Novel Scheme for Spatial Extrapolation of Multipath

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***Abstract*—It has been shown that the effects of multipath propagation in a mobile wireless communications system can be mitigated if the receiver can make predictions about the multipath fading. In this paper, we introduce a novel scheme for extrapolating multipath fields outwards in space, given field observations within a limited region. Whereas previous work has concentrated on simple multipath propagation with a finite number of plane wave scatterers, we use a less restrictive continuous model of scattering. The extrapolation scheme is based on a information-weighted modal expansion of the field, where modes containing too little information are penalized to minimize the extrapolation error. The performance of this scheme is shown to be far better than pessimistic error bounds derived in previous work.**

I. Introduction

Multipath fading is a major factor limiting the performance of wireless communications systems. Although progress has been made in exploiting the properties of multipath fields using spatial diversity of antennas [1], there is still considerable interest in multipath prediction schemes, particularly for mobile applications. These schemes allow a mobile receiver to extrapolate a multipath field outwards in space or time, allowing improved power control schemes to be negotiated with the transmitter based on future fading. In this paper, we propose a novel spatial extrapolation scheme.

Traditional approaches to the extrapolation problem have been based on simple models of multipath fading where a small number of plane wave scatterers are used to represent multiple paths [2]. In [3] and [4], extrapolation schemes were developed which recover this plane wave model using direction-of-arrival (DOA) algorithms. Other approaches to extrapolation have involved building adaptive auto-regressive models of the field [5] [6].

The problem with these schemes is that these simplistic scattering models cannot represent more complex multipath fields, such as fields generated from continuous scattering distributions [7] [8]. This means that these simpler schemes may give overly optimistic estimates of performance.

In [9], Teal et al. take a different approach based on a more complex physical model of wave propagation. Taking a fairly pessimistic approach, they demonstrate that wave equation constraints cause a worst case extrapolation error which grows rapidly with distance, and conclude that extrapolation beyond a wavelength is not practically feasible.

In this paper we use a similar physical model of multipath scattering, and show that the limitations imposed by the wave equation can actually be used to improve extrapolation performance. We introduce an improved scheme for multipath extrapolation based on a modal expansion, which allows us to represent multipath fields using an infinite set of modal coefficients. Given basic statistics of the signal and noise, we reconstruct the entire field from these modes, penalizing 'noisy' coefficients which we expect to contain little information about the field. This modal technique generalizes similar work on the problem of bandlimited extrapolation [10].

We show that the expected error of our extrapolation scheme is far lower than the worst case error predicted by [9].

In section II, we introduce a physical model of multipath scattering, and show that the resulting wavefields can be represented by an infinite modal expansion. In section III, we examine the effect of observing this field on a ring of radius r_S in the presence of spatially white noise. Importantly, the modal expansion coefficients can only be recovered approximately from the noisy field. In section IV, we demonstrate a scheme for optimally recombining the recovered modal coefficients to minimize the extrapolation error. The optimal combination involves imposing an exponential penalty on modes containing too little information about the multipath field.

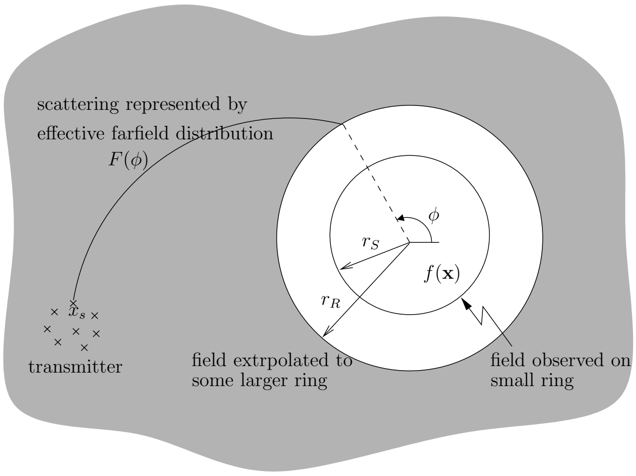

Fig. 1. The Geometry of the Multipath Extrapolation Problem

II. Multipath Fields

Consider a narrowband multipath field, $f(\mathbf{x})$, in two-dimensional space. As this field is a valid wavefield, it must be a solution to the Helmholtz equation [11],

$$\nabla^2 f(\mathbf{x}) + k^2 f(\mathbf{x}) = 0, \tag{1}$$

where $k \triangleq 2\pi/\lambda$ is the wavenumber, ∇^2 is Laplacian operator, and λ is the wavelength.

