A Space-time Channel Simulator using Angular Power Distributions

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***Abstract*— In this paper, we develop a channel simulator to generate the channel gains to an arbitrary array of receiver antennas, for a general class of non-line-of-sight channels. The channel scattering environment is defined by the angular power distribution as seen by the receiver. We derive the second order statistics of the channel gains in terms of the parameters of the angular power distribution. As an illustration of the channel simulator, we compare the performance of different direction-of-arrival techniques.**

I. Introduction

With recent development of practical multiple-input-multiple-output (MIMO) systems, there is need to quantify MIMO system performance over realistic channels. Many options for channel models are now available. However many models either require ray-tracing, complicated parametrizations, or restrict simulation to a single array geometry. We present here a simulator for arbitrary array geometries that generates channel realizations from readily available angular power distribution data.

A number of schemes have been proposed for simulating MIMO channels. Several authors propose ray tracing models [1]. However for non-line-of-sight channels, the channel gains are dominated by their second order statistics [2]. To model the second order channel statistics, many use oversimplified models such as Rayleigh fading and Kronecker models [3] which poorly estimate capacity [4]. Others use higher complexity data-dependent models (e.g. [5], [6]) which learn statistical parameters from a MIMO data set or geometric models [7] based on parametrizations of the directional power distribution.

In this paper, we describe a simulator to realize non-line-of-sight channels directly from directional power distributions. The advantage of our model is that (i) arbitrary distributions of receivers can be simulated from the same set of data, (ii) low simulator complexity and fast simulator time stemming from an efficient parametrization of the channel. Here we present a 2-D model, but it extends naturally to 3-D. The simulator is used to predict performance of competing direction-of-arrival (DOA) algorithms to distributed sources.

In Section II, we describe a general channel model for uncorrelated scatterers. In Section III, we derive a low order parametrization for such a channel and derive parameter statistics. Section IV reviews common power distributions. Section V summarizes the structure of the proposed channel simulator. In Section VI, we simulate channels for several power distributions.

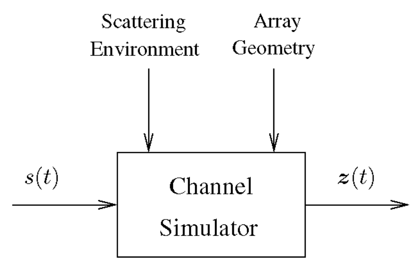

Fig. 1. Black box representation of the channel simulator.

II. Channel Model

Consider transmission of data from a transmitter over a flat fading channel to n_R receiver antennas. Let $\boldsymbol{x}_n$ be the nth receiver antenna position with respect to an arbitrary origin O (as shown in Fig. 2). Receiver antennas lie within a finite circular aperture $\mathbb{B}$ of radius R_R. The transmitter transmits a signal $s(t)$ over the time-varying channel with the transfer function $h_n(t)$ to receive a signal $z_n(t)$ at each antenna n. Collect transfer functions into vector $\boldsymbol{h}(t) \triangleq [h_1(t), h_2(t), \dots h_{n_R}(t)]^T$ where $\cdot^T$ is the vector transpose operator, and received signals $z_n(t)$ into vector $\boldsymbol{z}(t) \triangleq [z_1(t), z_2(t), \dots z_{n_R}(t)]^T$. Letting $\boldsymbol{w}(t) \triangleq [w_1(t), w_2(t), \dots w_{n_R}(t)]^T$ be additive white Gaussian noise at the receivers, we write:

$$\boldsymbol{z}(t) = \boldsymbol{h}(t)s(t) + \boldsymbol{w}(t). \tag{1}$$

Assume that all scatterers lie in the far-field. The scattering environment causes the transmitter signal to propagate in as plane waves with a different amplitude for each direction. Define the scattering gain $A(\phi, t)$ as the amplitude of the plane wave propagating in from direction ϕ at time t at the origin. We can then write:

$$h_n(t) = \int_0^{2\pi} A(\phi, t) e^{-ik\boldsymbol{x}_n \cdot \hat{\boldsymbol{\phi}}} d\phi, \tag{2}$$

where $\hat{\boldsymbol{\phi}}$ is the unit vector of polar coordinates $(1, \phi)$ and k is wave number. Assume slow fading so that the channel remains static over the symbol time. h_n and A are then not dependent on t over each symbol. For the remainder of the paper, the t-dependence is suppressed.

