Impact of Mobile Acceleration on the Statistics of Rayleigh Fading Channel

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***Abstract*— Clarke's model of the received signal statistics in a mobile isotropic scattering environment assumes a constant mobile velocity, a consequence of which is that the autocorrelation function of the received signal and the Power Spectral Density (PSD) are independent of the absolute time. In this contribution we relax the assumption of constant mobile velocity and analyze the statistics of the channel when the mobile receiver has a constant acceleration. First, we derive expressions for a general scattering environment and, then, specialize them to the case of isotropic scattering environment. The autocorrelation and PSD of the channel are not only a function of the lag, τ, but the absolute time index, n as well. There are now two kinds of PSDs: The conventional PSD , based on the well-known Wiener-Khintchine theorem, gives the spectrum in the τ domain. The second PSD is concerned with the variation of the channel with time and gives the spectrum in the n domain. The simulation results suggest that the two PSDs show a pattern of periodicity which can be explained by considering asymptotic approximation of the Bessel function. Moreover, the magnitudes of the PSDs diminish with increasing τ or n such that the conventional PSD approaches *uniform distribution* over 0 to 2π when time n is large whereas the PSD of channel variation with time approaches zero for large τ. We give results on the basis of simulations and justify analytically, or heuristically. We also discuss different implications of these results.**

I. INTRODUCTION

The performance of a communication system is strongly influenced by the underlying communication medium. In case of mobile wireless communication systems, the communication medium is a multipath radio channel in which the transmitted signal reaches the receiver by more than one path due to reflection, refraction and scattering of radio waves. In this paper we consider a narrow-band transmission model where this multipath phenomenon causes fluctuations in the received signal envelope and phase [1].

A typical narrow-band mobile radio situation is considered in [2] in which the transmitter is fixed while the receiver is moving in such a way that there is no direct line between the transmitter and the mobile receiver, and, therefore, the mode of propagation of electromagnetic energy from transmitter to receiver is largely by way of scattering. It is assumed that a sufficient number of scattered waves with phases independent of each other (and the Angles of Arrival, AOA), and uniformly distributed throughout 0 to 2π are impinging at the receive antenna. As a consequence of the Central Limit Theorem, the envelope of the resultant signal is Rayleigh distributed and the phase is uniformly distributed in $[0, 2\pi]$. In other words, the channel process is zero-mean complex Gaussian process with equal-variance independent real and imaginary parts.

Assuming that the mobile receiver equipped with omnidirectional antenna is moving with constant velocity and the AOA of waves are uniformly distributed over $[0, 2\pi]$, it is shown that the envelope of the received signal has strictly real-valued time-independent autocorrelation given by the Bessel function of order zero and PSD is bowl-shaped symmetric. In other words, the complex channel fading process is wide-sense stationary (WSS) and, hence, stationary. Since its introduction, this so-called Clarke's model has found widespread adoption, mainly due to its simplicity.

A communication scenario in which the transmitter and the receiver are stationary but the scatterers are moving has been considered in [3]. Unlike the bowl-shaped Doppler spectrum of the Clarke's model, the Doppler spectrum of the propagation channel is shown to be peaky centered around the carrier frequency. The user's (transmitter and/or receiver) acceleration has been modelled by a correlated Gaussian process in the context of adaptive handoff in CDMA networks in [4] and, tracking and prediction in wireless ATM networks in [5].

In this contribution we extend the Clarke's model to the case when the mobile receiver is moving with some constant acceleration which has not yet been analyzed and corresponds better to the physical reality because a mobile user may experience changes in velocity caused by traffic lights or road conditions. Due to constant acceleration, the mobile velocity changes continuously. The statistics of the channel, therefore, are time varying and hence non-stationary. Our main assumption in this contribution is that the scattering distribution remains fixed i.e., the number and the strengths of the scattered waves do not change with time and the channel is homogeneous which implies that gains corresponding to different AOA are zero-mean uncorrelated.

Throughout the paper, the following notation will be used: Bold lower (upper) letters denote vectors (matrices). * denotes the conjugate transpose. The $\oint$ denotes the integration over unit circle. The ceiling operator is denoted by $\lceil . \rceil$. The notation $E\{\cdot\}$ denotes the mathematical ensemble expectation.