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We consider multipath wavefields generated by the farfield scattering environment developed in [12], and represented in Fig. 1. The farfield scattering distribution, $F(\phi)$, represents the effective random complex gain of the scattered signal from direction ϕ. The field at a point $\mathbf{x} = (x, \theta)$ in polar coordinates can then be represented as a linear combination of plane waves from all directions:

$$f(x, \theta) = \int_{-\pi}^{\pi} F(\phi) e^{jkx\cos(\theta-\phi)} d\phi. \tag{2}$$

where $x = |\mathbf{x}|$ and $\theta = \angle \mathbf{x}$.

We can use the Jacobi-Anger expansion [13] to express the field as an infinite modal summation

$$f(x, \theta) = \sum_{n=-\infty}^{\infty} f_n u_n(x, \theta), \tag{3}$$

where the modes are given by

$$u_n(x, \theta) = j^n J_n(kx) e^{jn\theta}. \tag{4}$$

The coefficients of the modal expansion are the Fourier Series coefficients of the farfield distribution,

$$f_n = \int_{-\pi}^{\pi} F(\phi) e^{-jn\phi} d\phi. \tag{5}$$

Assuming a zero-mean uncorrelated scattering environment, these modal coefficients will be mutually independent, zero mean, random variables with variance

$$V_{f,n} = \mathrm{E}\left\{f_n \overline{f_n}\right\}, \tag{6}$$

where $\overline{(\cdot)}$ denotes complex conjugation, and $\mathrm{E}\{\cdot\}$ is the expectation operator.

In this work, we will deal solely with scattering distributions with a uniform, isotropic power density. Thus, we will always have $V_{f,n}$ equal to some constant, independent of n.

III. Measurement Model

Following [9], we allow the multipath field to be observed on a ring of radius r_S (see Fig. 1). In the absence of noise, we will have the field observation

$$s(\theta) = f(r_S, \theta). \tag{7}$$

Wavefields on the ring can be represented by an infinite set of orthonormal modes[1]

$$v_n(\theta) = \frac{u_n(r_S, \theta)}{\sqrt{\eta_n}}, \tag{8}$$

where the normalizing coefficient

$$\eta_n = 2\pi r_S J_n(kr_S)^2, \tag{9}$$

is chosen so that

$$\int_{-\pi}^{\pi} v_n(\theta) \overline{v_n(\theta)} r_S d\theta = 1. \tag{10}$$

[1]As long as the radius r_S is chosen such that $J_n(kr_S) \neq 0$ for all n. This constraint is not hard to fulfil.

We can now represent our observed field $s(\theta)$ as a modal decomposition

$$s(\theta) = \sum_{n=-\infty}^{\infty} s_n v_n(\theta), \tag{11}$$

where the expansion coefficients are given by

$$s_n = \int_{-\pi}^{\pi} s(\theta) \overline{v_n(\theta)} r_S d\theta. \tag{12}$$

Comparing (3) and (11) we can perfectly reconstruct the original field coefficients using the simple relationship

$$f_n = \frac{s_n}{\sqrt{\eta_n}}. \tag{13}$$

Thus, in the absence of noise we could perfectly reconstruct the entire field.

