Information Rates of Time-Varying Rayleigh Fading Channels in Non-Isotropic Scattering Environments

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Abstract — **In this contribution a temporally correlated rayleigh fading channel model applicable in general scattering environments, with Clarke's isotropic scattering environment as a special case, is considered. For a fixed direction of mobile travel, the results show that the information rate penalty incurred for not knowing the channel state information (CSI) in a non-isotropic scattering environment can be significantly less than that for the isotropic scattering environment for the same average received signal-to-noise power ratio (SNR). The results show that for a fixed mean angle of arrival, fading rate and direction of mobile travel, higher information rates are achievable in case of more non-isotropic scattering environment. The results are presented for different non-isotropic scattering environments in terms of the normalized fading rate and SNR, and are compared with those for isotropic scattering environments**

I. Introduction

In real world mobile communication scenarios either the transmitter and/or the receiver may be in motion. A mobile-radio situation in which transmitter is fixed in position while the receiver is moving, usually in such a way that the direct line between transmitter and receiver is obstructed by buildings, is more common. The amplitude fluctuations of the received signal in this communications scenario have been shown to follow Rayleigh distribution [4]. The receiver mobility manifests itself in small scale (on the scale of milliseconds) severe fading (signal level may drop to as low as -40 dB) [7]. The channel estimate on the basis of such deeply faded signal is supposed to be quite unreliable. In addition, in a high mobility environment the rate of channel variation might be too fast to limit enough observation period for reliable channel estimates. It is, therefore, of particular interest to explore the achievable information rates without CSI in a mobile Rayleigh fading channel.

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When the receiver has perfect CSI the capacity achieving distribution is Gaussian. If, in addition, the transmitter has CSI, the Gaussian distribution of the input alphabet is still optimal and capacity is achieved using the principle of waterfilling. In the absence of CSI, the fading channel capacity problem has been studied for some simplified channel models e.g., finite state markov channel model[13], discrete time memoryless channel model[11], [14], block Rayleigh fading channel[10], [15]. A detailed survey of capacity of fading channels can also be found in [2].

The problem of achievable information rates in a time-correlated Rayleigh fading channel without CSI has recently been considered in [16] and [5]. While [16] considers the problem in a more general setting allowing for different scattering environments, the results in [5] are based on the assumption of isotropic scattering environment around the mobile receive antenna. Bounds on achievable rates for constant power and Gaussian signaling were derived. Some useful asymptotic results were also given for information rate penalty, and achievable rates, when there is no CSI either at the receiver or the transmitter. It has been argued and experimentally demonstrated (see [1] and references therein) that the scattering encountered in many suburban and rural environments is non-isotropic. The non-isotropicity of the scattering might well result if directional antennas with non-uniform gain patterns are employed at the receiver. It is, therefore, of particular interest to explore the achievable information rates without CSI in a mobile Rayleigh fading channel when the scattering environment is non-isotropic. The purpose of this paper is to extend the results of [5] from isotropic scattering to general scattering environments.

We emphasize that the purpose of this paper is to complement the work of [5] to cover realistic non-isotropic (and isotropic) communication scenarios for design of practical communication systems. The main assumptions in this work are that the mobile antenna is isotropic but the scattering is non-isotropic (or isotropic), and the received average SNR is the same as would be if the scattering were isotropic.

Main contributions of this paper are as follows:

1. For a fixed direction of mobile travel, we show that the information rate penalty for unknown CSI in

non-isotropic scattering environments can be significantly less than the penalty in an isotropic scattering environment. The greater the degree of non-isotropicity, the lower the information rate cost for unknown CSI and, hence, higher the achievable information rates (Figs. 5 and 6). The autocorrelation and power spectral density (PSD) plots (Figs. 2 and 3) for different scattering scenarios help explain the reason behind less penalty for more non-isotropic scattering environments and vice versa.

2. The Clarke's model of 2D isotropic scattering is a special case of the generalized model being considered. The results presented in [5] for isotropic scattering distribution, therefore, are a special case of the generalized model.