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II. CHANNEL MODEL

We consider a mobile communications scenario in which the transmitter is stationary while, at time t, the receiver equipped with an omni-directional antenna is moving with velocity $\boldsymbol{v}$ and constant acceleration[1] α at angle ψ with respect to x-axis in a 2D scattering environment (Fig. 1). We assume that the scatterers are distributed in the far-field from the mobile receiver so that the electromagnetic waves impinging on the receive antenna can well be approximated as plane waves. We also assume that the channel between the transmitter and the mobile receiver always remains a strictly bandlimited process. In other words, we implicitly assume that the mobile velocity remains bounded[2].

For a fixed carrier frequency f_c, the Doppler spectrum of the channel fading process depends directly on the mobile velocity. In fact, we have the following relationship between a particular mobile velocity $\boldsymbol{v}$ and a particular Doppler frequency f_{d} present in the Doppler spectrum

$$f_{\mathrm{d}} = \frac{|\boldsymbol{v}|}{\lambda} \cos\,\gamma \qquad , \tag{1}$$

where $\lambda = c/f_c$ is the carrier wavelength (c is the speed of light), and γ is the angle between the angle of arrival (AOA) of a particular electromagnetic wave and the unit vector $\hat{\boldsymbol{v}}$ (pointing in the direction of mobile movement). Since in our case, the mobile receiver has constant acceleration α, the velocity of the mobile changes continuously with time. If, for example, $\boldsymbol{v}'$ denotes the mobile velocity at some time t', the velocity of the mobile at time $t > t'$ is given by

$$\boldsymbol{v}(t) = \boldsymbol{v}' + \alpha\,(t - t'). \tag{2}$$

The time-varying velocity implies that for the same γ in (1), the corresponding Doppler frequency is also time-varying. In other words, the whole Doppler spectrum would be time-varying. Since, for a fixed time, the Doppler spectrum and the channel autocorrelation function are the Fourier transform pair, a time-varying Doppler spectrum implies a time-varying Autocorrelation function. Thus, unlike [2], the channel statistics vary over time and are, hence, non-stationary. It is important to note that as long as the mode of propagation of the electromagnetic energy is by way of scattering, the Central Limit Theorem ensures that the assumption of zero-mean complex fading process $\{h(t)\}$ is valid.

Let $s(t)$ be the channel input at time t. Then the continuous-time complex baseband received signal $y(t)$ at time t, assuming the channel to be flat-fading, is

$$y(t) = h(t)s(t) + w(t), \tag{3}$$

where $s(t)$ is the complex channel input, $\{w(t)\}$ is a zero-mean circularly symmetric complex additive white Gaussian noise (AWGN) process with variance $1/2\rho$ per dimension (ρ denotes the average received signal-to-noise ratio (SNR)) and $\{h(t)\}$ is a zero-mean unit variance circularly symmetric complex Gaussian process with independent real and imaginary parts having equal variance. The equation (3) is normalized in such a way that $E\left\{|s(t)|^2\right\} = 1$.

If we assume that the channel output in (3) is processed through a unit-energy matched[3] filter, then the output y[n] at symbol time n in complex baseband equivalent form can be written as

$$y[n] = h[n]s[n] + w[n], \tag{4}$$

where $s[n]$ is the complex channel input symbol at signaling interval n. Assuming that the symbol rate is kept fixed at a value that corresponds to the worst channel conditions i.e., the maximum Doppler frequency, the other parameters in (3) and (4) are related in the following way:

$$y[n] = \frac{1}{\sqrt{T_s}} \int_{nT_s}^{(n+1)T_s} y(t)dt,$$
$$w[n] = \frac{1}{\sqrt{T_s}} \int_{nT_s}^{(n+1)T_s} w(t)dt,$$
$$h[n] = K \int_{nT_s}^{(n+1)T_s} h(t)dt,$$

where K in the last expression is a normalization constant to make variance of $\{h[n]\}$ equal to unity. As a result of discretization, the output, noise and channel spectra are scaled and 2π periodic[4] versions of the original spectra (see e.g., [6]). Since we are interested in the complex baseband equivalent model of (4), all replicas of the spectrum are discarded except in the fundamental period of $[-\pi, \pi]$. The discrete-time sequences $y[n]$, $w[n]$ and $h[n]$ retain, however, the stochastic properties of their continuous time counterparts. This point will further be discussed later in this Section.

