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Spatial soundfield reproduction with zones of quiet

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ABSTRACT

Reproduction of a spatial soundfield in an extended region of open space with a designated quiet zone is a challenging problem in audio signal processing. We show how to reproduce a given spatial soundfield without altering a nearby quiet zone. In this paper, we design a spatial band stop filter over the zone of quiet to suppress the interference from the nearby desired soundfield. This is achieved by using higher order spatial harmonics to cancel the undesirable effects of the lower order harmonics of the desired soundfield on the zone of quiet. We illustrate the theory and design by simulating a 2D spatial soundfield.

1. INTRODUCTION

Reproduction of a spatial soundfield over an extended region of open space with a designated quiet zone is a complex and challenging problem in audio signal processing. In this paper, we show how to reproduce a given spatial soundfield without altering a nearby quiet zone by designing a spatial band stop filter.

The main contribution of this paper is a novel method to cancel the interfering soundfiled on the quiet zone. We use higher order harmonics with respect to the origin of the desired spatial soundfield to cancel the lower order harmonics of the desired soundfield over the quiet zone. This is accomplished by using a harmonic translation theorem [1] to transform harmonic coefficients of a soundfield from one origin to another. Once we transformed the both the lower order harmonics coefficients of the desired spatial soundfield and its higher order harmonics coefficients from the desired spatial zone to the designated zone of quiet, we force the lower order modes of the quiet zone to zero to obtain the higher order harmonics. We term this operation as "spatial region filtering".

After calculating the higher order harmonic coefficients (with respect to the origin of the desired spatial soundfield) which cancel the lower order harmonics over the quiet zone, we can use any existing sounfiled reconstruction techniques such as Wave Field Synthesis (WFS) [2], [3], [4], [5], [6], higher order ambisonics (cylindrical harmonics) based [7], [8], [9], [10] or least squares based methods [11], [12], [13], [14] to reproduce the modified desired soundfield. We illustrate the theory and design by simulating a 2D spatial soundfield.

Notation

Throughout this paper, we use the following notations: matrices and vectors are represented by upper and lower bold face respectively, e.g., $\boldsymbol{T}$ and $\boldsymbol{\alpha}$. The inner product of two vectors is denoted by "$\cdot$". $e^{(\cdot)}$ is the exponential function. The imaginary unit is denoted by $j = \sqrt{-1}$.

2. PROBLEM FORMULATION

We consider 2D height invariant soundfields in this paper. Suppose the desired zone of quiet is a circular area with radius r_q with respect to the origin O_q and the area of the desired spatial soundfield is another circular area with radius r_d with respect to the origin O_d. Let the distance between the origins of the two zones be $R(> (r_q + r_d))$ and O_q located (R, θ_q) with respect to O_d. The system model is shown in Fig. 1. r_{c_d} is a radius for higher order mode threshold we will refer later in Section 3.2.

2.1. Soundfields: Harmonic Expansion

Consider a point (r, ϕ) in cylindrical coordinates with respect to an origin located within a source free region. Then, the soundfield at a point (r, ϕ) due to sources outside of the region of interest can be expressed in terms of a cylindrical harmonic expansion [7, 8] as

$$S(r, \phi; k) = \sum_{m=-\infty}^{\infty} \alpha_m(k) J_m(kr) e^{jm\phi} \tag{1}$$

where $\alpha_m(k)$ are the cylindrical harmonic coefficients of the soundfield, $k = 2\pi f/c$ is the wavenumber, f is the frequency, c is the speed of sound, $J_m(\cdot)$ are the Bessel functions of order m. Throughout this paper, we use k instead of f to represent frequency since we assume constant c. The soundfield coefficients $\alpha_m(k)$ can be used to describe a given spatial soundfield. Using the orthogonal property of $e^{jm\phi}$, the harmonic coefficients can be calculated using

$$\alpha_m(k) = \frac{1}{J_m(kr)} \int_0^{2\pi} S(r\phi; k) e^{-jm\phi} d\phi \tag{2}$$

provided $J_m(kr) \neq 0$.