3. The generalized channel model (equations (3) and (7)) given without proof due to paper length restriction can be used to analyze the interplay between different parameters i.e., the mobile direction of travel, the degree of non-isotropicity and the channel dynamics for arbitrary scattering distributions. Moreover, using the generalized model we can test whether the assumption of ergodicity of the channel fading process in general (unimodal) scattering environments is valid[1] (Fig. 3).

Throughout the paper, the following notation will be used: Bold lower (upper) letters denote vectors (matrices). * and ^{T} denote the conjugate transpose, and transpose, respectively. The symbol $\mathbb{C}$ denotes the unit circle. The log function is the natural logarithm so that the information rates and differential entropies are expressed in nats/symbol. The notation $E\{\cdot\}$ denotes the mathematical expectation and the matrix $\boldsymbol{I}$ is the $N \times N$ identity matrix.

II. CHANNEL MODEL

We consider a downlink SISO transmission system where the transmitter is stationary while the receiver is moving with some velocity $\boldsymbol{v}$. We consider the transmission of a sequence of N symbols, $\boldsymbol{x} = [x_1, x_2, \cdots, x_N]^T$. The received N$\times$1 vector $\mathbf{y}$ can be written in discrete-time complex baseband form as

$$\mathbf{y} = \sqrt{\rho}\, \boldsymbol{X}\boldsymbol{h} + \boldsymbol{n}, \tag{1}$$

where $\boldsymbol{X} = \mathrm{diag}(x_1, x_2, \cdots, x_N)$ is the diagonal matrix of average power constrained, $E\left\{|x_j|^2\right\} = 1$, transmitted symbols, $\boldsymbol{n} = [n_1, n_2, \cdots, n_N]^T$ is N-dimensional noise vector with zero mean vector and covariance matrix $\boldsymbol{I}$ and $\boldsymbol{h} = [h_1, h_2, \cdots, h_N]^T$ represents the samples of a band-limited, flat-fading (frequency non-selective) wide-sense stationary, zero-mean complex Gaussian process with Toeplitz positive semidefinite covariance matrix $\boldsymbol{C} \triangleq E\{\boldsymbol{h}\boldsymbol{h}^*\}$. Equivalently, the magnitude of the fading process is Rayleigh distributed and the phase is rectangularly distributed over $[0, 2\pi]$. The factor ρ is, then, the average received SNR.

Let $(j+k, j)$ entry of the channel covariance matrix be expressed as

$$[\boldsymbol{C}]_{j+k,j} = E\left\{h_{j+k}h_j^*\right\}. \tag{2}$$

It can be shown that the autocorrelation function of the channel fading process in a general (unimodal i.e., the scattering distribution has only one mode (dominant scatterer)) scattering environment is given by [2] for SISO as

$$\begin{aligned}\Phi(k) &= \sum_{n=-\infty}^{\infty} \gamma_n J_n(\eta v k T_s) e^{-in(\phi_v + \frac{\pi}{2})}, \\ &= \sum_{n=-\infty}^{\infty} \gamma_n J_n(\omega_d k T_s) e^{-in(\phi_v + \frac{\pi}{2})} = [\boldsymbol{C}]_{j+k,j}, \end{aligned} \tag{3}$$

where $J_n(\cdot)$ is the Bessel function of integer order n, T_s is the symbol time, $\omega_d = \eta v$ is the is the maximum Doppler spread with $\eta = 2\pi/\lambda$ being the free space phase constant and λ being the carrier wavelength, v is the magnitude of velocity of the mobile, and ϕ_v is the angle between the mobile direction of travel and the x-axis (Fig.1) and the coefficients [3] γ_n termed as scattering coefficients, are defined as

$$\gamma_n = \int_{\mathbb{C}} \Psi(\beta)\, e^{in\beta}\, \mathrm{d}\beta, \tag{4}$$

where $\Psi(\beta)$ is the average received power from angle of arrival β at the receive aperture.

Remarks:

1. Since we have assumed the channel fading process to be wide-sense stationary[4], the autocorrelation function, $\Phi(k)$, in (3) is only a function of the time lag k between the samples of the channel fading process.