We consider a mobile moving with velocity $\boldsymbol{v}_0$ and acceleration α be at the origin 0 at the signaling instant 0 (Fig. 1). Thus, at the signaling interval n, the mobile will be at the point (Λ_1, ψ) with respect to 0, where ψ is the direction of the mobile with respect to x-axis and, using the fact that velocity and the acceleration are aligned, $\Lambda_1 = v_0 nT_s + \frac{1}{2}\alpha(nT_s)^2$ in the direction of $\hat{\boldsymbol{v}}$. The channel gain at n^{th} symbol interval will be given by

$$h[n] = \oint g(\phi) e^{-i\kappa\Lambda_1 \hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{\phi}}} \mathrm{d}\phi, \tag{5}$$

where $\kappa = 2\pi/\lambda$ is the free space phase constant dependent on the carrier wave length, λ, $g(\phi)$ is the effective random scattering gain of the received signal from an angle ϕ, $\hat{\boldsymbol{\phi}}$ represents the unit vector in the direction of ϕ.

[1]We assume the acceleration to be aligned with velocity. The term 'acceleration' may, therefore, be interpreted as the magnitude of the acceleration.

[2]This assumption is necessary, and reasonable due to the physical limitations, because we will be considering a discrete time model so that we can analyze the impact of acceleration in terms of some practical parameters like symbol rate, normalized Doppler spread (and normalized Doppler frequency. The analysis given in this paper can be carried out in the continuous time domain.

[3]Since the spectrum of the channel is now time-varying due to the acceleration of the mobile receiver, it must be ensured that the symbol rate is greater than or equal to the Nyquist rate corresponding to the instantaneous channel spectrum. It can be done either by adapting the symbol rate to the current Doppler spectrum or transmitting at a fixed symbol rate that is greater than or equal to the Nyquist sampling rate corresponding to the worst case scenario.)

[4]The white noise $w(t)$ is an obvious exception because it is wideband with all frequencies present with equal strength. The discretization, therefore, results in the scaling of the spectrum only.

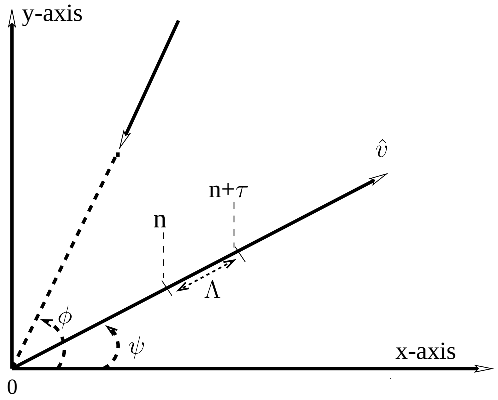


Fig. 1. The mobile is moving at an angle ψ with respect to x-axis with initial velocity v_0 at the origin and constant acceleration α in the direction of movement. A plane wave is shown incident on the receive antenna at an angle ϕ with respect to x-axis. Λ, given in (10), equals the difference of the distances covered by the mobile at $n+\tau$ and n signaling intervals with respect to the signaling instant 0.

Using (5), the normalized complex correlation between the channel gain at the signaling intervals n and $n+\tau$ (τ represents the lag in number of symbols) is given as

$$\begin{aligned}\Gamma(n,\tau) &= E\left\{h[n+\tau]h^*[n]\right\},\\ &= \oint\oint E\left\{g(\phi')g^*(\phi)\right\}\, e^{-i\kappa\Lambda_1\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{\phi}}}e^{i\kappa\Lambda_2\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{\phi}}'}\mathrm{d}\phi\mathrm{d}\phi',\end{aligned} \tag{6}$$

where $E\{\cdot\}$ stands for ensemble averaging over all possible situations implied by the assumed statistics of ϕ , * represents the complex conjugate, $\hat{\boldsymbol{v}}$ is the unit vector in the direction of mobile direction of travel and

$$\Lambda_2 = v_0(n+\tau)T_s + \frac{1}{2}\alpha\left((n+\tau)T_s\right)^2, \tag{7}$$

is the distance covered by the mobile, with respect to the signaling instant 0, at signaling instant $n+\tau$. If we assume that the scattering gains from two distinct directions are uncorrelated and are zero mean i.e., the channel is homogeneous, then [7]

$$E\left\{g(\phi)g^*(\phi'\right\} = E\left\{|g(\phi)|^2\right\}\delta(\phi-\phi'). \tag{8}$$