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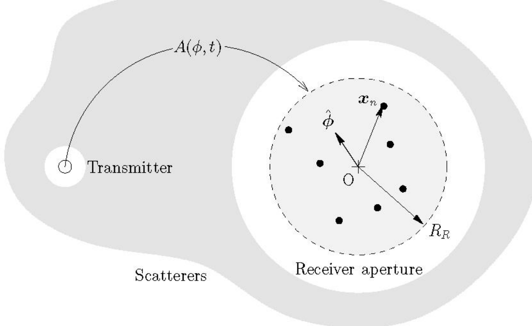

Fig. 2. The channel model. Scattering is modelled with the scattering gain $A(\phi, t)$. Each receiver is positioned at $\boldsymbol{x}_n$, within a circular aperture $\mathbb{B}$ shaped of radius R_R.

To investigate the statistics of non-line-of-sight scattering, $A(\phi)$ is assumed to be a zero mean Gaussian random variable at each angle ϕ. Further assume $A(\phi)$ uncorrelated between different angles:

$$E\{A(\phi)A^*(\varphi)\} = E\{|A(\phi)|^2\}\delta_{\phi\varphi}, \tag{3}$$

where $\delta_{\phi\varphi}$ is the Kronecker delta function and $\cdot^*$ is the complex conjugate. This assumption is referred to as the wide-sense stationary uncorrelated scatterer (WSSUS) assumption [8]. Without loss of generality, we assume $A(\phi)$ is normalized:

$$\int_0^{2\pi} E\{|A(\phi)|^2\}d\phi = 1.$$

The SIMO channel is then characterized by the power density of scatterers $\mathcal{P}(\phi)$ as seen by the receiver, defined by:

$$\mathcal{P}(\phi) \triangleq E\{|A(\phi)|^2\}. \tag{4}$$

$\mathcal{P}(\phi)$ is interpreted as the average energy arriving from the channel in direction ϕ. Statistics of $A(\phi)$ and hence $\boldsymbol{h}$ are dependent entirely upon $\mathcal{P}(\phi)$.

III. Channel Parametrization

We now derive a simple structure for the simulation of the above channel model. By transforming angular functions into the Fourier domain, we shall show how to represent the channel mixing vector $\boldsymbol{h}$ with a minimal number of parameters. We then derive the statistics of the Fourier parameters.

A. Fourier Expansion

Perform the Fourier expansion of $\mathcal{P}(\phi)$,

$$\mathcal{P}(\phi) = \frac{1}{2\pi}\sum_{m=-\infty}^{\infty} \gamma_m e^{im\phi},$$

where γ_m is the Fourier series coefficient of $\mathcal{P}(\phi)$,

$$\gamma_m = \int_0^{2\pi} \mathcal{P}(\phi)e^{-im\phi}d\phi. \tag{5}$$

Since $\mathcal{P}(\phi) \in \mathbb{R}$, we know that $\gamma_m^* = \gamma_{-m}$. Also note that $\mathcal{P}(\phi) \geq 0$ which corresponds to $2\sum_{m=1}^{\infty}|\gamma_m| < \gamma_0 = 1$.

Similarly write $A(\phi)$ as the Fourier expansion:

$$A(\phi) = \frac{1}{2\pi}\sum_{m=-\infty}^{\infty} \beta_m e^{im\phi}, \tag{6}$$

where β_m is the Fourier coefficient of $A(\phi)$,

$$\beta_m = \int_0^{2\pi} A(\phi)e^{-im\phi}d\phi. \tag{7}$$

Due to the limited size of $\mathbb{B}$, (6) can be truncated and from (2) each h_n can be written as a sum of a finite number of β_m coefficients. Drawing from [9], we state the following theorem:

Theorem (General SIMO Parametrization)
For a set of n_R receiver antennas, each positioned at $\boldsymbol{x}_n$ within the circular aperture of radius R_R, the vector of transfer functions $\boldsymbol{h}$ defined in SIMO model (1) can be decomposed:

$$\boldsymbol{h} = \boldsymbol{J_R}\boldsymbol{\beta}, \tag{8}$$

where $\boldsymbol{\beta} \triangleq [\beta_{-N_R}, \beta_{-N_R+1}, \ldots \beta_{N_R}]^T$ is a vector of the scattering function coefficients defined in (6), $\boldsymbol{J_R}$ is a function of the antenna positions,

$$\boldsymbol{J_R} \triangleq \begin{bmatrix} \mathcal{J}_{-N_R}(\boldsymbol{x}_1) & \ldots & \mathcal{J}_{N_R}(\boldsymbol{x}_1) \\ \mathcal{J}_{-N_R}(\boldsymbol{x}_2) & \ldots & \mathcal{J}_{N_R}(\boldsymbol{x}_2) \\ \vdots & \ddots & \vdots \\ \mathcal{J}_{-N_R}(\boldsymbol{x}_{n_R}) & \ldots & \mathcal{J}_{N_R}(\boldsymbol{x}_{n_R}), \end{bmatrix} \tag{9}$$

where for vector $\boldsymbol{x}$ defined in polar coordinates as (x, ϕ_x),

$$\mathcal{J}_m(\boldsymbol{x}) \triangleq i^m J_m(kx)e^{im\phi_x}, \tag{10}$$

and $J_m(\cdot)$ is the Bessel function of the first kind of order m and $N_R = \lceil ekR_R/2 \rceil$ is the dimensionality of the receiver aperture. □

This theorem is proved in the Appendix.

B. Statistical Relationships

We are interested in simulating the channel mixing vector $\boldsymbol{h}$. From (8), this can be done by realizing a random variable with the statistics of $\boldsymbol{\beta}$.

Since the scattering gain is zero mean Gaussian, the statistics of $\boldsymbol{\beta}$ are governed by its correlation matrix $\boldsymbol{R_\beta} \triangleq E\{\boldsymbol{\beta}\boldsymbol{\beta}^H\}$. From (7) each second order statistic $[\boldsymbol{R_\beta}]_{mm'} = E\{\beta_m\beta_{m'}^*\}$ can be calculated as:

$$E\{\beta_m\beta_{m'}^*\} = \int_0^{2\pi}\int_0^{2\pi} E\{A(\phi)A^*(\varphi)\}e^{-i(m\phi - m'\varphi)}d\phi d\varphi.$$

Applying the WSSUS property (3):

$$E\{\beta_m\beta_{m'}^*\} = \int_0^{2\pi} \mathcal{P}(\phi)e^{-i(m-m')\phi}d\phi, \tag{11}$$

from which we see by comparison with (5) that $E\{\beta_m\beta_{m'}^*\} = \gamma_{m-m'}$, from which is written:

Angular Power	$\mathcal{P}(\phi)$	γ_m
Jakes	$1/2\pi$	δ_{m0}
Uniform limited	$\begin{array}{ll} 1, & \lvert\phi-\varphi_0\rvert \le \Delta, \\ 0, & \text{otherwise.} \end{array}$	$e^{-m\varphi_0}\mathrm{sinc}(m\Delta)$
Von-Mises	$\begin{array}{ll} \frac{e^{\kappa\cos(\phi-\varphi_0)}}{2\pi I_0(\kappa)}, & \lvert\phi-\varphi_0\rvert \le \pi, \\ 0, & \text{otherwise.} \end{array}$	$e^{-m\varphi_0}\frac{I_{-m}(\kappa)}{I_0(\kappa)}$
Laplacian	$\begin{array}{ll} \frac{Q}{\sqrt{2}\pi}e^{-\sqrt{2}\lvert\phi-\varphi_0\rvert/\sigma}, & \lvert\phi-\varphi_0\rvert \le \pi, \\ 0, & \text{otherwise.} \end{array}$	$e^{-im\varphi_0}\frac{1-(-1)^{\lceil\frac{m}{2}\rceil}\xi F_m}{(1+\frac{\sigma^2 m^2}{2})(1-\xi)}$

TABLE I

POPULAR ANGULAR POWER DISTRIBUTIONS. $I_m(\cdot)$ IS THE MODIFIED BESSEL FUNCTION OF THE FIRST KIND. FOR THE LAPLACIAN DISTRIBUTION, $\xi = e^{-\pi/(\sqrt{2}\sigma)}$, $F_m = 1$ FOR m EVEN, $F_m = m\sigma\sqrt{2}$ FOR m ODD AND Q IS A NORMALIZATION CONSTANT.