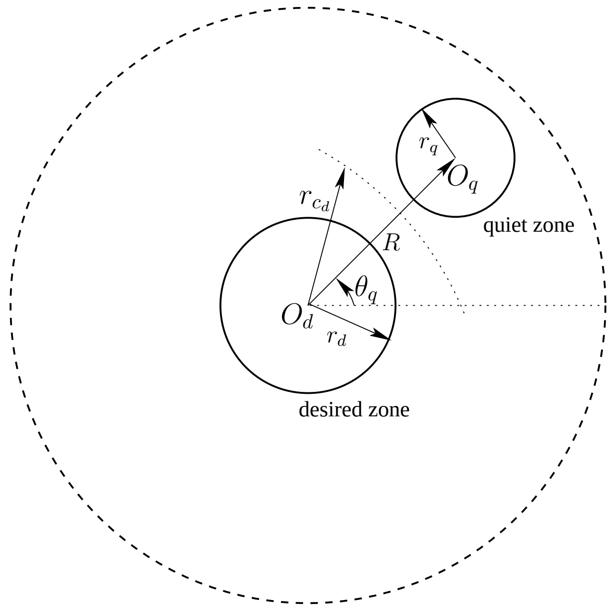

Fig. 1: Geometry of sound reproduction system with zones of quiet. The desired spatial soundfield is a circular area with radius r_d with respect to the origin O_d. The desired zone of quiet is another circular area with radius r_q with respect to the origin O_q. The distance between the origins of the two zones is $R(> (r_q + r_d))$ and O_q is located (R, θ_q) with respect to O_d.

2.2. Mode Limitedness

The representation (1) has an infinite number of terms. However, this series can be truncated to a finite number within a given region of interest due to the properties of the Bessel functions (see Fig. 2) and the fact that the soundfield has to be bounded within a spatial region where all sources are outside [15],[16]. Let R be the radius of the circular region of interest, then the soundfield inside this region can be represented by (1) with summation over m truncated to

$$M = \lceil ekR/2 \rceil \tag{3}$$

terms [17] with error in truncation is less than 67%.

2.3. Desired Soundfield

Let the desired soundfield confined to a limited spatial region with radius r_d, the desired soundfield at any observation point (r, ϕ) with respect to the origin O_d within

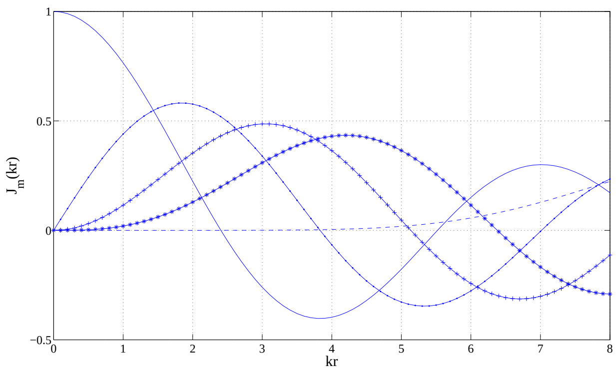

Fig. 2: Properties of the Bessel functions

the circular spatial region is

$$S^{\mathrm{d}}(r,\phi;k) = \sum_{n=-N_d}^{N_d} \alpha_n^{\mathrm{d}}(k) J_n(kr) e^{jn\phi}, \qquad (4)$$

where the truncation number N_d is upper bounded by the radius of the desired region r_d and the frequency k, i.e., $N_d = \lceil e r_d k/2 \rceil$. We use coefficients $\alpha_n^{\mathrm{d}}(k)$ to uniquely represent the desired soundfield.

