3. Assignment  
ENGN4226/6226 System Theory, Semester 1, 2007  
due: 04/04, 10am (end of lecture)

1. (25%) Consider the input/state/output system

\[
\begin{align*}
\frac{dx(t)}{dt} &= Ax(t) + Bu(t) \\
y &= Cx(t) + Du(t)
\end{align*}
\]

(S)

and prove that \((u, \begin{pmatrix} x \\ y \end{pmatrix})\) is an input/output splitting. What is the corresponding transfer function matrix? Does the system \((S)\) admit any other input/output splitting? Consider in particular the extreme case \(B = 0\) and \(D = I\).

2. (25%) Prove that \(x\) in system \((S)\) has the property of state.

**Hint:** Use the variation of constant formula.

3. (25%) Consider the forced undamped harmonic oscillator

\[
\frac{d^2x(t)}{dt^2} + kx(t) = F(t)
\]

and construct a state for this system.

**Hint:** You will need to introduce another variable and relate it to the given variables \(x\) and/or \(F\).

4. (25%) Consider an electrical circuit with a resistor \(R\), a capacitor \(C\) and an inductor \(L\) all connected in parallel to a source \((V, I)\). Write down a representation of the system and identify a state.