design requirement. To provide a comparison of how the proposed tuning method compares to a range of the common alternatives, data will be collected for an aural tuning by the client, a tuning using a piano (accounting for inharmoicity) ETD by the client and a professional tuning. This data is tabulated in Table 6.

Table 6: Data corresponding to the design requirements for various piano tuning options. The options are Railsback curve using the gear ratio mechanism (GRM) proposed in this report, a standard aural tuning by the client, a standard ETD tuning by the client, and a professional tuning.

Data	Benchmark	Railsback + GRM	Aural (client)	ETD (client)	Professional
Low beat frequency	<10	0	~1	N/A	N/A
High relative volume	>0.25	~1	0.75	N/A	N/A
Low upfront cost	<$33	$31.21 + GRM	$52.70	$1299.99	$165
Low ongoing cost	<$10	<$46	$10.54	$10.54	$264
Low tuning time	<2	1.5	3	2	1.5
Low learning time	<2	1.5	4	2	0

Note that while the cost for the professional option appears to exceed the benchmarks, none of the client's time (which was implicitly assumed for the benchmarks) is required and so theoretically the professional exactly meets the benchmarks. The data is justified as follows:

- Low beat frequency: the proposed scheme for the Railsback stretch was noted to only require reducing a beat frequency to 0 in Section 3.1. Within the scope of the lower partials, the seemingly best option for aural methods is comparing harmonics in the interval of a perfect fifth (7 semitones), as the beat between the second partial of the lower note and the first partial of the higher note is around 1Hz in the temperament octave for equal temperament.[2] For the last two options clearly no aural demand is made of the client.

- High relative volume: in Section 3.1 it was noted that the first partial (which is what needs to be detected aurally when tuning the other octaves) is approximately as loud as the loudest frequencies (the fundamental frequencies), so the ratio is thus approximately 1. The highest partial for aural techniques is a second partial which is about has a normalised amplitude of about 0.75 (Schuck 1943). Again the last two options require no aural capabilities of the client.

- Low upfront cost: The minimum muting and ETD tools were taken to be one wedge mute, one felt strip and a standard universal ETD, which cost $1.19, $6.75, and $23.27 respectively totalling

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[2]For example, consider the perfect fifth D4 to A4. D4 is 7 semitones below A4 which has a fundamental frequency of 440Hz, and so in equal temperament D4 has a fundamental frequency of $440 \times 2^{-\frac{7}{12}} \approx 293.665$Hz. Since the beat frequency is the difference of the frequencies, the beat frequency will be the difference between the second partial of D4 ($(2+1) \times 293.665 \approx 880.994$Hz) and the first partial of A4 ($(1+1) \times 440 = 880$Hz) giving the beat frequency to be ≈ 0.994. Other perfect fifths in the temperament octave yield similar results.

$32.21 (Supplies 2015b; Supplies 2015c; Center 2015). In the table the additional cost of the gear ratio mechanism is denoted by GRM, which because hypothetical no real cost exists, but clearly it will be more than \$2 and so it can be assumed that the benchmark will be not be met. The upfront cost for the aural tuning is the cost of a economic piano tuning kit (Supplies 2015a). The price of the ETD is the software cost for the Reyburn CyberTuner (Apple 2015). The price of the tuning tools was disregarding considering that the software is so expensive that it would be irrelevant. The professional piano tuner rates are that of a Canberra professional (Leslie 2015).

- Low ongoing cost: the assumption made for any tool based option was that the tools needed replacing every 10 tunings, which at 2 tunings a year gives their upfront cost divided by 5 to get a yearly cost. Since the GRM is hypothetical an arbitrary upper bound price of \$200 was assumed. The ETD option was based on the assumption that the same economic kit was purchased as for the aural option. The professional tuner is based on the existing client rate \$132 for twice a year.

- Low tuning time: the tuning time for the proposed method was based on the fact that for Millard and Tizhoosh's method the tuning time "greatly exceeds the performance of an amateur piano tuner using a traditional tuning hammer" (Millard and Tizhoosh 2007). This is reasonable because the proposed method is as 'straightforward' as the one they propose. An estimate for the aural method was given as twice that of the professional given the difficulty. The ETD figure is the half an hour more than the professional, because while no aural difficulties are present, using the tuning hammer inefficiently will increase the time taken.

