Observers for linear time-varying systems

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Extended abstract

In a general sense, observation is the problem of finding estimates for the current values of a set of signals given the current and the past values of another set of signals, where both signal sets are interconnected by the action of a dynamical system. In the work of Luenberger [6, 7] a method is described how this can be done in the context of linear time-invariant control systems, where the observed signals are the input and the output and the to be estimated signals are (parts of) the state. The main idea is to feed the observed signals into an auxiliary system, the observer, and to use its output as the desired estimate.

One desired property of such an estimate is that it is asymptotically accurate, in other words, it converges to the actual value of the observed signals when time goes to infinity. An observer that achieves this is usually called an asymptotic observer. A characterisation of all asymptotic observers, given a controllable linear time-invariant system and a to be observed linear function of its state, has already been given by Fortmann and Williamson [2]. However, their proof is rather incomplete (see [9] for a discussion) and it is only recently that a full proof has been given by Fuhrmann and Helmke [4]. A full characterisations for the existence of such observers has been given by Schumacher [8] in terms of conditioned invariant subspaces.

A generalisation of some of these results to the setting of linear behaviors in ARMA form has been given by Valcher and Willems [10], who also rigorously define observers and their desirable properties in terms of sets of observed and to be estimated system variables and their past, current and future values.

The purpose of this talk is to generalise some of these ideas to the case of linear finite-dimensional time-varying systems of the form

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)u(t) \\ y(t) &= C(t)y(t) \end{aligned} \tag{1}$$

where A, B and C are sufficiently well-behaved matrix functions. An *observer* for $z(t) = V(t)x(t)$, where V is again a matrix function, will be an auxiliary system of the form

$$\begin{aligned}\dot{v}(t) &= K(t)v(t) + L(t)y(t) + M(t)u(t)\\ w(t) &= P(t)v(t)\end{aligned} \tag{2}$$

where K, L, M and P are matrix functions and w is an estimate for z.

In the time-invariant case the fundamental property of such an observer turns out to be the *tracking property*: for every starting state $x(0)$ of the observed system there exists a starting state $v(0)$ of the observer such that $w(t) = z(t)$ for all t, i.e. such that the observer tracks the to be estimated signal. This property is fundamental in the sense that for controllable observed time-invariant systems it is already implied by the asymptotic property described before. An observer will be called a *tracking observer* if it has the tracking property. Results we will present for the time-varying case include the following theorems.

Theorem 1. *Let all the matrix functions be piecewise smooth. System* (2) *is a tracking observer for $z(t) = Vx(t)$ if and only if there exists a piecewise smooth matrix function $U(t)$ such that*

$$\begin{aligned}\dot{U}(t) &= K(t)U(t) - U(t)A(t) + L(t)C(t)\\ M(t) &= U(t)B(t)\\ V(t) &= P(t)U(t)\end{aligned}$$

It is an asymptotic observer if additionally $\dot{e}(t) = K(t)e(t)$ is stable.

As in the time-invariant case, existence conditions can be given in terms of conditioned invariant subspaces. This notion has been generalised to the time-varying case by Ilchmann [5]. There is also recent work by Balas, Bokor and Szabó [1] which in a sense is less general.

Theorem 2. *Let all the matrix functions be piecewise analytic. There exists a tracking observer for $z(t) = V(t)x(t)$ of order q if and only if there exists a time-varying conditioned invariant family $\mathcal{V}(t) \subset \operatorname{Ker} V(t)$ with $\operatorname{codim} \mathcal{V}(t) = q$. There exists an asymptotic observer of order q if additionally $\mathcal{V}(t)$ is outer detectable.*

References

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