Now we consider the effect of a spatially white additive noise field $w(\theta)$. Our field observation is

$$\tilde{s}(\theta) = s(\theta) + w(\theta). \tag{14}$$

We now examine how spatially white noise affects the modal decomposition (11). Based on Gallager's model for 1D noise fields [14], we model the effect of white noise by its projection into a modal basis. This means that noise field, $w(\theta)$, can be partitioned into a component representable by modal expansion,$w_M(\theta)$, and an orthogonal component, $w_{\perp M}(\theta)$.

$$\begin{aligned} w(\theta) &= w_M(\theta) + w_{\perp M}(\theta) && (15)\\ &= \sum_{n=-\infty}^{\infty} w_n v_n(\theta) + w_{\perp M}(\theta), && (16) \end{aligned}$$

where

$$w_n \triangleq \int_{-\pi}^{\pi} w(\theta) \overline{v_n(\theta)} r_S d\theta. \tag{17}$$

In Gallager's model, white noise projects equally into orthonormal modes. We generalize this model for the spatial case [15]. Where the Gallager model defines a noise power independent of region size, we consider a noise field, $w(\theta)$ that is spatially white with a homogeneous (albeit infinite) power distribution in space. This means that the power projected into each mode will grow with the area of the observation region,

$$\begin{aligned} V_{w,n} &= \mathrm{E}\{w_n \overline{w_n}\} && (18)\\ &= \sigma(2\pi r_S), && (19) \end{aligned}$$

where σ is the noise variance per unit area.

Using this noise model, the observed field (14) can be represented

$$\tilde{s}(\theta) = \sum_{n=-\infty}^{\infty} (s_n + w_n) v_n(\theta) + w_{\perp M}(\theta). \tag{20}$$

When we try to recover the modal coefficients using (12), we will introduce a noise component,

$$\tilde{s}_n = s_n + w_n \tag{21}$$

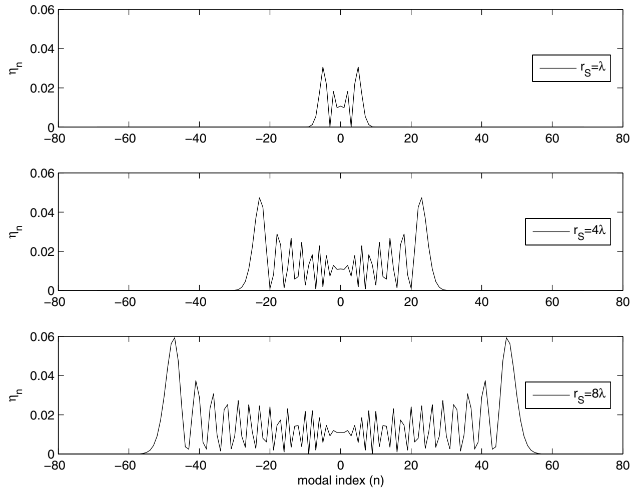

Fig. 2. Comparison of the behaviour of the normalizing constant η_n with increasing order for three different region radii.

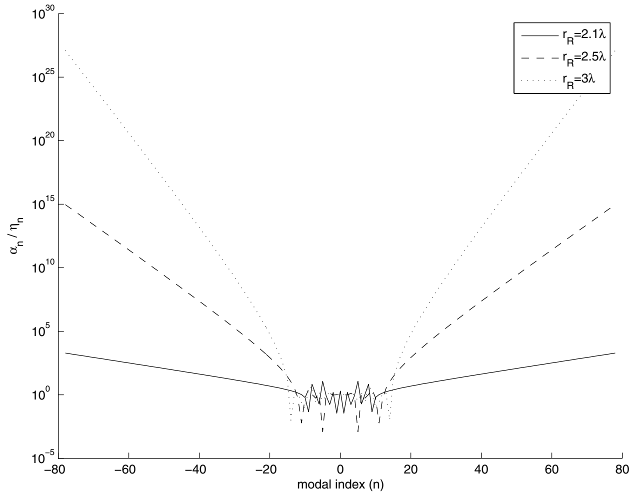

Fig. 3. Comparison of the behaviour of the ratio α_n/η_n with increasing order for three different region radii. The observation radius is $r_S = 2\lambda$

Thus, the optimal unbiased estimates of the original field coefficients f_n are

$$\tilde{f}_n = \frac{\tilde{s}_n}{\sqrt{\eta_n}} = f_n + \frac{w_n}{\sqrt{\eta_n}}. \tag{22}$$

Using this noise model, the optimal estimator of the field coefficients f_n will be a normally distributed Gaussian process

$$\tilde{f}_n \sim \mathcal{N}\left(f_n, \frac{V_{w,n}}{\eta_n}\right). \tag{23}$$

Although the noise field is spatially white, its effect on the modal coefficients is nonuniform with order, n. Fig. 2 shows the value of η_n for three different observation ring radii. For large $|n| \gg kr_S$, η_n decays rapidly according to [16]

$$\eta_n \sim \frac{k^{2n} r_S^{2n}}{(n!)^2 4^n}, \text{ as } |n| \to \infty. \tag{24}$$

This means that our estimates of the higher order coefficients will rapidly become noisy with $1/\eta_n$.