Using (8), we can write (6) as

$$\Gamma(n,\tau) = \oint G(\phi)e^{-i\kappa\Lambda\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{\phi}}}\mathrm{d}\phi, \tag{9}$$

where $G(\phi) = E\left\{|g(\phi)|^2\right\}$ is the angular power distribution [8] of the received signal and

$$\begin{aligned}\Lambda &= \Lambda_2 - \Lambda_1,\\ &= v_0\tau T_s + \frac{1}{2}\alpha(\tau T_s)^2 + \alpha(\tau T_s)(nT_s).\end{aligned} \tag{10}$$

It can be observed that the complex channel autocorrelation in (9) not only depends on the time difference τ between samples but the absolute time n as well. It can also be observed that if the mobile were moving with constant velocity ($\alpha = 0$), Λ in (10) would reduce to $\tau T_s v_0$ implying that the channel autocorrelation is only a function of the lag variable τ and the channel process would have been wide sense stationary (WSS).

Notice that $G(\phi)$ is periodic in ϕ. It can, therefore, be expanded using a Fourier series with orthogonal circular harmonics as the basis set. Let us define

$$\beta_m = \oint G(\phi)\, e^{-im\phi}\,\mathrm{d}\phi, \tag{11a}$$

$$G(\phi) = \frac{1}{2\pi}\sum_{m=-\infty}^{\infty}\beta_m\, e^{im\phi}, \tag{11b}$$

where β_m are the coefficients of the Fourier series expansion of $G(\phi)$.

The factor $e^{-i\kappa\Lambda\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{\phi}}}$ in (9) can be expanded using the Jacobi-Anger expansion [9] as

$$e^{-i\kappa\Lambda\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{\phi}}} = \sum_{m=-\infty}^{\infty} i^m\, J_m(\kappa\Lambda)e^{im(\phi-\psi)}, \tag{12}$$

where ϕ and ψ both are measured with respect to x-axis (Fig. 1) and $J_m(\cdot)$ is the Bessel function of integer order m. Let us define the following:

$$\omega_{d,0} = \eta v_0, \tag{13a}$$

$$\omega_d^\tau = \eta\,\alpha\,\tau\,T_s\,, \tag{13b}$$

where $\omega_{d,0}$ is the maximum angular Doppler spread at signaling instant 0, and ω_d^τ is the maximum Doppler spread at lag τ from the current time index n. Combining equations (9), (11a), (12), (13a) and (13b), after some simplification, we get

$$\Gamma(n,\tau) = \sum_{m=-\infty}^{\infty} i^m\beta_m J_m(\tau T_s\omega_{d,0} + \tau T_s 2\omega_d^\tau + nT_s\omega_d^\tau)\, e^{-im\psi}. \tag{14}$$

Equation (14) is the average instantaneous[5] complex autocorrelation function of the channel fading process as a function of lag τ and the absolute time index n. In fact, it represents the sum of contributions from all harmonics of the scattering distribution. We have assumed that the scattering environment remains fixed and, therefore, independent of the absolute time n so that the Fourier coefficients β_m depend only on a particular pdf of AOA and are independent of time, n. As can be observed from (14), the autocorrelation of the fading process, in general, not only depends on the distribution of AOA through β_m but also on the direction of mobile travel.

We make use of the following addition theorem for Bessel functions [10] twice

$$J_m(x_1+x_2) = \sum_{k=-\infty}^{\infty} J_k(x_1)J_{m-k}(x_2), \tag{15}$$

to rewrite (14) as

$$\begin{aligned}\Gamma(n,\tau) = &\sum_{m=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} i^m\beta_m J_p(\tau T_s\omega_{d,0})\\ &J_{k-p}(\tau T_s\omega_{d,1}^\tau/2)\, J_{m-k}(nT_s\omega_{d,1}^\tau)e^{-im\psi},\end{aligned} \tag{16}$$

[5]The autocorrelation is average in the sense that expectation is taken over all possible situations implied by the assumed statistics of the AOA ((6)), and instantaneous in the sense that it is dependent on the absolute time index n ((14)).