2. The function $\Psi(\beta)$, termed as *Angular Power distribution*[8], is normalized such that

$$\int_{\mathbb{C}} \Psi(\beta)\mathrm{d}\beta = 1. \tag{5}$$

In the literature, mathematically convenient distributions such as Von-Mises, truncated Laplacian, truncated Gaussian distributions have been used to

[1] For a channel fading process to be ergodic, the spectrum of the fading process must be continuous[3] in the range $-\pi \le \omega \le \pi$.

[2] We assume that the channel fading process is sampled at least at the Nyquist rate.

[3] For isotropic power distribution, $\gamma_n = 0$ for $n \neq 0$

[4] Since the channel fading process is complex Gaussian, the assumption of wide-sense stationarity implies stationarity in our case.

model $\Psi(\beta)$. In [18], the Fourier series is used to expand an arbitrary $\Psi(\beta)$, i.e,

$$\Psi(\beta) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \gamma_n \, e^{-in\beta} \tag{6}$$

and γ_n for popular distributions were given. Note that (4) and (6) form a Fourier transform pair.

By taking the discrete-time Fourier transform (DTFT) of (3), it can be shown that the PSD of the fading process is given as

$$\Phi(\omega) = \frac{1}{\omega_D} \sum_{n=-\infty}^{\infty} \gamma_n \, e^{-in(\phi_v+\pi)} \, F_n\left(\frac{\omega}{\omega_D}\right), \tag{7}$$

where [17]

$$F_\mu(x) \triangleq 2\frac{\cos(\mu \cos^{-1}(x))}{\sqrt{1-x^2}}, \tag{8}$$

and $\omega_{\mathrm{D}} = \omega_d T_s = \omega_d/(1/T_s)$ is the maximum Doppler spread normalized by the symbol rate, and $f_{\mathrm{D}} = \omega_{\mathrm{D}}/2\pi = \omega_d T_s/2\pi$ is the normalized maximum Doppler frequency which is also called the normalized fading rate. Equation (7) gives the distribution of power among different modes of the distribution function as a function of doppler frequency and may be called *Power Modal Spectral Density.*

III. Clarke's model as a special case of the generalized model

Consider the scattering scenario (Fig. 1) in which the scattered power (or the probability of angles of arrival) is uniformly distributed over a sector with mean β_0 and the maximum deviation[5] Δ_r on each side of the mean, i.e.,

$$\Psi(\beta) = \begin{cases} \frac{1}{2\triangle_r}, & |\beta - \beta_0| \le \triangle_r; \\ 0, & \text{otherwise,} \end{cases} \tag{9}$$

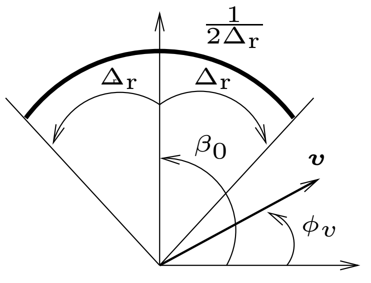

Fig. 1: Uniform-limited scattering scenario where the scattered power is uniformly distributed with magnitude $1/2\Delta_r$ over a circular sector with mean angle of arrival β_0 and maximum deviation of Δ_r on each side of the mean. The mobile is moving with velocity $\boldsymbol{v}$ at an angle of ϕ_v with x-axis.

[5]The relationship between angular spread, σ, and the maximum deviation about the mean, Δ_r is given by $\sigma = \Delta_r/\sqrt{3}$.