- Low learning time: the time taken for the proposed method and the ETD was taken to be the time for one tuning, as one tuning would provide sufficient time to become accustomed to the tuning devices. The aural option was considered to require four tunings to learn correctly given the difficulty due to the aural demands. In the case of the professional obviously no learning is required by the client.

4.2.1   Evaluation

With benchmarks established and data gathered, a simple direct comparison between the options can be performed. For each design requirement, the option that satisfies it the best is given a 1, the second best a 2, and so on. The sum over all of the design requirements is then computed for each option, and the lower totals are theoretically the better options. This process is carried out in Table 7.

Table 7: Direct Ranking for each of the options.

Data	Railsback + GRM	Aural (client)	ETD (client)	Professional
Low beat frequency	2	3	1	1
High relative volume	2	3	1	1
Low upfront cost	2	1	4	3
Low ongoing cost	2	1	1	3
Low tuning time	1	3	2	1
Low learning time	2	4	3	1
Total	11	15	12	10

It can be seen by the highlighted cell that direct ranking identifies the professional option as the best. This is due to the non existent aural demands and time spent learning and tuning, for which the professional was the best option. It should be noted that this is a very simplified comparison and doesn't take into account the relative importance of the design requirements. For instance, if a weighting greater than 2 was placed on the cost requirements then the professional option would become worse than most other options. Additionally, if the degree to which the options exceeded the bench marks was taken into

account then it is clear that the ETD option would become the worst by far due to the enormous upfront cost.

Of the options considered, the aural and ETD options should be regarded as not feasible. For the latter this is because the software cost exceeds to benchmark by too much to be considered reasonable. For the former it is the aural demands that rule it out. While it meets the benchmarks as shown in Table 6, this does not accurately represent how well the requirements are met. This is because while the faintest partial has a relative amplitude of 0.75, the duration of this partial wasn't considered. The partial decays very quickly (Schuck 1943). This means that in addition to the difficulty of identifying the beat, the beat last only a very short time. This makes tuning the string much harder, because one cannot be turning the tuning pin while listening to the beat, which allows for more accurate pin adjustments, causing the tuning process to be much slower and more difficult. Additionally, since a non zero beat needs to be achieved, another tool such as a metronome is required to decide when the beat is at the correct frequency. This is not required in the case when the beat needs to be zero, this is easily achieved by ear.

Considering the eliminations, only the proposed solution and the professional option are left. Both of these options have a very close direct ranking score, making identification of the best option ambiguous. Generally, it can be seen that the proposed solution is the better option for cost, and the professional solution is the better option for skill and time demand. This means that for a given client the better option depends on the relative importance of these factors. Additionally, the matter of taste was not considered in the preceding analyses; the client may prefer a "fine tuning" performed aurally by a professional over the approximation of the Railsback stretch. This was identified as out of the scope of the project in Section 2.1, and so will not be factored into the conclusion of the analyses.

It is possible to attempt to remove this ambiguity by performing an importance ranking for the design requirements, which if used would have appeared in Section 2.2. This was not done in the attempt to keep the conclusions of the report more general. The relative importance of aural difficulty, and especially cost and time required varies too greatly between individuals to apply a meaningful ranking of their importance without significantly narrowing the client base.

5   Conclusion

The design process for a piano tuning system that would allow the client to be able to tune their instrument as easily as one can tune a guitar was documented. The tuning scheme used the Railsback stretch and an equally tempered temperament octave for its simplicity and quality. The proposed solution uses a universal ETD to tune the temperament octave and the other octaves are tuned by ear. The pin turning mechanism uses gear ratios to allow for fine and consistent pin adjustments. When compared with other user based tuning methods the proposed solution was found to be the best option. When compared with hiring a professional tuner the two options were found to be both desirable, with the professional being favoured when time and skill requirements are more important, and the proposed solution being favoured when cost is more important.

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