Another way to consider the effect of the noise is the mutual information between $\tilde{f}_n$ and f_n. This is effectively the information we can recover about f_n from $\tilde{f}_n$ [17],

$$I_n(\eta_n) = \frac{1}{2}\ln\left(1 + \frac{\eta_n V_{f,n}}{V_{w,n}}\right). \tag{25}$$

For $|n| \to \infty$, η_n decays, and the information about f_n that is contained in $\tilde{f}_n$ rapidly approaches zero.

IV. Multipath Extrapolation

We have seen so far that an entire multipath wavefield can be represented using an infinite set of modal coefficients f_n. Given a noisy field observation on a ring of radius r_S, we can use (12) and (22) to recover estimates of the coefficients $\tilde{f}_n$.

In this section, we deal with the problem of multipath extrapolation — reconstructing the entire multipath field from these approximate coefficients.

A naive approach to extrapolation would be to use the recovered coefficients in the modal expansion (3),

$$\tilde{f}(x,\theta) = \sum_{n=-\infty}^{\infty} \tilde{f}_n u_n(x,\theta) = \sum_{n=-\infty}^{\infty} \left(f_n + \frac{w_n}{\sqrt{\eta_n}}\right) u_n(x,\theta). \tag{26}$$

To test the quality of this extrapolation, we evaluate the mean-squared extrapolation error (MSE) on some larger ring of radius $r_R > r_S$,

$$\begin{aligned}\text{MSE} &= \text{E}\left\{\int_{-\pi}^{\pi} |\tilde{f}(r_R,\theta) - f(r_R,\theta)|^2 r_R d\theta\right\} &(27)\\ &= \frac{\text{E}\{w_n \overline{w_m}\}}{\sqrt{\eta_n\eta_m}} \int_{-\pi}^{\pi} u_n(r_R,\theta)\overline{u_m(r_R,\theta)} r_R d\theta &(28)\\ &= \sum_{n=-\infty}^{\infty} \frac{\alpha_n}{\eta_n} V_{w,n}, &(29)\end{aligned}$$

where

$$\begin{aligned}\alpha_n &= \int_{-\pi}^{\pi} u_n(r_R,\theta)\overline{u_n(r_R,\theta)} r_R d\theta &(30)\\ &= 2\pi r_R J_n(kr_R)^2. &(31)\end{aligned}$$

The ratio α_n/η_n is basically a ratio of squared bessel functions with the same order. Fig. 3 shows how this ratio grows rapidly with increasing order, n, for different extrapolation radii r_R. Thus, the error performance of this naive extrapolation scheme will be extremely bad, even for small extrapolations, due to the huge errors in the higher order modal coefficients. These bad results tend to agree with the worst case predictions made in [9].

As the statistics of the signal and noise are independent for each mode, we can design a better extrapolation method by introducing a set of coefficients, c_n, into (26) to individually weight each modal contribution:

$$\tilde{f}_c(x,\theta) = \sum_{n=-\infty}^{\infty} c_n \tilde{f}_n u_n(x,\theta). \tag{32}$$

In this case, the MSE is

$$\begin{aligned}\text{MSE}_c &= \text{E}\left\{\int_{-\pi}^{\pi} |\tilde{f}_c(r_R,\theta) - f(r_R,\theta)|^2 r_R d\theta\right\} &(33)\\ &= \sum_{n=-\infty}^{\infty} (1-c_n)^2 \alpha_n V_{f,n} + c_n^2 \frac{\alpha_n}{\eta_n} V_{w,n} &(34)\end{aligned}$$