For this distribution, called uniform-limited power distribution[6], values of γ_n were derived in closed form in [18] and are given as

$$\gamma_n = e^{in\beta_0}\mathrm{sinc}(n\Delta_r). \tag{10}$$

Using (10) in (3) and (7), we can write

$$\Phi(k) = \sum_{n=-\infty}^{\infty} \mathrm{sinc}(n\Delta_r)\, J_n(\omega_{\mathrm{D}} k)\, e^{in(\beta_0-\phi_v-\pi/2)} \tag{11a}$$

$$\Phi(\omega) = \frac{1}{\omega_{\mathrm{D}}} \sum_{n=-\infty}^{\infty} \mathrm{sinc}(n\Delta_r)\, F_n\left(\frac{\omega}{\omega_{\mathrm{D}}}\right) \, e^{in(\beta_0-\phi_v-\pi)} \tag{11b}$$

where (11b) is a consequence of the fact that $\Psi(\beta)$ is real and $\gamma_n = \gamma_{-n}^*$. Notice that, in general, the autocorrelation is complex valued and depends on the mean angle of arrival (AOA) and the mobile direction of travel unlike isotropic scattering environments in which channel autocorrelation is strictly real valued and is independent of AOA or the direction of mobile travel.

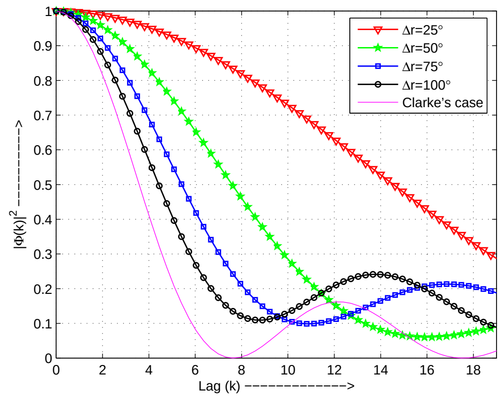

Fig. 2: The squared magnitude of the autocorrelation function of the channel fading process given in (11a) as a function of time lag k in number of symbols for different Δ_r when $\beta_0 = 90°$, $\phi_v = 45°$, SNR=10dB and $f_D = 0.05$.

Equations (11a) and (11b) give the second order statistics for different Δ_r corresponding to different scattering scenarios, and will be used, in the next section, in determining the information rate penalty and achievable information rates when no CSI is available at the transmitter and the receiver. The autocorrelation and PSD of the fading process based on (11a) and (11b) have been plotted for different non-isotropic scattering environments in Figs. 2 and 3, respectively.

In the limiting case of $\Delta_r = \pi$, which corresponds to the isotropic power distribution, (11a) and (11b) collapse,

[6]We have considered uniform-limited scattering scenario because it is easy to appreciate the equivalence of the Clarke's model to the generalized scattering model as a special case.

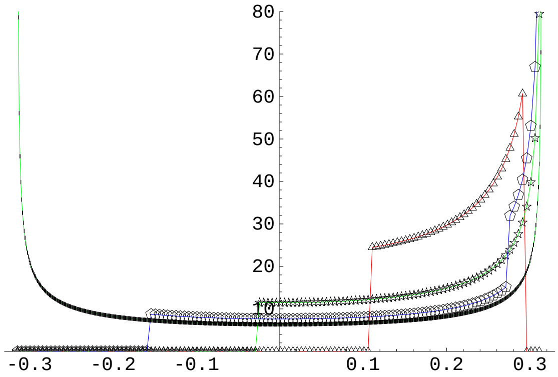

Fig. 3: The PSD of the channel fading process as a function of the doppler frequency for different Δ_r when $\beta_0 = 90°$, $\phi_v = 45°$, SNR=10dB and $f_{\rm D} = 0.05$. Horizontal axis is the normalized doppler spread (ω) in radians and the vertical axis is the magnitude of PSD in watts/Hz. Red-Triangle, Green-Star, Blue-Star and Green lines, respectively, correpond to the values of $\Delta_r = 25°, 50°, 75°, 180°$. These plots show the discontinuous spectrum of the fading process for each non-isotropic scenario and, hence, the assumption of ergodicity of the fading process can not valid [3].

respectively, to

$$\Phi(k) = J_0(2\pi f_{\rm D} k), \tag{12}$$

$$\Phi(\omega) = \frac{1}{\pi f_{\rm D}\left(1 - (f/f_{\rm D})^2\right)}, \tag{13}$$

where $|\omega| \leq 2\pi f_{\rm D}$ in (11b) and (13) as a consequence of the bandlimitedness of the channel fading process. Equations (12) and (13) are the well-known Clarke's 2D model for the autocorrelation and PSD of the fading process, respectively.

