FARFIELD ARRAY WEIGHT REDESIGN FOR NEARFIELD BEAMFORMING

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ABSTRACT

This paper presents a new method for nearfield linear array beamforming that achieves a desired beampattern (as a function of direction) at any nominal finite distance from the array origin. Given a set of array weights which achieves the desired beampattern for farfield sources, we device a linear transformation to obtain another set of array weights which achieves the same beampattern for sources in the nearfield. A simulation example is presented to demonstrate the effectiveness of this method in producing a nearfield beampattern using a standard farfield design technique and the proposed transformation.

1. INTRODUCTION

Sensor arrays provide an efficient means to detect and process signals arriving from different directions. The majority of array processing literature deals with the case in which the source is assumed to be in the farfield of the array, which simplifies the beamforming design problem. However, in many practical situations, the source is in the nearfield of the array and using farfield assumption to design the beamformer results in severe degradation in the beampattern. In the simplest case, the beamforming problem consists of finding array weights that satisfy a set of specifications on the beampattern. In this paper, we propose a method to “refocus” a farfield beamformer to nearfield sources by a simple matrix transformation of array weights.

There appears to be little work in the literature on nearfield beamforming. In [1] the curvature of a spherical wavefront of a nearfield source is approximated by a quadratic surface and in [2] time delays were applied to compensate for differing propagation delays. However, both of these methods do not accurately achieve the desired response over all angles. Other related work we are aware of dealing with nearfield arrays can be found in [3–7].

In this paper, a new method of nearfield beamforming is proposed in which a desired arbitrary beampattern in angle may be produced using standard farfield design techniques. The design methodology relies on three key ideas: 1) a relationship between nearfield response and farfield response of a same theoretical continuous sensor based on modal expansion; 2) a Fourier transform relationship between farfield beampattern and a continuous aperture function; and 3) an expression relating modal coefficients of a beampattern in terms of farfield array weights. Using these ideas, we redesign the farfield array weights using a linear transformation to produce the desired beampattern in the nearfield.

2. PROBLEM FORMULATION

Suppose, there exists a linear array of $(2M+1)$ sensors with array weight vector

$$\mathbf{W}^{(\infty)} = [w_{-M}^{(\infty)}, \cdots, w_{M}^{(\infty)}]^T$$

aligned to the x axis, such that the response to plane waves from a farfield source, impinging at an angle θ to the array axis, is

$$a(\theta) = \sum_{m=-M}^{M} w_m^{(\infty)} e^{-ikx_m \cos\theta} \tag{1}$$

where $i = \sqrt{-1}$ and $k \triangleq 2\pi f/c = 2\pi/\lambda$ is the wave number which can be expressed in terms of the propagation speed c and the frequency f, or the wavelength λ. We assume that the propagation speed c is a constant, implying k is a constant multiple of frequency f and throughout this paper we will often refer to k as “frequency”. Let

$$\mathbf{W}^{(r)} = [w_{-Q}^{(r)}, \cdots, w_{Q}^{(r)}]^T$$

be the weights of a linear array of $(2Q+1)$ sensors, which achieved the desired beampattern specification $a(\theta)$ for a nearfield source at an angle θ and distance r

from the array origin. Hence,

$$a(\theta) = \sum_{q=-Q}^{Q} w_q^{(r)} \frac{r}{d(r, x_q, \theta)} e^{ik(d(r,x_q,\theta)-r)}, \quad (2)$$

where

$$d(r, x, \theta) \triangleq (r^2 - 2rx\cos\theta + x^2)^{1/2}$$

is the distance from the source to a sensor position x in the array axis.

The problem we consider is determining the "farfield-nearfield" transformation matrix $\mathbf{A}$ such that

$$\mathbf{W}^{(r)} \approx \mathbf{A}\mathbf{W}^{(\infty)} \quad (3)$$

and identify the nature of the approximation in (3).