$$\Gamma(\nu,\omega) = \sum_{m=-\infty}^{\infty}\sum_{p=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}\sum_{\tau=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} i^m\beta_m e^{-im\psi} J_p(\tau T_s\omega_{d,0})J_{p-k}(\tau T_s 2\omega_d^\tau)J_{m-k}(nT_s\omega_d^\tau)\, e^{-iT_s(\omega\tau+\nu n)} \ldots\ldots (A)$$

$$\Gamma_{\rm iso}(\nu,\omega) = \sum_{p=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}\sum_{\tau=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} J_p(\tau T_s\omega_{d,0})J_{p-k}(\tau T_s 2\omega_d^\tau)J_{-k}(nT_s\omega_d^\tau)\, e^{-iT_s(\omega\tau+\nu n)} \ldots\ldots (B)$$

where $J_p(\cdot)$ does not contain the acceleration factor, $J_{k-p}(\cdot)$ shows the effect of acceleration with τ only and $J_{m-k}(\cdot)$ contains the absolute time index n and τ.

The discrete-time Fourier transform (DTFT) of (16) with respect to τ gives the instantaneous PSD of the process in the conventional sense (Wiener-Khintchine theorem [11])

$$\Gamma(n,\omega) = \sum_{m=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}\sum_{p=-\infty}^{\infty}\sum_{\tau=-\infty}^{\infty} i^m\beta_m e^{-im\psi}J_p(\tau T_s\omega_{d,0}) J_{k-p}(\tau T_s 2\omega_d^\tau)\, J_{m-k}(nT_s\omega_d^\tau)e^{-i\omega\tau Ts}, \tag{17}$$

and Fourier transformation with respect n gives the spectrum of the time variation

$$\Gamma(\nu,\tau) = \sum_{m=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}\sum_{p=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} i^m\beta_m e^{-im\psi}J_p(\tau T_s\omega_{d,0}) J_{k-p}(\tau T_s 2\omega_d^\tau)\, J_{m-k}(nT_s\omega_d^\tau)e^{-i\nu nTs}. \tag{18}$$

The 2-D DTFT [6] of (16) given by the equation (A) at the top of this page gives the joint spectrum of τ and n domains. Since the underlying channel fading process has been assumed to be zero-mean proper complex Gaussian, equations (16) and (A) completely characterize the statistics of the channel under general 2D (isotropic and non-isotropic) scattering environments with a valid pdf of AOA.

The Bessel function of order n, $J_n(x)$, starts small and reaches to its maximum at arguments $x \approx O(n)$ before starts decaying slowly. It was shown in [13] that $J_n(x) \approx 0$ for $|n| > 2\lceil x/2\rceil + 1$ with $e = 2.7183\ldots$. Therefore, for finite τ and n, (14) and (16) always reduce to finite summations so as to give the exact autocorrelation in the closed form. PSDs in (17), (18) and (A), on the other hand, rely on some kind of numerical approximation because, even if one of the variables is kept constant, we have to perform infinite summation over the other to transform it to Fourier domain which is not possible.

As an application, we specialize (16) and (A), for the case of isotropic scattering environment for which [12] $\beta_m = 0$ for $m \neq 0$ and $\beta_0 = 1$ implying that the autocorrelation and PSD are no more dependent on the mobile direction of movement. We can write (16) as

$$\Gamma_{\rm iso}(n,\tau) = \sum_{k=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} J_p(\tau T_s\omega_{d,0}) \times J_{k-p}(\tau T_s 2\omega_d^\tau)\, J_{-k}(nT_s\omega_d^\tau) \tag{19}$$

and equation (A) as (B) given at the top of the page. If we do not apply the addition theorem of (15), the autocorrelation would simply be given by

$$\Gamma_{\rm iso}(n,\tau) = J_0(\tau T_s\omega_{d,0} + \tau T_s 2\omega_d^\tau + nT_s\omega_d^\tau), \tag{20}$$

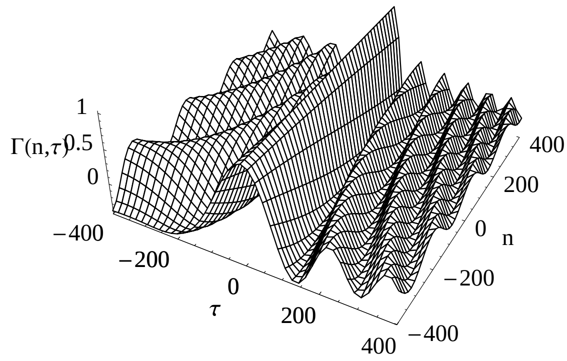


Fig. 2. The autocorrelation function as a function of the lag τ and time n. Increasing τ and n have the effect of reducing the autocorrelation faster.