Since the autocorrelation and PSD of channel fading process in a non-isotropic scattering environment is dependent on the mobile direction of travel, it is of interest to find out the autocorrelation and PSD if the mobile direction of travel is averaged out. If the mobile direction of travel is equiprobable in all directions i.e., $p(\phi_v) = 1/2\pi$, then, from (11a), the average autocorrelation, $\Phi_{\rm avg}(k)$ is given by

$$\Phi_{\rm avg}(k) = \sum_{n=-\infty}^{\infty} \mathrm{sinc}(n\Delta_r)\, J_n(\omega_{\rm D} k) \times e^{in(\beta_0 - \pi/2)} \int_0^{2\pi} e^{-in\phi_v} \mathrm{d}\phi_v, \tag{14}$$

and using (11b), the average PSD, $\Phi_{\rm avg}(\omega)$, is given by

$$\Phi_{\rm avg}(k) = \frac{1}{\omega_{\rm D}} \sum_{n=-\infty}^{\infty} \mathrm{sinc}(n\Delta_r)\, F_n\left(\frac{\omega}{\omega_{\rm D}}\right) \times e^{in(\beta_0 - \pi/2)} \int_0^{2\pi} e^{-in\phi_v} \mathrm{d}\phi_v, \tag{15}$$

It is not hard to see that, irrespective of Δ_r and the β_0, the integrals in equations (14) and (15) are zero for all $n \neq 0$ so that these two equations converge, respectively, to (12) and (13) i.e., the Clarke's isotropic case.

IV. Information rates in non-isotropic scattering environments

In order to make this paper self-contained , we review in the following the information rate bounds derived in [5]. From equation (1), the mutual information, $I(\mathbf{y};\mathbf{x})$, between the output vector $\mathbf{y}$ and the input vector $\mathbf{x}$ can be expressed using the chain rule as follows

$$\begin{aligned} I(\mathbf{y};\mathbf{x}) &= I(\mathbf{y};\mathbf{x},\mathbf{h}) - I(\mathbf{y};\mathbf{h}|\mathbf{x}), && \text{(16a)}\\ &= I(\mathbf{y};\mathbf{x}|\mathbf{h}) - \{I(\mathbf{y};\mathbf{h}|\mathbf{x}) - I(\mathbf{y};\mathbf{h})\}, && \text{(16b)} \end{aligned}$$

where the first term in (16b) is the mutual information with perfect CSI and, therefore,

$$P_\delta \triangleq I(\mathbf{y};\mathbf{h}|\mathbf{x}) - I(\mathbf{y};\mathbf{h}), \tag{17}$$

is the penalty in information rate due to unknown CSI. Since $I(\mathbf{y};\mathbf{h})$ is nonegative,

$$P_\delta \leq I(\mathbf{y};\mathbf{h}|\mathbf{x}). \tag{18}$$

Thus $I(\mathbf{y};\mathbf{h}|\mathbf{x})$ is the upper bound on the penalty. Making use of the Jensens's inequality and the determinant identity, $\det(\boldsymbol{I} + \boldsymbol{AB}) = \det(\boldsymbol{I} + \boldsymbol{BA})$, it was shown in [5] that

$$\begin{aligned} P_\delta &\leq \mathrm{logdet}(\boldsymbol{I} + \rho\boldsymbol{C}), && \text{(19a)}\\ &= \sum_{i=1}^{N} \log\left(1 + \rho\lambda_i\right), && \text{(19b)} \end{aligned}$$

where $\lambda_i, i = 1, 2, \cdots, N$, are the eigenvalues of the covariance matrix $\boldsymbol{C}$ and equality holds in for constant power signaling (M-PSK signaling). As we have seen in section III that the non-isotropicity of the scattering environment affects the second order channel statistics and, hence, the eigenvalues of the channel correlation matrix, P_δ as given in (19b) will be different for different non-isotropic scattering environments.