3. BEAMPATTERN FORMULATION

3.1. Nearfield Modal Analysis

Consider a theoretical continuous sensor aligned to the x axis with an aperture illumination function $\rho^{(r)}(x,k)$, where x is the distance to a point in the sensor from the sensor origin. Then the response of the sensor to a source at an angle θ and distance r from the array origin is

$$b_r(\theta) = \int_{-\infty}^{\infty} \rho^{(r)}(x,k) \frac{r}{d(r,x,\theta)} e^{ik(d(r,x,\theta)-r)} dx, \quad (4)$$

provided the function $d(r,x,\theta) \neq 0$ (which is the distance from the source to a point x in the sensor). Also we assume that $\rho^{(r)}(x,k) \approx 0$ for $|x| > r$. Under these mild conditions, we can use the convergent series expansion [8, page 366],

$$\frac{e^{ikd(r,x,\theta)}}{d(r,x,\theta)} = ik \sum_{n=0}^{\infty} (2n+1) h_n^{(1)}(kr) j_n(kx) P_n(\cos\theta)$$

where $P_n(\cdot)$ are the Legendre functions and $j_n(\cdot)$ and $h_n^{(1)}(\cdot)$ are the so called *spherical Bessel and Hankel functions* of first kind which are defined as [9, page 125]

$$\begin{aligned} j_n(t) &= \sqrt{\frac{\pi}{2t}}\, J_{n+\frac{1}{2}}(t), \\ h_n^{(1)}(t) &= \sqrt{\frac{\pi}{2t}}\, (J_{n+\frac{1}{2}}(t) + iY_{n+\frac{1}{2}}(t)) \end{aligned}$$

where $J_{n+\frac{1}{2}}(\cdot)$ and $Y_{n+\frac{1}{2}}(\cdot)$ are the half integer order Bessel functions of the first and second kind respectively. By substituting above expansion in (4) and taking integration term by term of the series, on account of its convergence, we obtain the nearfield sensor response as

$$b_r(\theta) = ikr\, e^{-ikr} \sum_{n=0}^{\infty} (2n+1)\, \alpha_n^{(r)}(k) \, h_n^{(1)}(kr)\, P_n(\cos\theta), \quad (5)$$

where

$$\alpha_n^{(r)}(k) \triangleq \int_{-\infty}^{\infty} \rho^{(r)}(x,k) j_n(kx) dx. \quad (6)$$

Equation (5) is the standard modal representation of a beampattern [7], but it has a different derivation in [7]. Also we can evaluate the coefficients $\alpha_n^{(r)}(k)$ of the series (5) for a fixed frequency k using the orthogonally property of Legendre functions [10, page 85] as

$$\alpha_n^{(r)}(k) = \frac{e^{ikr}}{2ikr} \frac{1}{h_n^{(1)}(kr)} \int_{-1}^{1} b_r(u) P_n(u) du, \quad (7)$$

where $u = \cos\theta$. Here we have replaced $b_r(\theta)$ by $b_r(u)$, since the beampattern $b_r(\theta)$ is a function of $\cos\theta$.

Suppose the response $b_r(\theta)$ of the continuous sensor is equal to the desired beampattern specification $a(\theta)$. Hence, we can substitute (1) into (7) and interchange the integration and summation to obtain

$$\alpha_n^{(r)}(k) = \frac{e^{ikr}}{2ikr} \frac{1}{h_n^{(1)}(kr)} \sum_{m=-M}^{M} w_m^{(\infty)} \int_{-1}^{1} e^{-ikx_m u} P_n(u)\, du.$$

The integral of the above equation can be evaluated to get

$$\alpha_n^{(r)}(k) = \frac{e^{ikr}}{kr} \frac{(-i)^{n+1}}{h_n^{(1)}(kr)} \sum_{m=-M}^{M} w_m^{(\infty)}\, j_n(kx_m). \quad (8)$$

Thus we have established an expression relating model coefficients $\alpha_n^{(r)}(k)$ of a beampattern in terms of its farfield array weights.

3.2. Behaviour of Nearfield Design in Farfield

Response of the same continuous sensor $\rho^{(r)}(x,k)$, that was considered in the last section, to waves from a farfield source, impinging at an angle θ to the sensor, is given by

$$b_\infty(\theta) = \int_{-\infty}^{\infty} \rho^{(r)}(x,k)\, e^{-ikx\cos\theta}\, dx. \quad (9)$$

Again using a Legendre series expansion, we can write [8, page 368]

$$e^{-ikx\cos\theta} = \sum_{n=0}^{\infty} (-i)^n (2n+1) j_n(kx) P_n(\cos\theta). \quad (10)$$

By substituting (10) in to (9) and interchanging the summation and integration, we obtain the model representation of a farfield beampattern as

$$b_\infty(\theta) = \sum_{n=0}^{\infty} (-i)^n (2n+1) \alpha_n^{(r)}(k) P_n(\cos\theta). \quad (11)$$