2.4. Desired Zone of Quiet

We assume the origin of the desired zone of quiet is located (R,θ_q) with respect to O_d and the desired zone of quiet is also confined to a limited spatial region with radius r_q, we can write the soundfield at (R^q,ψ) with respect to the origin O_q as

$$S^{\mathrm{q}}(R^q,\psi;k) = \sum_{n=-N_q}^{N_q} \alpha_n^{\mathrm{q}}(k) J_n(kr) e^{jn\psi} = 0, \qquad (5)$$

where $\alpha_n^{\mathrm{q}}(k)$ uniquely represents the desired zone of quiet. If the zone is quiet, then we have

$$\alpha_n^{\mathrm{q}}(k) = 0 \text{ for } n = -N_q, \ldots, N_q. \qquad (6)$$

2.5. Problem Setup

The problem we consider in this paper is as follows: Given the desired spatial soundfield by coefficients $\alpha_m^{\mathrm{d}}(k)$ as well as the size and location of the zone of quiet, how can we design a loudspeaker array to realize both the desired spatial soundfield and the zone of quiet?

3. SPATIAL REGION FILTERING

In this paper, we propose a novel method to cancel the interfering soundfield on the quiet zone. We use higher order harmonics with respect to the origin O_d to cancel the lower order N_d harmonics of the desired soundfield over the quiet zone. This is accomplished by using a harmonic translation theorem [1] to transform harmonic coefficients of a soundfield from one origin to another. Once we transformed the both N_d and its higher order harmonics coefficients from O_d to O_q, we force first N_q modes of the quiet zone to zero to obtain the higher order harmonics. We term this operation as "spatial region filtering".

3.1. Spatial harmonic coefficients translation theorem

Theorem 1 *Let $\{\alpha_n^q(k)\}$ be a set of coefficients of a soundfield with respect to a co-ordinate systems whose origin is at (R,θ_q) with respect to O_d, where the two co-ordinate systems have the same angular orientation. Then the corresponding soundfield coefficients with respect to O_d is given by*

$$\alpha_n^q(k) = \alpha_n^{\mathrm{d}}(k) * T_n^{(dq)}(R,\theta_q;k), \qquad (7)$$

where $T_n^{(qd)}(R,\theta_q;k) \triangleq J_n(kR)e^{-jn\theta_q}$ [18], [19], [20] and '$*$' denotes the convolution.

Details please refer to [1].

3.2. Design of spatial band stop filter

We consider a set of higher order coefficients with respect to O_d are denoted by $\alpha_n^{\mathrm{d(h)}}(k)$ for $N_h < \|m\| < N_0$, where we define the mode numbers $N_h = \lceil ker_h/2 \rceil$ and $N_0 = \lceil keR/2 \rceil$ [15] as the higher order modes turning on between the spatial region of radii r_{c_d} and R respectively (referred in Fig. 1). We also assume the number of higher order coefficients is $N = N_0 - N_h$, where $N \geq N_q$.

Applying the coefficients translation theorem, the undesirable interference on the quiet zone due to the lower order N_d harmonics is given by $\alpha_m^{\mathrm{d}}(k) * T_m^{(dq)}$, and the induced soundfield on the quiet zone due to the higher order coefficients is given by $\alpha_m^{\mathrm{d(h)}}(k) * T_m^{(dq)}$. Using the higher order harmonics to cancel the lower order harmonics leads to

$$\alpha_m^{\mathrm{d(h)}}(k) * T_m^{(dq)} = -\alpha_m^{\mathrm{d}}(k) * T_m^{(dq)}. \qquad (8)$$

The convolution in (8) can be expressed in summation form

$$\sum_{N_0 \le \|n_h\| \le N_h} \alpha_{n_h}^{\mathrm{d(h)}}(k) T_{m-n_h}^{(dq)} = \sum_{n_d=-N_d}^{N_d} \alpha_{n_d}^{\mathrm{d}}(k) T_{m-n_d}^{(dq)}. \tag{9}$$