Let $I_C(\mathbf{y};\boldsymbol{x})$ and $I_G(\mathbf{y};\boldsymbol{x})$ denote the achievable information rates without CSI for constant power and Gaussian signaling, respectively. Making use of (16a), (16b) and (19b), the following upper bound results for constant power signaling [5]

$$I_C(\mathbf{y},\mathbf{x}) \leq C_{\rm AWGN}(\rho) - P_\delta, \tag{20}$$

where $\mathrm{C}_{\rm AWGN}(x) \triangleq \log(1 + x)$. The Gaussian signaling lower bound is given as

$$I_G(\mathbf{y},\mathbf{x}) \geq C_{\rm Rayleigh}(\rho) - P_\delta, \tag{21}$$

where $C_{\rm Rayleigh}$ is the ergodic Rayleigh capacity [9] with perfect CSI given as

$$C_{\rm Rayleigh} \triangleq E_h \log\left(1 + x|h_n|^2\right) = -\exp\left(\frac{1}{\rho}\right) E_i\left(-\frac{1}{\rho}\right)$$

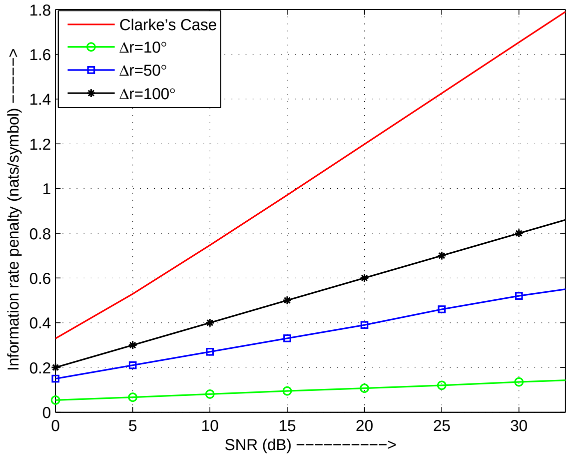

Fig. 4: Information Rate Penalty, P_δ, versus SNR for different Δ_r when $\beta_0 = 90°$, $\phi_v = 45°$, $f_{\mathrm{D}} = 0.05$ and infinite block length.

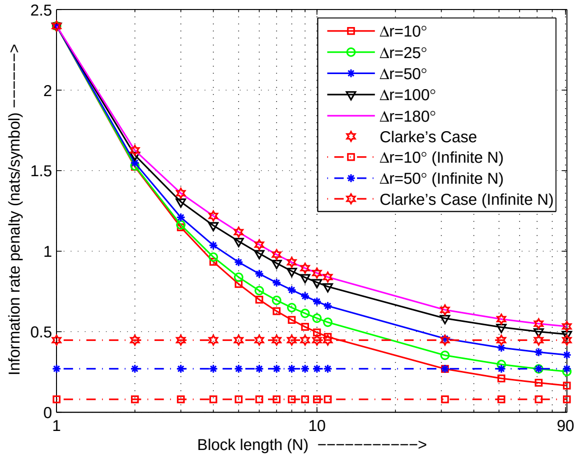

Fig. 5: Information Rate Penalty, P_δ, versus block length for different scattering scenarios when $\beta_0 = 90°$, $\phi_v = 45°$, SNR=10dB and $f_{\mathrm{D}} = 0.05$.

where E_i is the exponential integral. In this bound we can see that C_{Rayleigh} is the ergodic capacity and is independent of a particular scattering scenario.

A. Analysis of Information Rate Penalty and Numerical Results

As we can see that the information rate penalty for unknown CSI (19b) is a function of the block length and SNR, it is of information-theoretic interest to find out the penalty asymptotics for large (possibly infinite) block length and high (possibly infinite) SNR. From a practical standpoint, to find out the behavior of penalty with increasing transmission block length is of interest.