Note that from (5) and (11),

$$\lim_{r\to\infty} b_r(\theta) = b_\infty(\theta), \tag{12}$$

since the asymptotic behavior of the spherical Hankel functions for large arguments is given by [8, page 201]

$$h_n^{(1)}(t) = (-i)^{n+1}\frac{e^{it}}{t}\,\{1+O\left(\frac{1}{t}\right)\},\ t\to\infty. \tag{13}$$

Thus, we can view $b_\infty(\theta)$ as the beampattern $b_r(\theta)$ evaluated at $r=\infty$. What we can grasp from the above result is that the farfield and nearfield responses of the same continuous sensor differ only by a simple factor in each mode n which depends on the operating radius.

3.3. Aperture Illumination

In order to derive an expression for aperture function $\rho^{(r)}(x,k)$ in terms of the array weights, we rewrite the farfield array response (9) as

$$b_\infty(u) = \int_{-\infty}^{\infty} \rho^{(r)}(x,k)\, e^{-ikux}\, dx \tag{14}$$

where $u=\cos\theta$. Note that (14) is the standard Fourier transform relating a farfield aperture response $b_\infty(u)$ to the aperture illumination function $\rho^{(r)}(x,k)$ for frequency k. Thus the inverse Fourier transform corresponding to (14) is given by

$$\rho^{(r)}(x,k) = \frac{k}{2\pi}\int_{-1}^{1} b_\infty(u)\, e^{ikux}\, du. \tag{15}$$

We substitute (11) in (15), perform the intergration after interchanging integral and summation to get an exact expression for aperture illumination

$$\rho^{(r)}(x,k) = \frac{k}{\pi}\sum_{n=0}^{\infty}(2n+1)\alpha_n^{(r)}(k)\, j_n(kx). \tag{16}$$

For practical purposes, we assume that there are only $N+1$ significant terms in the infinite series expression (16) for the continuous sensor $\rho^{(r)}(x,k)$. From (8) and truncated series (16) we get

$$\rho^{(r)}(x,k) = \sum_{n=0}^{N}(2n+1)\,\frac{(-i)^{n+1}e^{ikr}}{\pi r h_n^{(1)}(kr)}\, j_n(kx) \sum_{m=-M}^{M} w_m^{(\infty)}\, j_n(kx_m), \tag{17}$$

which relate the continuous aperture function required to produce the beampattern $a(\theta)$ for a nearfield source to the weights of a linear array of sensors which produce the same beampattern $a(\theta)$ for a farfield source. In the following section, we truncate and discretize this continuous aperture to obtain the desired transformation matrix between farfield and nearfield.

4. TRANSFORMATION MATRIX

It can be shown that the Fourier transform of $\rho^{(r)}(x,k)$ with respect to x is bandlimited by k. This implies that we can represent $\rho^{(r)}(x,k)$ by its samples if the sampling distance is less than $\lambda/2\,(=\pi/k)$. Further we have assumed that $\rho^{(r)}(x,k)$ has only finite support in x; hence we can approximate the integral in (4) by a finite summation to obtain

$$b_r(\theta) \approx \sum_{q=-Q}^{Q} g_q\,\rho^{(r)}(x_q,k)\,\frac{r}{d(r,x_q,\theta)}\, e^{ik(d(r,x_q,\theta)-r)}dx, \tag{18}$$

where $[x_{-Q},\cdots,x_Q]$ is a possible set of sampling points (sensor locations) and g_q depend on the sensor separations. By comparing (2) with (18) we can observe that

$$w_q^{(r)} \approx g_q\rho^{(r)}(x_q,k)\ \text{for}\ q=-Q,\cdots,Q$$

so that

$$\mathbf{W}^{(r)} \approx \begin{bmatrix} g_{-Q}\rho^{(r)}(x_{-Q},k)\\ \vdots \\ g_Q\rho^{(r)}(x_Q,k)\end{bmatrix}. \tag{19}$$