Theorem (WSSUS Parameter Correlation)
In a WSSUS Gaussian channel possessing angular power distribution $\mathcal{P}(\phi)$, the correlation matrix of $\boldsymbol{\beta}$ defined in (7) possesses the Toeplitz structure:

$$\boldsymbol{R}_{\boldsymbol{\beta}} = \begin{bmatrix} \gamma_0 & \gamma_{-1} & \cdots & \gamma_{-2N_R} \\ \gamma_1 & \gamma_0 & \cdots & \gamma_{-(2N_R-1)} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{2N_R} & \gamma_{2N_R-1} & \cdots & \gamma_0 \end{bmatrix}, \tag{12}$$

where γ_m is defined in (5). □

By virtue of the $\gamma_{-m} = \gamma_m^*$ property, to completely describe the channel statistics for any geometry of antennas within $\mathbb{B}$, only $2N_R + 1$ complex parameters $\{\gamma_0, \ldots, \gamma_{2N_R+1}\}$ are required.

IV. Angular Power Distributions

In this section, we present some popular choices for the angular power distribution $\mathcal{P}(\phi)$ in the simulator. These distributions are parametrised by the mean direction φ_0 and an angular spreading parameter:

- Jakes model [10] where scatterers are isotropically arranged to yield rich scattering.
- Uniform limited angular distribution [11] where energy arrives uniformly from a restricted range of angles $(\varphi_0 - \Delta, \varphi_0 + \Delta)$.
- Von-Mises distribution [12] having degree of nonisotropy $\kappa > 0$.
- Laplacian distribution [13] with measure σ of angular spread.

The functional forms of angular power and corresponding γ_m coefficients are summarized in Table I. Derivation of these coefficients was made in [14].

V. Simulator

We now describe a simple simulator algorithm that outputs realizations of the channel mixing vector over a WSSUS channel. This simulator generates a set of received sensor data $\{\widetilde{\boldsymbol{z}}(n)\}_{n=1}^N$ statistically representative of the channel simulation parameters, namely the angular power distribution $\mathcal{P}(\phi)$, noise variance σ_w and array geometry $\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_{n_R}$. As summarized in Fig. 3, the procedure for producing the sensor data is:

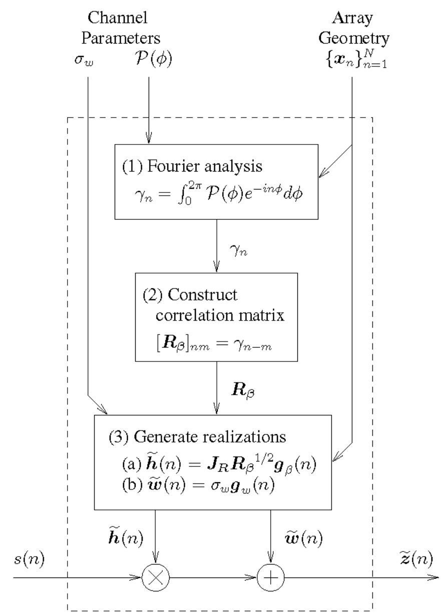

Fig. 3. Internal structure of the WSSUS channel simulator.

1) Calculate coefficients $\{\gamma_n\}_{n=-2N_R}^{N_R}$ from (5).
2) Construct the correlation matrix $\boldsymbol{R}_{\boldsymbol{\beta}}$ using (12).
3) Generate N realizations of $\boldsymbol{h}$ and $\boldsymbol{w}$, $\{\widetilde{\boldsymbol{h}}(n), \widetilde{\boldsymbol{w}}(n)\}_{n=1}^N$ from the channel correlation parameters.
4) Calculate signal realizations $\{\widetilde{\boldsymbol{z}}(n)\}_{n=1}^N$ from (1).

To perform step 3), we define the $n_R \times 1$ independent and identically distributed Gaussian random variables $\boldsymbol{g}_{\boldsymbol{\beta}}$ and $\boldsymbol{g}_{\boldsymbol{w}}$ from which we generate realizations of $\boldsymbol{\beta}$ and $\boldsymbol{w}$ as: $\widetilde{\boldsymbol{\beta}}(n) = \boldsymbol{R}_{\boldsymbol{\beta}}^{1/2}\widetilde{\boldsymbol{g}}_{\boldsymbol{\beta}}(n)$, and $\widetilde{\boldsymbol{w}}(n) = \sigma_w\widetilde{g}_{\boldsymbol{w}}(n)$, where by calculating $E\{\widetilde{\boldsymbol{\beta}}(n)\widetilde{\boldsymbol{\beta}}^H(n)\}$, we see that $\boldsymbol{R}_{\boldsymbol{\beta}}^{1/2}[\boldsymbol{R}_{\boldsymbol{\beta}}^{1/2}]^H = \boldsymbol{R}_{\boldsymbol{\beta}}$, so that $\boldsymbol{R}_{\boldsymbol{\beta}}^{1/2}$ is the Cholesky decomposition of $\boldsymbol{R}_{\boldsymbol{\beta}}$. Then from (8), $\widetilde{\boldsymbol{h}}(n) = \boldsymbol{J}_{\boldsymbol{R}}\widetilde{\boldsymbol{\beta}}(n)$.