Equation (9) can be written in matrix form as

$$\boldsymbol{\alpha}^{\mathrm{d(h)}}(k) T_h(k) = -\boldsymbol{\alpha}^{\mathrm{d}}(k) T_\ell(k), \tag{10}$$

where

$$\boldsymbol{\alpha}^{\mathrm{d}}(k) \triangleq [\alpha_{-N_d}^{\mathrm{d}}(k), \ldots, \alpha_{N_d}^{\mathrm{d}}(k)]^T, \tag{11}$$

$$\begin{aligned}\boldsymbol{\alpha}^{\mathrm{d(h)}}(k) \triangleq [&\alpha_{-N_0}^{\mathrm{d(h)}}(k), \ldots, \alpha_{-N_h}^{\mathrm{d(h)}}(k), \\ &\alpha_{N_h}^{\mathrm{d(h)}}(k), \ldots, \alpha_{N_0}^{\mathrm{d(h)}}(k)]^T,\end{aligned} \tag{12}$$

$$T_\ell(k) \triangleq \begin{pmatrix} T_{-N+N_d}^{(qd)} & \cdots & T_{-N-N_d}^{(qd)} \\ T_{-N+1+N_d}^{(qd)} & \cdots & T_{-N+1-N_d}^{(qd)} \\ \vdots & \ddots & \vdots \\ T_{N+N_d}^{(qd)} & \cdots & T_{N-N_d}^{(qd)} \end{pmatrix}, \tag{13}$$

and we omit the arguments of $T_m^{(dq)}$ for convenience.

The matrix $T_h(k)$ is given in (15) displayed on the top of next page. Note that number of entries in (15) is forced to zero due to the translation property,

$$T_m^{(dq)} \triangleq J_m(kR) e^{-jm\theta^{(dq)}} = 0 \text{ for } \|m\| > N_0. \tag{14}$$

By applying the least squares method, we can obtain the higher order harmonic coefficients

$$\boldsymbol{\alpha}^{\mathrm{d(h)}}(k) = -[T_h(k)^T T_h(k)]^{-1} T_h(k)^T T_\ell(k) \boldsymbol{\alpha}^{\mathrm{d}}(k). \tag{16}$$

After obtaining the higher order coefficients $\alpha_n^{\mathrm{d(h)}}(k)$, now we can expressed the soundfield coefficients in the following form:

$$\tilde{\alpha}_m^{(d)}(k) = \begin{cases} \alpha_m^{\mathrm{d}}(k) \text{ for } -N_d < m < N_d \\ \alpha_m^{\mathrm{d(h)}}(k) \text{ for } N_h < |m| < N_0 \end{cases}, \tag{17}$$

where the soundfield coefficients $\tilde{\alpha}_m^{(d)}$ contain both the desired soundfield coefficients of spatial zone and higher order coefficients canceling the undesirable effects over the desired zone of quiet.

4. LOUDSPEAKER ARRAY DESIGN

knowing the soundfield coefficients $\tilde{\alpha}_m^{(d)}$ to produce the desired spatial region together with the zone of quiet, we can apply any of the existing soundfield reproduction technique to reconstruct the soundfield. It does not necessarily require a circular loudspeaker array to reproduce the soundfield and the system could be extended to a general loudspeaker setting. For simplicity, here we used the circular loudspeaker array to reproduce the multizone soundfield. We assume that the loudspeakers are placed on a circle with radius $R_p \ge R + r_q$ from the origin of the desired spatial soundfield O_d. The loudspeaker weight at angle ϕ is denoted as $\rho_p(\phi, k)$.

4.1. soundfield reproduction using the continuous loudspeaker method

Since the underlying structure of the loudspeaker weights is a function of the desired soundfield, here we use the continuous loudspeaker method [10]. By using the desired sound coefficients for the entire soundfield $\tilde{\alpha}_m^{(d)}$, we can use P discrete loudspeakers equally $\Delta\phi$ spaced on a circle of radius R_p, provided $P > 2M_{R_p}$, where $M_{R_p} = \lceil keR_p/2 \rceil$. The loudspeaker weights are given by [9]

$$\rho_p(\phi, k) = \sum_{m=-M_{R_p}}^{M_{R_p}} \frac{2}{j\pi H_m^{(1)}(k\|R_p\|)} \tilde{\alpha}_m^{(d)} e^{jm\phi_p} \Delta\phi. \tag{18}$$

The corresponding reproduced soundfield is given by

$$S^{\mathrm{a}}(r, \phi; k) = \sum_{p=1}^{P} \rho_p(k) \frac{j}{4} H_0^{(1)}(k\|R_p \hat{\boldsymbol{\phi}}_p - r\phi\|), \tag{19}$$

where $\hat{\boldsymbol{\phi}}_p = (1, \phi_p)$ and $R_p \hat{\boldsymbol{\phi}}_p$ denotes the loudspeaker position.