and the conventional PSD would be given as

$$\Gamma_{\rm iso}(n,\omega) = \sum_{\tau=-\infty}^{\infty} J_0(\tau T_s\omega_{d,0} + \tau T_s 2\omega_d^\tau + nT_s\omega_d^\tau)e^{-iT_s\tau\omega}, \tag{21}$$

and PSD of time variation would be

$$\Gamma_{\rm iso}(\nu,\tau) = \sum_{n=-\infty}^{\infty} J_0(\tau T_s\omega_{d,0} + \tau T_s 2\omega_d^\tau + nT_s\omega_d^\tau)e^{-iT_s n\nu} \tag{22}$$

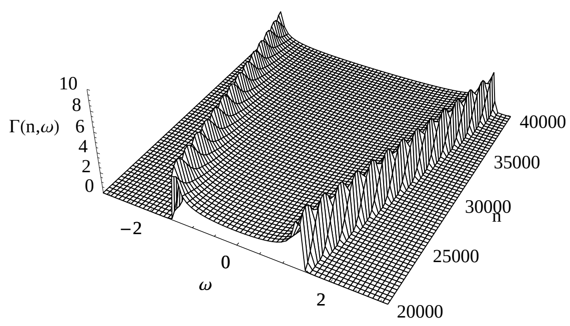
Equations (19) and (20) give the autocorrelation, and equations (21), (22) and (B) give different PSDs of the channel fading process for the case of isotropic scattering environment. It can easily be verified that with acceleration equal to zero ($\alpha = 0$), the above two equations collapse to the well-known Clarke's model.

The multiplication in the time domain implies convolution in the frequency domain [6]. Equation (B) can therefore be written as

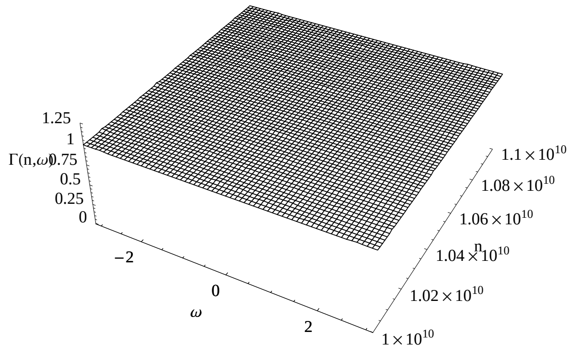
$$\Gamma_{\rm iso}(\nu,\omega) = \sum_{p=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} \{\mathcal{F}_p \otimes \mathcal{F}_{p-k} \otimes \mathcal{F}_\nu, \}, \tag{23}$$

where $\otimes$ represents the periodic convolution[6], $\mathcal{F}_p$ is the Fourier transform of $J_p(\cdot)$, $\mathcal{F}_{p-k}$ is the Fourier transformation of $J_{k-p}(\cdot)$ and $\mathcal{F}_\nu$ represents the double Fourier transform of $J_{-k}(nT_s\omega_{d,1}^\tau)$, first, with respect to n and , then, with respect to τ. The difficulty in finding a closed form solution to different equations for PSD is obvious. In section III we will resort to some numerical means to describe the autocorrelations and the PSD of the channel fading process.

[6]All Fourier transforms are DTFT. The periodic convolution of these DTFTs would result in a 2D 2π periodic DTFT.



(a) The effect of constant acceleration on PSD



(b) The PSD for large n

Fig. 3. The effect of the mobile acceleration on the PSD of the fading process. Figure (a) shows that the effect of acceleration on the maximum Doppler spread and Figure (b) shows the convergence of PSD to a uniform distribution in the limit of large n.