In order to find out the behavior of the penalty with increasing transmission block length in different non-isotropic environments, we consider four different uniform-limited scattering scenarios (keeping all other parameters fixed),i.e, $\Delta_r = \{10°, 25°, 50°, 180°\}$ from very non-isotropic to isotropic environment (Clarke's case), and then use (19b) and the autocorrelation function in (11a) to compute the penalty for each scattering scenario as a function of block length (Fig.5).

Since the penalty in (19b) is a non-increasing sequence in N (the block length), it has a limit as $N \to \infty$. In order to find that limit for different non-isotropic scattering scenarios under consideration, we make use of (11b) and the Szegö's limit theorem [6, 64-65] to get

$$\lim_{N\to\infty} P_\delta = \frac{1}{2\pi}\int_{-\pi}^{\pi} \log\left(1+\rho\,\Phi(\omega)\right)d\omega. \tag{22}$$

Since $\Phi(\omega)$ given in (11b) involves summation, it is not possible to find a closed form solution for P_δ due to the summation appearing within log in (22).

Fig.5 gives the behavior of penalty with increasing block length and the asymptotic penalty for each scattering scenario. The Clarke's case is also shown for comparison. In the limiting case of $\Delta_r = \pi$, the results for information rate penalty for finite and infinite block lengths are consistent with those for Clarke's reported in [5]. It can be observed from Fig. 5 that the information rate penalty for not knowing CSI in the non-isotropic mobile communications scenario under consideration is significantly less than that for isotropic scattering distribution.

Using (17), (20), (21) the achievable information rates have been plotted versus SNR in Fig.6 for two non-isotropic scattering scenarios along with the Clarke's isotropic environment. It can be seen that the information rate bounds in non-isotropic scattering environment are higher than those for the isotropic scattering environment, for the same fading rate and SNR.

V. Conclusion and Future Work

Information rates achievable without CSI in a time-varying Rayleigh fading channel were investigated for a particular non-isotropic scattering environment keeping the mobile direction of travel fixed. It was observed that greater degree of non-isotropicity of scattering distribution resulted in higher correlation of the channel fading process over time which in turn resulted in less cost for not knowing the CSI for the same block length and, hence, higher achievable rates. For any fixed scattering scenario and fading rate, the autocorrelation and PSD of the fading process are identical to those for the Clarke's case when the direction of mobile travel is averaged out. In other words, the achievable rates in a non-isotropic scattering scenario, with mobile direction of travel averaged out, are the same as for the isotropic scattering environment.

In this contribution we assumed that the mobile antenna is isotropic while the scattering environment is non-isotropic. If a directional antenna is employed at the receiver in an isotropic environment, there is an apparent

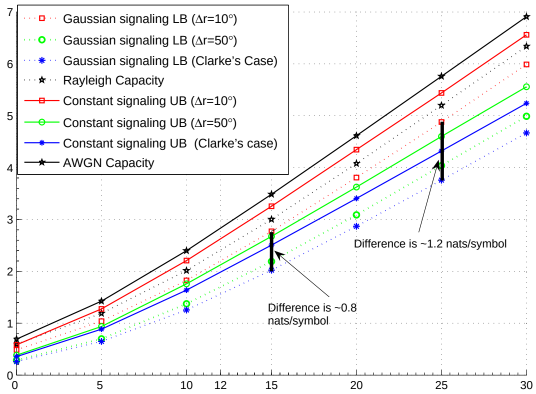

Fig. 6: Information rate bounds for Gaussian and Constant power signaling as a function of SNR when $\beta_0 = 90°$, $\phi_v = 45°$, SNR=10dB, $f_\mathrm{D} = 0.05$ and block length is infinite.

trade-off between the increased correlation of the fading process and reduced SNR (as only a portion of the available SNR is being intercepted) as seen by the receiver. It seems interesting to look for some optimal trade-off between these two parameters. Also unlike isotropic scattering environment in which the autocorrelation is real-valued, the autocorrelation in non-isotropic environments is complex valued. It also seems interesting to look for an answer to the question: Can we make use of the additional correlation dimension?

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