4.2. Soundfield reproduction using the least squares method

We can also use the least squares method to calculate the loudspeaker weights. We define the loudspeaker weights matrix $\boldsymbol{\rho}(k)$ as $\boldsymbol{\rho}(k) = [\rho_1(k), \ldots, \rho_P(k)]^T$, and it becomes [9]

$$\boldsymbol{\rho}(k) = \boldsymbol{H}(k)^{-1} \tilde{\boldsymbol{\alpha}}^d(k), \tag{20}$$

where $\tilde{\boldsymbol{\alpha}}^d(k) = [\tilde{\alpha}_{-M_{R_p}}^{(d)}(k), \ldots, \tilde{\alpha}_{M_{R_p}}^{(d)}]^T$ and

$$\boldsymbol{H}(k) = \frac{j}{4} \begin{pmatrix} H_{-M}^{(1)}(kR_p) e^{jM\phi_1} & \cdots & H_{-M}^{(1)}(kR_p) e^{jM\phi_P} \\ \vdots & \ddots & \vdots \\ H_M^{(1)}(kR_p) e^{-jM\phi_1} & \cdots & H_M^{(1)}(kR_p) e^{-jM\phi_P} \end{pmatrix}. \tag{21}$$

$$T_h(k) = \begin{pmatrix} T^{(dq)}_{N_0-N} & T^{(dq)}_{N_0-N-1} & \cdots & T^{(dq)}_{N_0-2N} & T^{(dq)}_{-N_0} & 0 & \cdots & 0 \\ T^{(dq)}_{N_0-N+1} & T^{(dq)}_{N_0-N} & \cdots & T^{(dq)}_{N_0-2N+1} & T^{(dq)}_{-N_0+1} & T^{(dq)}_{-N_0} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots & \vdots & \vdots & \ddots & 0 \\ T^{(dq)}_{N_0} & T^{(dq)}_{N_0-1} & \cdots & T^{(dq)}_{N_0-N} & T^{(dq)}_{-N_0-N} & T^{(dq)}_{-N_0-N+1} & \cdots & T^{(dq)}_{-N_0} \\ 0 & T^{(dq)}_{N_0} & \cdots & T^{(dq)}_{N_0-N+1} & T^{(dq)}_{-N_0-N-1} & T^{(dq)}_{-N_0-N} & \cdots & T^{(dq)}_{-N_0-1} \\ \vdots & \ddots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & T^{(dq)}_{N_0} & T^{(dq)}_{-N_0-2N} & T^{(dq)}_{-N_0-2N+1} & \cdots & T^{(dq)}_{-N_0-N} \end{pmatrix}. \quad (15)$$

The corresponding reproduced soundfield is given by (19).

5. DESIGN EXAMPLE

In this paper, we use a simple example to illustrate the ability to reproduce a 2D soundfields with zones of quiet. We consider a desired circular spatial zone of radius 0.3m and a desired circular quiet zone of radius 0.3. The distance between O_d and O_q is 1.5m and $\theta_q = 0°$. The desired soundfield is monochromatic plane wave of frequency of 1000 Hz arriving from $60°$. According to the dimensionality of spatial soundfield reconstruction [1], the minimum number of loudspeakers P required to reconstruct the field is $2 * \lceil ke(R + r_q) \rceil + 1$. In this case, we place 57 loudspeakers on a circle of 1.8m while we calculate loudspeaker weights using the least squares method, and the resulting reproduced field is shown in Figure 3.