III. NUMERICAL RESULTS AND DISCUSSION

In order to gain insight into different phenomena due to the mobile moving with constant acceleration in an isotropic scattering environment, we use the simulation parameters given in the following table:

Carrier frequency, f_c	2 GHz
Carrier wavelength, λ	0.15 m
Free Space phase constant, η	0.94 rad/m
Symbol Time, T_s	2.65 $\times 10^{-3}$ sec
Initial Velocity, v_0	20 m/s
Acceleration, α	10 m/s/s

The effect of acceleration in the sense of conventional PSD (Wiener-Khintchine theorem) is shown in Fig. 3. Notice that the maximum normalized Doppler spread is increasing linearly with the time index n unlike the classical Clarke's case of constant mobile velocity where the maximum Doppler spread (and, more precisely, the whole PSD) is constant over time. We emphasize that this is an estimate of the PSD given in (17) for the case of isotropic scattering environment. Only finite number of terms of infinite summation over τ were considered (Blackman-Tukey algorithm) using Hamming window. For fixed n, increasing the number of terms in the summation over τ has the effect of increasing the resolution of the ω axis. Since the statistics of the channel under consideration are non-stationary, the terms at large lag, τ, are of little interest. The PSD shown in 3a can well, therefore, be thought of as instantaneous PSD.

Fig. 3b shows the effect of increasing n for a fixed maximum τ. It can be observed that the PSD tends to be approximately uniform over $[-\pi, \pi]$ on ω (Normalized Doppler Spread) axis for large n. This is because as the time index increases, the mobile instantaneous velocity also increases due to the acceleration of the mobile. Since the instantaneous autocorrelation is inversely proportional to the instantaneous velocity ((14)), the autocorrelation becomes increasingly smaller. This trend of decrease in the autocorrelation continues till the time when the channel becomes uncorrelated with every other symbol and, thus, Bessel function becomes a $\delta\ (\tau)$ function in (21). All summation terms over τ would be zero except the zeroth term. Fourier transformation of $\delta(\tau)$ with respect to τ is equal to 1. This explains the result in Fig. 3b for large n. One may be tempted to think that this result is independent of n. The autocorrelation becomes a delta function only at some large (perhaps infinite) value of n. The factor n is therefore included indirectly in the foregoing reasoning. This is in contrast to the stationary case of Clarke [2] where the correlation is independent of time and, therefore, the foregoing reasoning does not apply. It is interesting to note that uniform PSD is obtained in an isotropic environment in 3D [14–16] when the scattered power is uniformly distributed over a sphere. The convergence of the PSD in Fig. 3b to a uniform distribution, therefore, implies that the long term effect of acceleration in a 2D scattering environment is to make it look like 3D environment. It is somewhat similar to the effect observed in [3] for the case of stationary transmitter and receiver with moving scatterers.

The spectral density along ν axis that corresponds to the spectrum of the variation of the channel autocorrelation with time is given in Fig. 4a. The linear increase in the maximum normalized Doppler spread can be observed which is similar to what is observed in Fig. 3a. The effect of increasing the lag τ on the spectrum along ν axis is shown in Fig. 4b. Two observations can be made from Fig. 4b. Firstly, there is some periodicity in the PSD pattern along τ axis (similar behavior is present, though not shown, in case of conventional PSD). Secondly, there is a gradual decrease in the strengths of the maximum normalized Doppler spread peaks along τ axis. In the limit of large τ, the PSD on ν axis approaches zero (it has been verified by the authors). This behavior can be explained if we consider (20) for the case of isotropic scattering. The argument of the Bessel function of order zero is

$$x = \tau T_s \omega_{d,0} + \frac{\eta}{2}\alpha(\tau T_s)^2 + \eta\alpha(nT_s)(\tau Ts). \qquad (24)$$

The following relationships for Bessel function of order zero with argument x exist [17]:

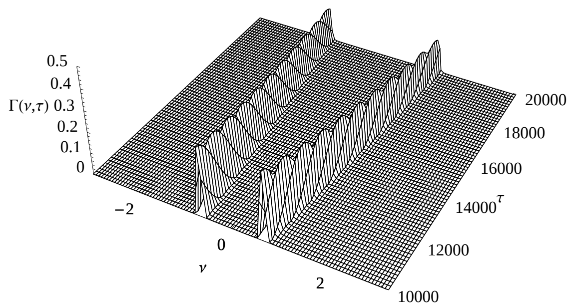
$$J_0(x) \approx \begin{cases} 1, & x \ll 1 \\ \sqrt{\frac{2}{\pi x}}\cos(x - \frac{\pi}{4}), & x \gg 1, \end{cases} \tag{25}$$

For values of τ and n such that $x \ll 1$ keeping other parameters fixed, the autocorrelation in (20) would, therefore, essentially be one. Fourier transform of 1 with respect either to τ or n (or both) is a $\delta(\cdot)$ function at the origin. This approximation is valid for short distances over which the effect of acceleration can almost be ignored. This approximation determines the extent of validity of quasi-stationarity assumption usually employed while evaluating the performance of different communication systems through non-stationary channels [18], and depends on the carrier frequency and symbol interval.