Comments:

1) If we choose a scattering environment with a power distribution in Table I, and associated mean angle and angular spread.
2) The simulator yields $\boldsymbol{h}$ vectors with the WSSUS statistical properties preserved. The spatial correlation between positions in the WSSUS field are presented in [14].

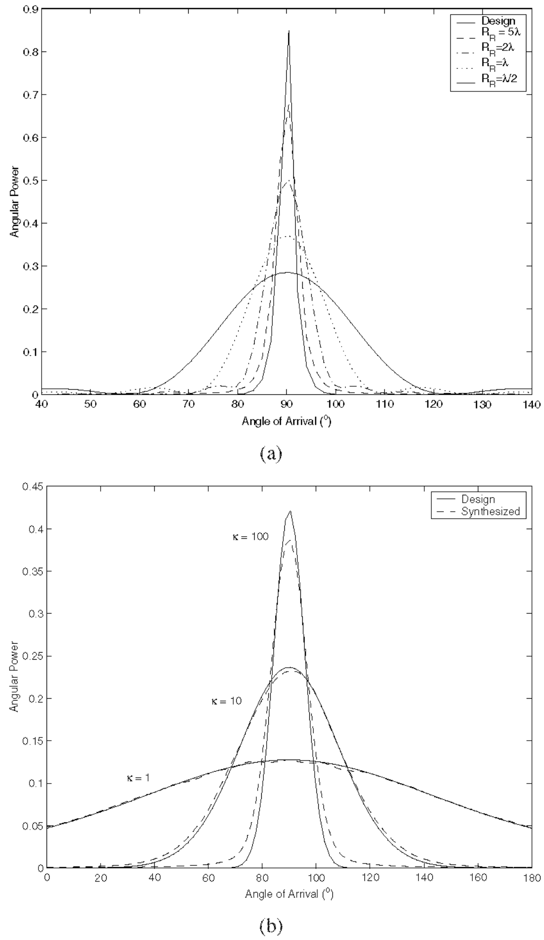

Fig. 4. Simulation of Synthesized versus Design angular power distributions. (a) Laplacian distribution for $\sigma = 2^o$ in apertures of radii $R_R = [.5\lambda, \lambda, 2\lambda, 5\lambda]$. (b) Von-Mises distribution for spreading parameter $\kappa = [1, 10, 100]$ in an aperture of radius $R_R = 2\lambda$.

3) Due to the finite size of the receiver aperture, the mean synthesized power distribution $\mathcal{P}_{\text{synth}}(\phi) \triangleq E\{|\widetilde{A}(\theta; n)|^2\}$ where $\widetilde{A}(\theta; n)$ is a realized scattering gain, is a low bandwidth approximation to the design power distribution $\mathcal{P}(\phi)$. Inserting the N_R truncation of (6) for $\widetilde{A}(\theta; n)$, we can show:

$$\begin{aligned}\mathcal{P}_{\text{synth}}(\phi) &= \frac{1}{(2\pi)^2} \sum_m \sum_{m'} E\{\widetilde{\beta}_m \widetilde{\beta}^*_{m'}\} e^{i(m-m')\phi} \\ &= \frac{1}{2\pi} \sum_{m=-2N_R}^{2N_R} t(m) \gamma_m e^{im\phi}.\end{aligned}$$

where in the first equation m and m' are summed from $-N_R$ to N_R and $t(m) = 1 - |m|/(2N_R + 1)$ is a triangular window function. Measuring with a finite aperture, the higher order γ_m coefficients are attenuated. However in the limit of large N_R, $\mathcal{P}_{\text{synth}}(\phi) = \mathcal{P}(\phi)$.