By combining (19) and (17) we can obtain the following matrix equation,

$$\mathbf{W}^{(r)} \approx \mathbf{D_1 J_1 D_2 J_2}^T\mathbf{W}^{(\infty)},$$

where

$$\mathbf{D_1} = \begin{bmatrix} g_{-Q} & 0 & \cdots & 0\\ 0 & \ddots & & \vdots\\ \vdots & & \ddots & 0\\ 0 & \cdots & 0 & g_Q\end{bmatrix}$$

is a $(2Q+1)\times(2Q+1)$ diagonal matrix,

$$\mathbf{J_1} = \begin{bmatrix} j_0(kx_{-Q}) & \cdots, & j_N(kx_{-Q})\\ \vdots & & \vdots\\ j_0(kx_Q) & \cdots, & j_N(kx_Q)\end{bmatrix}$$

is a $(2Q+1)\times(N+1)$ matrix,

$$\mathbf{D_2} = \frac{e^{ikr}}{\pi r}\begin{bmatrix} \frac{(2\times0+1)(-i)^{0+1}}{h_0^{(1)}(kr)} & 0 & \cdots & 0\\ 0 & \ddots & & \vdots\\ \vdots & & \ddots & 0\\ 0 & \cdots & 0 & \frac{(2N+1)(-i)^{N+1}}{h_N^{(1)}(kr)}\end{bmatrix}$$

is a $(N+1)\times(N+1)$ diagonal matrix, and

$$\mathbf{J_2} = \begin{bmatrix} j_0(kx_{-M}) & \cdots, & j_N(kx_{-M})\\ \vdots & & \vdots\\ j_0(kx_M) & \cdots, & j_N(kx_M)\end{bmatrix}$$

is a $(2M+1)\times(N+1)$ matrix, Hence we can conclude that the farfield-nearfield transformation matrix $\mathbf{A} = \mathbf{D_1J_1D_2J_2}^T$.

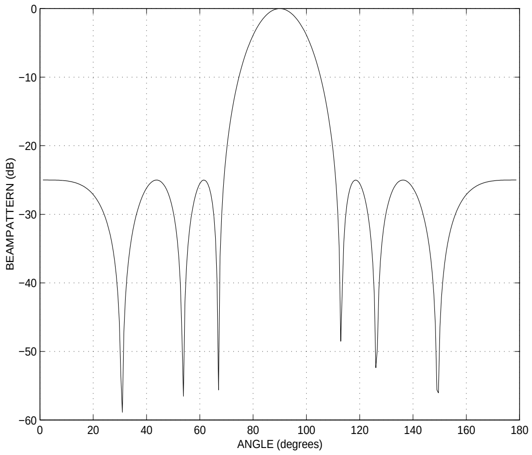

Figure 1: Desired nearfield beampattern.

5. EXAMPLE

The following example illustrates the use of above transformation technique for nearfield beamforming in comparison with a technique in [1, page 36]. We wish to design a nearfield beamformer having the response in Figure 1 at a distance of 3 wavelengths from the array origin.

A set of weights for array of 7 half wave-length spaced sensors is designed according to [11] to produce the required beampattern in the farfield. For the actual array for nearfield operations, we choose 13 sensors with uniform sensor separation of half a wavelength. Then we calculate the transformation matrix $\mathbf{A}$ with the maximum number of modes $N = 15$ for this example. Next we evaluate the corresponding nearfield weight vector using (3). The resulting beamformer is simulated in the nearfield and the response is depicted (solid) in Figure (2). Also shown is the desired beampattern (dotted), and the response of the nearfield method [1] (dashed). We note that the proposed nearfield design technique provides a close realization of the desired beampattern over all angles, not just at angles close to broadside as for the nearfield method of [1].

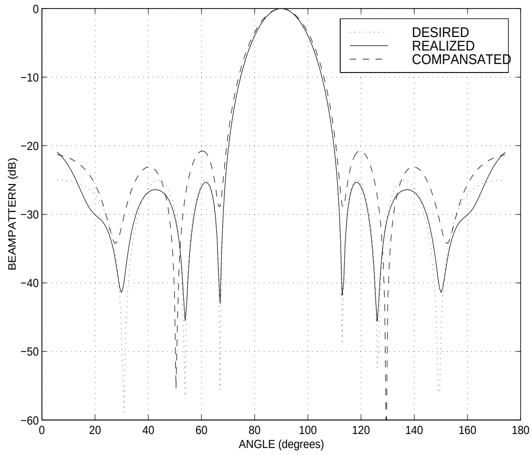

Figure 2: Resulting beamformer performance in the nearfield at a distance of 3 wavelengths from the array origin. The desired beampattern (dotted) and the response of the beamformer designed using nearfield quadratic compensation method (dashed) are also shown for comparison.

6. REFERENCES

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