We determine the optimal c_n that will minimize the MSE. Taking the derivative of the error function,

$$\frac{\partial}{\partial c_n}\mathrm{MSE}_c = 2\sum_{n=-\infty}^{\infty} \alpha_n V_{f,n}(c_n - 1) + \frac{\alpha_n}{\eta_n} V_{w,n} c_n. \quad (35)$$

We set the derivative to zero to obtain the optimal set of weighting coefficients as

$$c_{n,\mathrm{opt}} = \frac{\eta_n V_{f,n}}{\eta_n V_{f,n} + V_{w,n}} = 1 - e^{-2I_n(\eta_n)}, \quad (36)$$

where $I_n(\eta_n)$ is the mutual information given in (25).

Thus, the optimal set of coefficients imposes an exponential penalty on the higher-order approximate coefficients, $\tilde{f}_n$, containing too little mutual information about the original modal coefficients, f_n. Note that the optimal weighting coefficients, $c_{n,\mathrm{opt}}$, are independent of r_R - in effect we can use the same set of coefficients to optimally extrapolate to any desired radius. The coefficients do, however, depend on the observation radius r_S through the η_n.

The minimized MSE is

$$\mathrm{MSE}_{c,\mathrm{opt}} = \sum_{n=-\infty}^{\infty} \frac{\alpha_n V_{w,n} V_{f,n}}{V_{w,n} + \eta_n V_{f,n}} \quad (37)$$

To allow a fair comparison between different extrapolation radii, we normalize the MSE by dividing by the signal power we would expect to find on a ring of that radius. That is,

$$\text{Normalized MSE} = \frac{\mathrm{MSE}}{S_R}, \quad (38)$$

where the expected signal power is given by

$$S_R = \mathrm{E}\left\{\int_{-\pi}^{\pi} |f(r_R,\theta)|^2 r_R d\theta\right\} \quad (39)$$

$$= \sum_{n=-\infty}^{\infty} \alpha_n V_{f,n}. \quad (40)$$

This normalized error is plotted in Fig. 4. The figure shows that the expected extrapolation error grows much more slowly with r_R than the exponential growth predicted in the worst case scenario presented in [9], and the naive reconstruction (26) which corresponds to $c_n = 1$.

Fig. 4 also shows the effect on extrapolation performance of increasing the radius of the observation region (r_S). In order to make a fair comparison between radii, we fix the ratio $V_{f,n}/V_{w,n}$ for all cases. As we would expect, as the observation radius is increased, more coefficients contain useful information about the field and extrapolation performance improves. Despite this, extrapolation performance is reasonably invariant to the size of the observation region.

V. Conclusion

We have demonstrated a multipath extrapolation scheme which uses knowledge of the signal and noise statistics, and the physical properties of wavefields, to extrapolate multipath fields in space. The expected extrapolation error is far better than the worst case predictions made in previous work.

An extension of this work to nonuniform scattering distributions will be reported in a future journal article.

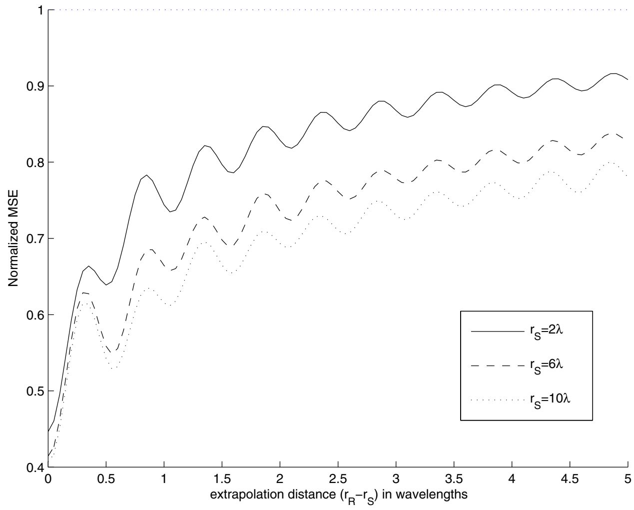

Fig. 4. Effect of varying the radius of the observation region (r_S), on the normalized mean squared extrapolation error.

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