VI. Examples

In this section, we use the WSSUS simulator to perform Monte-Carlo simulations with $N = 10000$ trials for different power distributions.

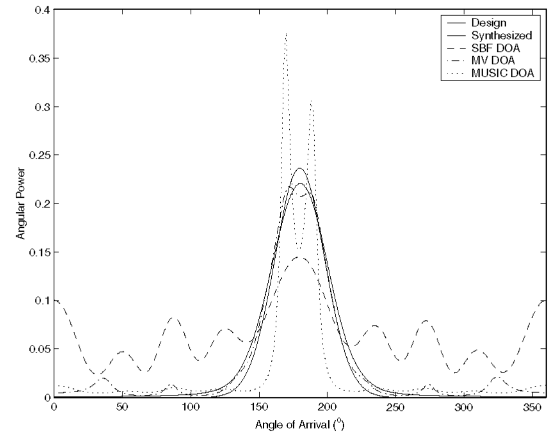

Fig. 5. Simulation of direction-of-arrival estimation techniques from data generated from a Von-Mises power distribution with $\kappa = 10$. Techniques shown are steered beamforming (SBF), minimum variance (MV) and multiple signal classification (MUSIC), using a 5 element uniform circular array of radius 0.8λ.

In Fig. 4 we simulate the Laplacian and Von-Mises power distributions described in Table I, plotting the synthesized power $\mathcal{P}_{\text{synth}}(\phi)$ in each case. Fig. 4(a) plots the Laplacian distribution synthesized by a finite aperture. The cusp of the Laplacian distribution here possesses a large angular bandwidth that requires a large aperture to observe. Fig. 4(b) illustrates its ability to reproduce different Von-Mises distributions.

In Fig. 5, we use of the simulator data to compare the performance of basic DOA estimation schemes. Due to underlying assumptions (e.g. limited numbers of multipath directions), different schemes will perform better or worse in different environments. In the case shown, MUSIC erroneously estimates the single peak as two distinct multipaths.

VII. Conclusion

In this paper, we present a single-input-multiple output (SIMO) channel simulator that generate channel data given particular parameters of an angular power distribution. This simulator implements a wide-sense stationary uncorrelated scatterer flat fading channel. A Matlab implementation of this simulator is available at $\mathrm{http://rsise.anu.edu.au/\sim terenceb/code/simo\_sim.zip}$. We are currently extending this simulator to multiple-input-multiple output (MIMO) channels, using double directional angular power distributions, simulating Doppler fading, and removing the uncorrelated scatterer assumption.

Acknowledgment

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APPENDIX

Proof: We start by writing for the vectors $\boldsymbol{x} \triangleq (x, \phi_x)$ and $\hat{\boldsymbol{\phi}} \triangleq (1, \phi)$ the Jacobi-Anger expression [15],

$$e^{ik\boldsymbol{x}\cdot\hat{\boldsymbol{\phi}}} = \sum_{m=-\infty}^{\infty} i^n J_m(kx) e^{im(\phi_x - \phi)}. \qquad (13)$$

Substituting (6) and (13) into (2) followed by rearranging and introducing $\mathcal{J}_m(\boldsymbol{x})$ as defined in (10):

$$\begin{aligned} h_n &= \sum_{m=-\infty}^{\infty} \sum_{m'=-\infty}^{\infty} \beta_m \mathcal{J}_{-m'}(\boldsymbol{x}_n) \times \\ & \quad \frac{1}{2\pi} \int_0^{2\pi} e^{i(m+m')\phi} d\phi. \end{aligned}$$

Evaluating the integral:

$$h_n = \sum_{m=-\infty}^{\infty} \beta_m \mathcal{J}_m(\boldsymbol{x}_n). \qquad (14)$$

It has been shown in [16] that (13) can accurately be truncated to $|n| \leq N_R$ for $N_R = \lceil ekR_R/2 \rceil$ since successive terms of the series decay exponentially. For a bounded scattering function, the β_n coefficients are bounded and (14) can be truncated to $|n| \leq N_R$. Writing truncated (14) in vector product form $h_n = \boldsymbol{j}_n \boldsymbol{\beta}$ where

$$\boldsymbol{j}_n \triangleq [\mathcal{J}_{-N_R}(\boldsymbol{x}_n) \ldots \mathcal{J}_{N_R}(\boldsymbol{x}_n)].$$

Result follows from noting that $\boldsymbol{j}_n$ is a row of (9).

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