It is calculated at 80×80 points and displayed as a "density plot" which means that the numerical values are represented by different shades of gray. The top two plots show the real and imaginary parts of the desired soundfield with zone of quiet, and the bottom two plots show the soundfield reproduced by the loudspeaker array. The reproduced soundfield corresponds well to the desired soundfield with zone of quiet where the zone boundaries of the two reproduction regions are indicated in two circles. We defined the reproduction error as

$$\epsilon \triangleq \frac{\int_0^{2\pi} |S^{\mathrm{d}}(r, \phi; k) - S^{\mathrm{a}}(r, \phi; k)|^2 d\phi r dr}{\int_0^{2\pi} |S^{\mathrm{d}}(r, \phi; k)|^2 d\phi r dr} \qquad (22)$$

where $S^{\mathrm{a}}(r, \phi; k)$ is the reproduced desired spatial soundfield with corresponding coefficients $\tilde{\alpha}_m^{(d)}(k)$. The reproduced error in this case is 2.59%.

6. CONCLUSION

In this paper, we propose a novel method to cancel the interfering soundfield on the quiet zone. We use higher order harmonics with respect to the origin of the desired spatial soundfield to cancel the lower order harmonics of the desired soundfield over the quiet zone. This is accomplished by using a harmonic translation theorem to transform harmonic coefficients of a soundfield from one origin to another. We term this operation as spatial region filtering. Simulation result demonstrates favorable reproduction performance.

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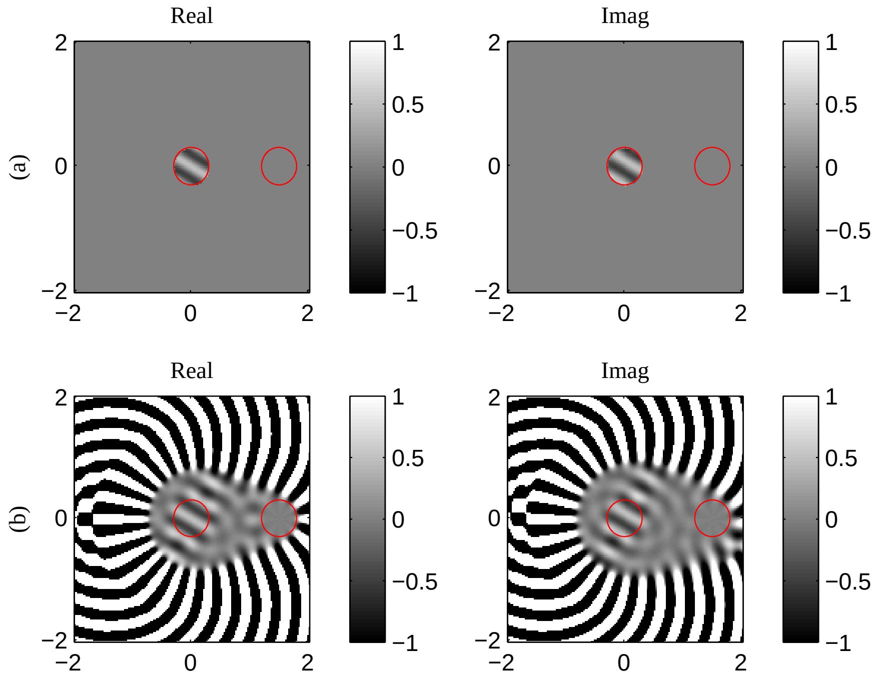

Fig. 3: Reproduction of a soundfield with zone of quiet. The radius of the desired spatial zone is reproduction zone is 0.3m and the radius of the desired quiet zone is 0.3m. The desired soundfield is monochromatic plane wave of frequency of 1000 Hz arriving from $60°$. $\theta_q = 0°, R = 1.5$m. (a) desired field, and (b) reproduction field. 57 loudspeakers are equally spaced on a circle of $R = 1.8$m. The reproduction error is 2.59%.

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