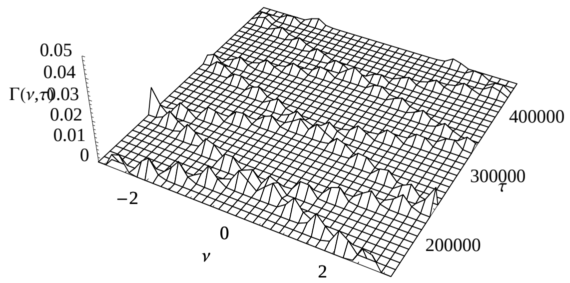
For the case of $x \gg 1$, the bessel function turns into a $\cos(\cdot)$ function with "dying" amplitude as in (25). Now the Fourier transform of the $\cos(\cdot)$ is a pair of impulses[7] located symmetrically around the origin at a distance equal to its frequency [6]. The spikes appearing in different figures are just due to this fact. Consistent with the approximation (25), the PSDs (though not shown for the purpose of brevity) start with impulses at the origin, split into decaying impulses at some time along both τ and n suggesting the emergence of some sinusoidal behavior. The periodic pattern occurs only along n when range of τ is fixed ((21)), and along τ axis when n is fixed ((22)). For fixed τ, changing n only changes the argument of the Bessel function, and a change of τ for fixed n again changes the argument of the Bessel function. Mathematically, therefore, this periodicity seems to be due to the periodic functions involved.

Apart from periodicities that require the help from mathematics to be explained, the gradual decrease in the magnitude of PSD along ν axis to zero along τ can be explained intuitively. First, it is important to point out that a window of finite length was used on the n axis. For increasingly larger lag τ, the autocorrelation becomes increasingly smaller. In other words, at larger τ, the information provided by current time index n becomes increasingly less important. We can therefore move around the current index n (to an extent that depends obviously on current time index n and the lag τ) without observing any noticeable changes in the correlation values at large lag, τ. The more away we are from the current time index, the less the changes we would observe in correlation due to a shift of the time index. In the light of these arguments, the result shown in Fig. 4b is justified: For the same observation window, larger lags will be less sensitive to the channel variations along n axis.

[7]The frequency is the first derivative of the phase. It is not hard to see from (24) that the frequency is a linear function of τ for fixed n and is constant for all n with τ fixed. Therefore, for $x \gg 1$, (21) would involve summation of $\cos$ functions of different frequencies because frequency is different for different τ. On the other hand, equation (22) would involve the summation of $\cos$ functions of the same frequency. This different behavior for τ and n may be apparent from comparison of 3a and 4a where somewhat bowl-shaped behavior in the former is different from the later. The presence of well-defined impulses in 4a is suggestive of the fact that there is some sinusoidal function only.



(a) The spectrum of time variation of the channel versus lag, τ



(b) The spectrum of time variation for large τ

Fig. 4. The PSD of the channel variation with time. Figure (a) gives an idea of the effect of acceleration on the spectrum of variation of channel in the time domain versus τ. Figure (b) shows the spectrum for large τ. Even though there is periodicity of the spectrum along the τ axis, a gradual decrease in the strengths of spikes (impulses) is evident suggesting that larger lags are more insensitive to the time variation of the channel.

IV. Conclusions

The effect of mobile acceleration on the statistics of the complex Gaussian fading process was analyzed. We showed that, under certain assumptions, the classical Clarke's model that assumes constant mobile velocity (zero acceleration) extends naturally to the case of mobile moving with some constant acceleration. It turned out that the autocorrelation of the channel now becomes time and lag dependent. The PSD of the channel process in the conventional sense i.e., the Fourier transformation of the autocorrelation with respect to the lag variable τ and the PSD of the channel variation with time n are affected due to the mobile acceleration in somewhat similar manner: The maximum Doppler spread in both domains, ω corresponding to τ and ν corresponding to the time n, increases and decreases with n and τ, respectively, in a periodic manner. However, the strength of the Doppler spectrum diminishes with n and τ. The conventional Doppler spectrum converges to the uniform distribution over 0 to 2π whereas the Doppler spectrum due to the time variation approaches zero. The results were given using simulations and justified analytically, where ever possible.

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